

C12 - 7.0 - Exponential Review

Interest

KEY

$$F = P(1 \pm r)^t$$

F : Future Amount

P : Present Amount

r : Interest rate as decimal

t : time

$$F = \frac{P^* \left(\left(1 + \frac{r}{n} \right)^{tn} - 1 \right)}{\frac{r}{n}}$$

*;+ = Growth
-= Decay*

$$F = P \left(1 \pm \frac{r}{n} \right)^{tn}$$

; with Compounding

n : # of compounding periods per year

r : Rate per period

n : number of periods

$$n = \frac{1}{T}$$

*Yearly; n = 1
Monthly; n = 12
Weekly; n = 52*

Growth & Decay

$$F = P(r)^{\frac{t}{T}}$$

; Growth with "T"

T : Time/Amount for Rate to OCCUR

$$F = 100(0.87)^t$$

$$F = 100 \left(\frac{1}{2} \right)^{\frac{t}{5}}$$

$$F = Pe^{kt}$$

; Continuous Growth

e: constant ≈ 2.718

k: proportional constant

<i>Growth</i>	2% = .02	15% = .15	40% = .4	50% = .5	60% = .6	100% = 1.00	<i>Double</i>
(1 + r)	(1 + .02)	(1 + .15)	(1 + .4)	(1 + .5)	(1 + .6)	(1 + 1.00)	
(....)	(1.02)	(1.15)	(1.4)	(1.5)	(1.6)	(2)	(2)

<i>Decay</i>	10% = .1	15% = .15	40% = .4	50% = .5	60% = .6	95% = .95	<i>Half - Life</i>
(1 - r)	(1 - .1)	(1 - .15)	(1 - .4)	(1 - .5)	(1 - .6)	(1 - .95)	
(....)	(.9)	(.85)	(.6)	(.5)	(.4)	(.05)	($\frac{1}{2}$)

Method: Arbitrarily set P = 100% or 100 or 1

Remember: The exponent is the time or the number of time periods.

Intensity

$$I = 10^{b-s}$$

; Earthquakes, pH

KEY

$$I = \frac{I_b}{I_s} : \text{Intensity}$$

b – Larger Richter, Decibel, pH etc

s – Smaller Richter, Decibel, pH etc

$$pH = -\log(H^+)$$

H⁺ - Concentration of Hydrogen

Change of Base

$$x^m \times x^n = x^{m+n} \quad \frac{x^m}{x^n} = x^{m-n} \quad (x^m)^n = x^{m \times n} \quad x^{-a} = \frac{1}{x^a} \quad \frac{1}{x^{-a}} = x^a \quad x^{\frac{m}{n}} = \sqrt[n]{x^m} \quad \left(\frac{x}{y}\right)^{-m} = \frac{y^m}{x^m}$$

Blank

C12 - 7.1 - Exponent Laws Notes

Simplify

$$5^2 \times 5^3 = 5^5 \quad \text{Add Exponents}$$

$$\frac{3^5}{3^2} = 3^3 \quad \text{Subtract Exponents}$$

$$(2^2)^3 = 2^6$$

$$(3 \times 4)^2 = 3^2 \times 4^2$$

$$(2x)^3 = 2^3 x^3 = 8x^3$$

$$\left(\frac{3}{5}\right)^2 = \frac{3^2}{5^2}$$

Multiply/Distribute
Exponents

$$5^{-2} = \frac{1}{5^2}$$

$$\frac{1}{3^{-2}} = 3^2$$

$$3a^{-2} = \frac{3}{a^2}$$

$$(2x)^{-3} = \frac{1}{(2x)^3}$$

Negative
Exponents

$$3^{-1} = \frac{1}{3}$$

$$\frac{1}{3^1} = 3^{-1}$$

$$3^{-3}a^{-2} = \frac{1}{3^3a^2}$$

$$\left(\frac{5}{3}\right)^{-2} = \frac{3^2}{5^2}$$

$$\frac{1}{25} = \frac{1}{5^2} = 5^{-2}$$

$$9 = 3^2$$

$$25 = 5^2$$

$$4^2 = (2^2)^2 = 2^4$$

$$27^4 = (3^3)^4 = 3^{12} \quad \text{Change Base}$$

$$5^{\frac{3}{4}} = \sqrt[4]{5^3}$$

$$8^{\frac{1}{3}} = \sqrt[3]{8}$$

$$\begin{aligned} \frac{8^{\frac{2}{3}}}{\sqrt[3]{8^2}} &= \\ 2^2 &= 4 \end{aligned}$$

$$\begin{aligned} \sqrt[4]{\frac{1}{16}} &= \\ \frac{1}{\sqrt[4]{16}} &= \frac{1}{2} \end{aligned} \quad \text{Radicals}$$

$$\begin{aligned} \frac{3^4 \times 3^{-3}}{9} &= \\ \frac{3^1}{3^2} &= \\ 3^{-1} &= \\ \frac{1}{3^1} &= \frac{1}{3} \end{aligned}$$

Add Exponents
Change Base
Subtract Exponents
Negative Exponents
Simplify

$$\begin{aligned} \frac{4^2 \times 16^3}{128^2} &= \\ \frac{((2^2)^2 \times (2^4)^3)}{(2^7)^2} &= \\ \frac{2^4 \times 2^{12}}{2^{14}} &= \\ \frac{2^{16}}{2^{14}} &= \\ 2^{(16-14)} &= \end{aligned}$$

Change of base
Multiply Exponents
Add Exponents
Subtract Exponents
Simplify

$$2^2 = 4$$

C12 - 7.1 - Simplifying/Separating Exponents Notes

Simplify

$$3^x \times 3 = \\ 3^x \times 3^1 = 3^{x+1}$$

Add Exponents

$$(5^2)^x = 5^{2x}$$

Multiply Exponents

$$\frac{6^x}{6} = \\ \frac{6^x}{6^1} = 6^{x-1}$$

$$\frac{3}{3^x} = \\ \frac{3^1}{3^x} = 3^{1-x}$$

Subtract Exponents

Separate into a multiplication/division/or use brackets with the same base. (*Isolate #^x*)

$$6^{x+1} = 6^x(6^1) = 6(6^x)$$

$$7^{x-1} = 7^x \times 7^{-1} = \frac{7^x}{7^1}$$

$$4^{1-x} = 4^1(4^{-x})$$

$$5^{2x} = (5^x)^2 = (5^2)^x$$

$$= \frac{4}{4^x}$$

$$3^{2x+1} = 3^{2x}3^1 \\ = (3^x)^23^1$$

$$6^x = (2 \times 3)^x$$

$$= 3(3^x)^2$$

$$= 2^x \times 3^x$$

$$\frac{2^{7x+5} \times 8^{x+1}}{4^{x-2}} = \\ \frac{2^{7x+5} \times (2^3)^{x+1}}{(2^2)^{x-2}} = \\ \frac{2^{7x+5} \times 2^{3x+3}}{2^{2x-4}} = \\ \frac{2^{10x+8}}{2^{2x-4}} = 2^{8x+12}$$

Change Base
Multiply Exponents
Add Exponents
Subtract Exponents

Subtracting
Negative
Exponents!

$$\frac{2^{7x+5} \times 8^{x+1}}{4^{x-2}} = 8^{(x+1)} = \\ (2^3)^{(x+1)} = 2^{3x+3}$$

$$\frac{2^{7x+5} \times 2^{3x+3}}{2^{2x-4}} = 4^{x-2} = \\ (2^2)^{x-2} = 2^{2x-4}$$

C12 - 7.2 - Separate/Factoring/Solving Exponents Notes

Solve for x

$$\begin{aligned} 2(3^x) + 3^x &= 243 & \text{let } m = 3^x \\ 2m + m &= 243 \\ 3m &= 243 \\ m &= 81 \end{aligned}$$

$$\begin{aligned} 3^x &= 81 \\ 3^x &= 3^4 \end{aligned}$$

$$x = 4$$

Check Answer:

$$\begin{aligned} 2(3^x) + 3^x &= 243 \\ 2(3^4) + 3^4 &= 243 \\ 2(81) + 81 &= 243 \\ 243 &= 243 \end{aligned}$$



$$\begin{aligned} 7^x + 7^{x+1} &= 392 & \text{Let } m = 7^x \\ 7^x + 7^x 7^1 &= 392 \\ m + 7m &= 392 \\ 8m &= 392 \end{aligned}$$

$$m = 49$$

$$\begin{aligned} 7^x &= 49 \\ 7^x &= 7^2 \end{aligned}$$

$$x = 2$$

$$\begin{aligned} 7^x + 7^{x+1} &= 392 \\ 7^2 + 7^{2+1} &= 392 \\ 49 + 343 &= 392 \\ 392 &= 392 \end{aligned}$$



$$\begin{aligned} (2^x)^2 - 12(2^x) + 32 &= 0 & \text{let } m = 2^x \\ m^2 - 12m + 32 &= 0 \\ (m - 4)(m - 8) &= 0 \end{aligned}$$

$$\begin{aligned} m - 4 &= 0 & m - 8 &= 0 \\ m &= 4 & m &= 8 \end{aligned}$$

$$\begin{aligned} 2^x &= 4 & 2^x &= 8 \\ 2^x &= 2^2 & 2^x &= 2^3 \end{aligned}$$

$$x = 2$$

$$x = 3$$

$$\begin{aligned} (2^x)^2 - 12(2^x) + 32 &= 0 \\ (2^2)^2 - 12(2^2) + 32 &= 0 \\ 16 - 48 + 32 &= 0 \end{aligned}$$



$$\begin{aligned} (2^x)^2 - 12(2^x) + 32 &= 0 \\ (2^3)^2 - 12(2^3) + 32 &= 0 \\ 64 - 96 + 32 &= 0 \end{aligned}$$



$$\begin{aligned} 9^{2x} - 2(9^x) - 3 &= 0 & \text{let } m = 9^x \\ (9^x)^2 - 2(9^x) - 3 &= 0 & m^2 - 2m - 3 = 0 \\ m^2 - 2m - 3 &= 0 & (m - 3)(m + 1) = 0 \end{aligned}$$

$$\begin{aligned} m - 3 &= 0 & m + 1 &= 0 \\ m &= 3 & m &= -1 \end{aligned}$$

$$\begin{aligned} 9^x &= 3 & 9^x &= -1 \\ (3^2)^x &= 3^1 & (3^2)^{\frac{1}{2}} &= 3^1 \\ 3^{2x} &= 3^1 & 3^1 &= 3^1 \\ 2x &= 1 & x &= 1 \end{aligned}$$

No Solution

$$x = \frac{1}{2}$$

$$\begin{aligned} 9^{2x} - 2(9^x) - 3 &= 0 \\ 9^{2(\frac{1}{2})} - 2(9^{(\frac{1}{2})}) - 3 &= 0 & 9^{\frac{1}{2}} = \sqrt[2]{9^1} = 3 \\ 9^1 - 2(3) - 3 &= 0 \\ 9 - 6 - 3 &= 0 \\ 0 &= 0 \end{aligned}$$



C12 - 7.2 - Separate/Factoring/Solving Exponents Notes

Solve for x

$$4^x - 4^{x-1} - 24 = 0$$

$$4^x - \frac{4^x}{4^1} - 24 = 0$$

$$\left(4^x - \frac{4^x}{4^1} - 24 = 0\right) \times 4$$

$$4(4^x) - 4^x - 96 = 0$$

$$4m - m - 96 = 0$$

$$3m = 96$$

$$m = 32$$

$$4^x = 32$$

$$(2^2)^x = 2^5$$

$$2^{2x} = 2^5$$

$$2x = 5$$

$$x = \frac{5}{2}$$

$$\begin{aligned} 4^x - 4^{x-1} - 24 &= 0 \\ 4^{\left(\frac{5}{2}\right)} - 4^{\left(\frac{5}{2}\right)-1} - 24 &= 0 \\ 32 - 8 - 24 &= 0 \end{aligned}$$

$$3^x - 3 = 4(3^{-x}) \quad 3^{-x} = \frac{1}{3^x}$$

$$3^x - 3 = \frac{4}{3^x}$$

$$m - 3 = \frac{4}{m}$$

$$\left(m - 3 = \frac{4}{m}\right) \times m$$

$$m^2 - 3m = 4$$

$$m^2 - 3m - 4 = 0$$

$$(m - 4)(m + 1) = 0$$

$$m - 4 = 0 \quad m + 1 = 0$$

$$m = 4 \quad m = -1$$

$$3^x = 4 \quad 3^x = -1$$

$$x = 1.2619$$

No Solution

Calc $y_1 = y_2$

$$3^x - 3 = \frac{4}{3^x}$$

$$3^{1.2619} - 3 = \frac{4}{3^{1.2619}}$$

$$\begin{aligned} 4 - 3 &= \frac{4}{4} \\ 1 &= 1 \end{aligned}$$



$$\begin{aligned} 4^x + 4^{1-x} &= 5 \\ 4^x + 4(4^{-x}) &= 5 \\ 4^x + \frac{4}{4^x} &= 5 \\ \text{Let } m = 4^x & \quad m + \frac{4}{m} = 5 \\ \left(m + \frac{4}{m}\right) \times m & \quad m^2 + 4 = 5m \\ m^2 + 4 &= 5m \\ m^2 + 4 - 5m &= 0 \\ m^2 - 5m + 4 &= 0 \\ (m - 1)(m - 4) &= 0 \end{aligned}$$

$$\begin{aligned} m - 1 &= 0 & m - 4 &= 0 \\ m = 1 & & m = 4 & \end{aligned}$$

$$\begin{aligned} 4^x &= 1 & 4^x &= 4 \\ 4^x &= 4^0 & 4^x &= 4^1 \end{aligned}$$

$$x = 0$$

$$x = 1$$

$$\begin{aligned} 4^x + 4^{1-x} &= 5 \\ 4^0 + 4^{1-0} &= 5 \\ 1 + 4 &= 5 \\ 5 &= 5 \end{aligned}$$

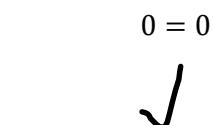
$$\begin{aligned} 2(2^x)^2 - 3(2^x) + 1 &= 0 \\ 2m^2 - 3m + 1 &= 0 \\ (2m - 1)(m - 1) &= 0 \end{aligned}$$

$$\begin{aligned} 2m - 1 &= 0 & m - 1 &= 0 \\ m = \frac{1}{2} & & m = 1 & \end{aligned}$$

$$\begin{aligned} 2^x &= 1 & 2^x &= 2^0 \\ 2^x &= \frac{1}{2} & 2^x &= 2^{-1} \\ 2^x &= 2^{-1} & x = 0 & \end{aligned}$$

$$x = -1$$

$$\begin{aligned} 2(2^x)^2 - 3(2^x) + 1 &= 0 & 2(2^x)^2 - 3(2^x) + 1 &= 0 \\ 2(2^{-1})^2 - 3(2^{-1}) + 1 &= 0 & 2(2^0)^2 - 3(2^0) + 1 &= 0 \\ 2\left(\frac{1}{2}\right)^2 - 3\left(\frac{1}{2}\right) + 1 &= 0 & 2(1)^2 - 3(1) + 1 &= 0 \\ 2\left(\frac{1}{4}\right) - \frac{3}{2} + 1 &= 0 & 2 - 3 + 1 &= 0 \\ 0 &= 0 & 0 &= 0 \end{aligned}$$



C12 - 7.2 - Separate/Factoring/Solving Exponents Notes

Solve for x

$$3^{2x+1} - 4(3^{x+1}) + 9 = 0$$

$$3^{2x}3^1 - 4(3^x3) + 9 = 0$$

$$(3^x)^23 - 4(3^x)3 + 9 = 0$$

$$3(3^x)^2 - 12(3^x) + 9 = 0$$

$$3m^2 - 12m + 9 = 0$$

$$m^2 - 4m + 3 = 0$$

$$(m-1)(m-3) = 0$$

Divide both sides by 3!

let $m = 3^x$

$$m-1 = 0 \quad m-3 = 0$$

$$m=1$$

$$m=3$$

$$3^x = 1$$

$$3^x = 3^0$$

$$3^x = 3$$

$$3^x = 3^1$$

$$\text{oval } x = 0$$

$$\text{oval } x = 1$$

$$\begin{aligned} 3^{2x+1} - 4(3^{x+1}) + 9 &= 0 & 3^{2x+1} &= 3^{2x}3^1 & 4(3^{x+1}) &= 4(3^x3^1) \\ 3(3^x)^2 - 12(3^x) + 9 &= 0 & &= (3^x)^23^1 & &= 12(3^x) \\ 3m^2 - 12m + 9 &= 0 & &= 3(3^x)^2 & & \end{aligned}$$

$$\begin{aligned} 3^{2x+1} - 4(3^{x+1}) + 9 &= 0 \\ 3^{2(0)+1} - 4(3^{(0)+1}) + 9 &= 0 \\ 3^1 - 4(3) + 9 &= 0 \\ 3 - 12 + 9 &= 0 \\ 0 &= 0 \end{aligned}$$



$$\begin{aligned} 3^{2x+1} - 4(3^{x+1}) + 9 &= 0 \\ 3^{2(1)+1} - 4(3^{(1)+1}) + 9 &= 0 \\ 3^3 - 4(3^2) + 9 &= 0 \\ 27 - 36 + 9 &= 0 \\ 0 &= 0 \end{aligned}$$



$$\begin{aligned} 10^x - 4(5^x) - 5(2^x) + 20 &= 0 \\ 2^x \times 5^x - 4(5^x) - 5(2^x) + 20 &= 0 \end{aligned}$$

$$\begin{aligned} mn - 4n - 5m + 20 &= 0 \\ (mn - 4n)(-5m + 20) &= 0 \\ n(m - 4) - 5(m - 4) &= 0 \\ (n - 5)(m - 4) &= 0 \end{aligned}$$

$$\begin{aligned} \text{let } m = 2^x & \quad \text{let } n = 5^x & 10^x &= (2 \times 5)^x \\ & & &= 2^x \times 5^x \end{aligned}$$

$$\begin{aligned} n-5=0 & \quad m-4=0 \\ n=5 & \quad m=4 \end{aligned}$$

$$\begin{aligned} 5^x=5 & \quad 2^x=4 \\ 5^x=5^1 & \quad 2^x=2^2 \end{aligned}$$

$$\text{oval } x = 1$$

$$\text{oval } x = 2$$

Group

$$\begin{aligned} 10^x - 4(5^x) - 5(2^x) + 20 &= 0 \\ 10^1 - 4(5^1) - 5(2^1) + 20 &= 0 \\ 10 - 20 - 10 + 20 &= 0 \\ 0 &= 0 \end{aligned}$$



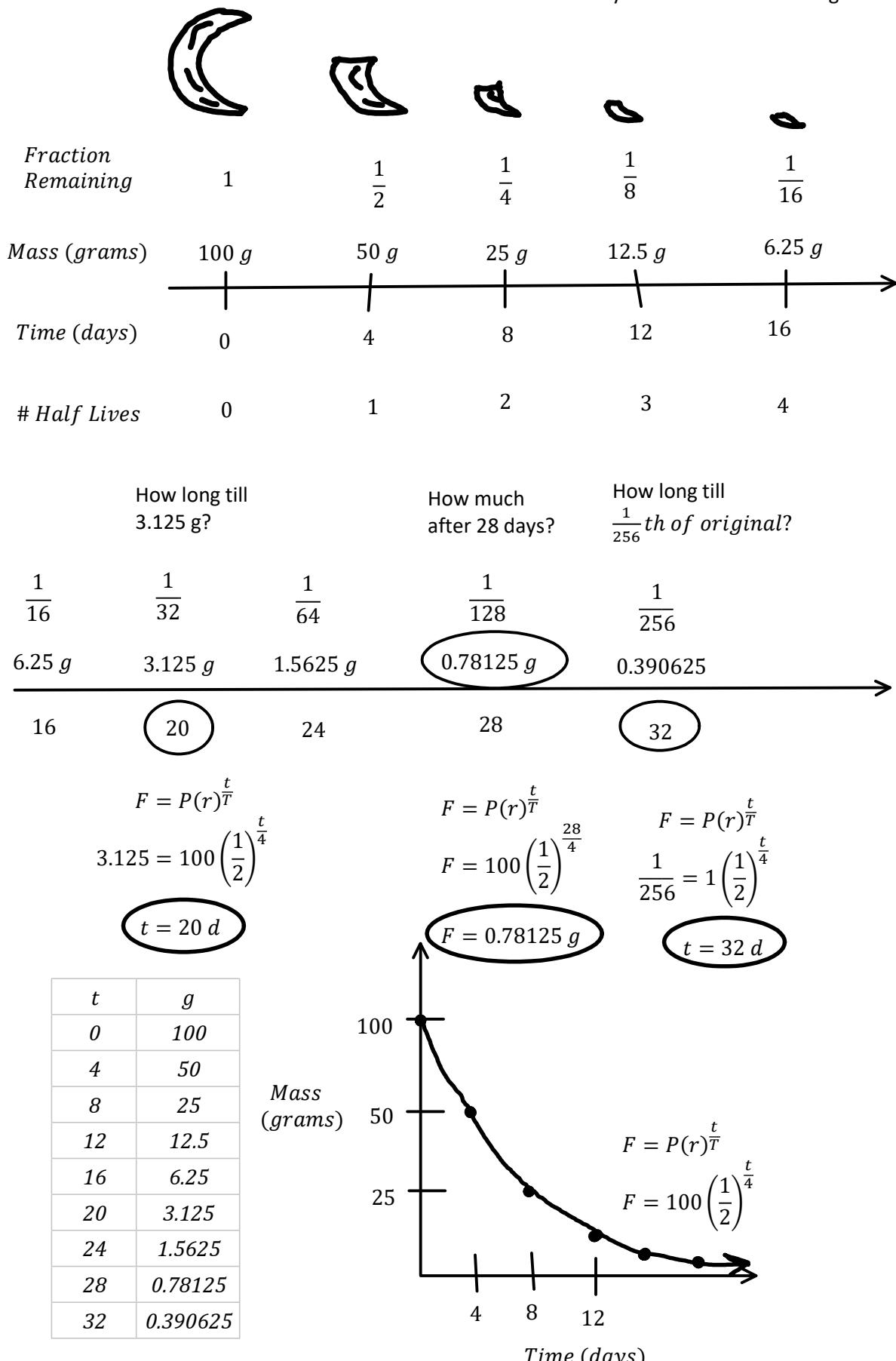
$$\begin{aligned} 10^x - 4(5^x) - 5(2^x) + 20 &= 0 \\ 10^2 - 4(5^2) - 5(2^2) + 20 &= 0 \\ 100 - 100 - 20 + 20 &= 0 \\ 0 &= 0 \end{aligned}$$



C12 - 7.3 - Half Life (HL) Theory

Bananas have a half life of 4 days.

Half Life: Time to decay to half of the remaining mass.



C12 - 7.3 - Word Problems Notes

If you deposit \$2000 in the bank at 12% interest how much will you have after 8 years?

$$F = P(1 \pm r)^t$$

$$F = 2000(1 + 0.12)^8$$

$$F = 4951.93$$

Find the rate to triple your money in 10 years.

$$F = P(1 + r)^t$$

$$3 = 1(1 + r)^{10}$$

$$(3)^{\frac{1}{10}} = ((1 + r)^{10})^{\frac{1}{10}}$$

$$1.116 = 1 + r$$

$$r = 0.1116 = 11.6\%$$

$$F = P \left(1 \pm \frac{r}{n}\right)^{tn}$$

$$F = 5000 \left(1 + \frac{0.08}{4}\right)^{6 \times 4}$$

$$F = 8042.19$$

If a population starts at 1000 and triples every 4 hours, how large will the population grow in 25 hours?

$$F = P(r)^{\frac{t}{T}}$$

$$F = 1000(3)^{\frac{25}{4}}$$

$$F = 959417 \text{ pop}$$

If the population starts at 300 and grows continuously at a rate of 0.06, how large will it grow after 20 days?

$$F = Pe^{kt}$$

$$F = 300e^{0.06 \times 20}$$

$$F = 996.03 \text{ pop}$$

How many times as intense is an earthquake of 6.0 than 3.0?

$$I = 10^{b-s}$$

$$I = 10^{6-3}$$

$$I = 10^3$$

$$I = 1000 \text{ times}$$

Find the present value of deposit worth \$2000 in the bank at 10% interest how much will you have after 4 years?

$$F = P(1 \pm r)^t$$

$$2000 = P(1 + 0.1)^4$$

$$2000 = P(1.4641)$$

$$P = \frac{2000}{1.4641}$$

$$P = \$1366.03$$

Find the rate of a \$1000 deposit worth \$1100 after 2 years.

$$F = P(1 \pm r)^t$$

$$1100 = 1000(1 + r)^2$$

$$\frac{1100}{1000} = (1 + r)^2$$

$$1.1 = (1 + r)^2$$

$$(1.1)^{\frac{1}{2}} = ((1 + r)^2)^{\frac{1}{2}}$$

$$1.0488 = 1 + r$$

$$r = 0.0488$$

$$r = 4.9\%$$

$$F = P(1 \pm r)^t$$

$$400 = 100(1 + 0.08)^t$$

$$\frac{400}{100} = 1.08^t$$

$$4 = 1.08^t$$

$$y_1 = y_2$$

Calc Intersection or "logs"

$$t = 18.01 \text{ yrs}$$

If you deposit \$100 in the bank, how long will it take to grow to \$6400 if it doubles each year?

$$F = P(r)^{\frac{t}{T}}$$

$$6400 = 100(2)^{\frac{t}{1}}$$

$$\frac{6400}{100} = 2^t$$

$$64 = 2^t$$

$$2^6 = 2^t$$

$$t = 6s$$

An earth quake in California of Richter 8.5 Magnitude was 100 times as strong as an earth quake in Vancouver of what Richter Magnitude.

$$I = 10^{b-s}$$

$$100 = 10^{8.5-s}$$

$$10^2 = 10^{8.5-s}$$

$$2 = 8.5 - s$$

$$s = 6.5 R$$

Light diminishes by 10% every 5 meters. Find the depth of 1% light.

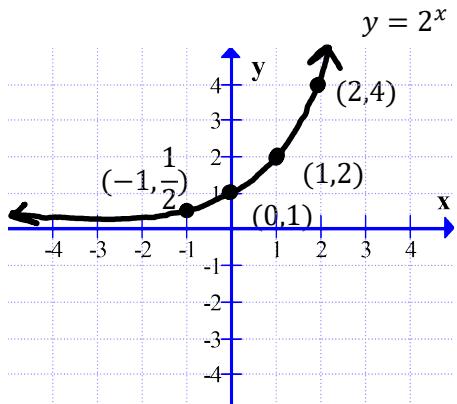
$$F = P(1 \pm r)^{\frac{t}{T}}$$

$$1 = 100(1 - 0.1)^{\frac{d}{5}}$$

$$0.01 = 0.9^{\frac{d}{5}}$$

$$d = 218.5 \text{ m}$$

C12 - 7.4 - Exponent Reflections Graphs Notes



x	y
-1	$\frac{1}{2}$
0	1
1	2
2	4

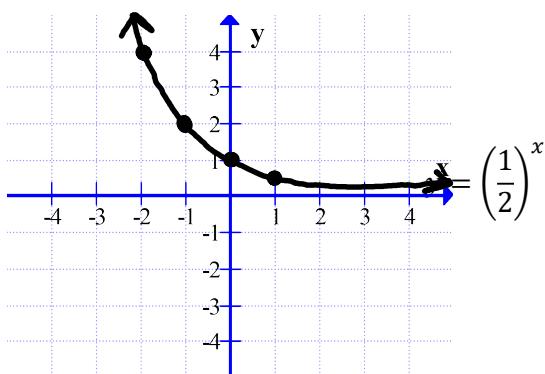
$$\begin{aligned} 2^{-1} &= \frac{1}{2} & (-1, \frac{1}{2}) \\ 2^0 &= 1 & (0, 1) \\ 2^1 &= 2 & (1, 2) \\ 2^2 &= 4 & (2, 4) \end{aligned}$$

End Behavior
 $x \rightarrow +\infty$
 $y \rightarrow +\infty$
 $x \rightarrow -\infty$
 $y \rightarrow 0$
HA:
 $y = 0$

Domain: $x \in \mathbb{R}$

Eg. Time* $t \geq 0$

$y = 2^{-x}$ Horizontal Reflection

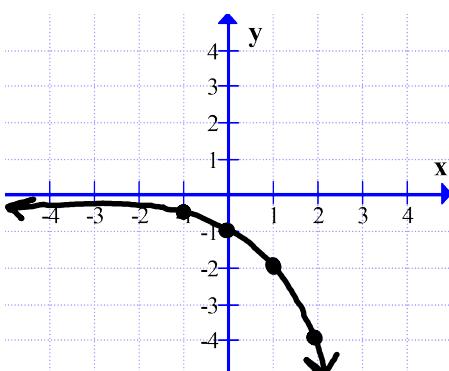


$$\left(\frac{1}{2}\right)^x = (2^{-1})^x = 2^{-x}$$

$x \rightarrow +\infty$
 $y \rightarrow 0$
 $x \rightarrow -\infty$
 $y \rightarrow +\infty$

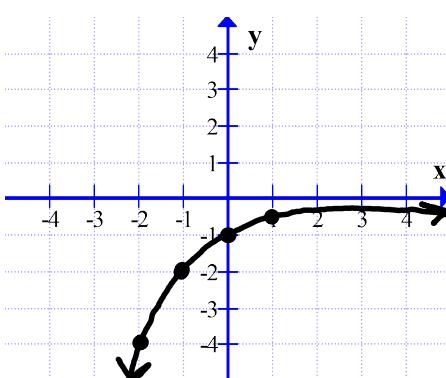
Remember: Positive Open up to the Right

Remember: Negative exponents and fractions with positive exponents Down to the Right



Vertical Reflection

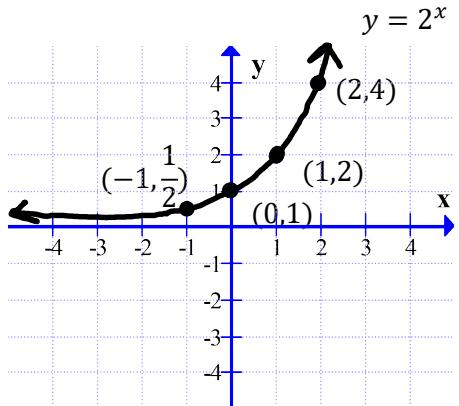
$$y = -2^x$$



Vertical Reflection and
Horizontal Reflection

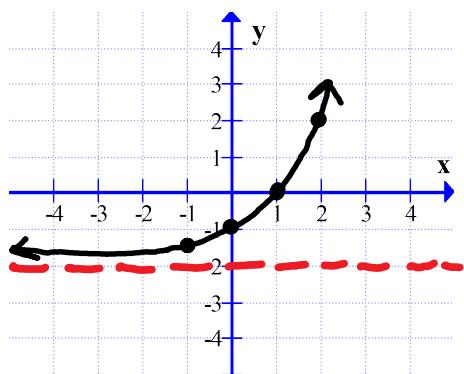
$$y = -2^{-x}$$

C12 - 7.4 - Exponent Transformations Graphs Notes



x	y
-1	$\frac{1}{2}$
0	1
1	2
2	4

$$\begin{array}{ll} 2^{-1} = \frac{1}{2} & (-1, \frac{1}{2}) \\ 2^0 = 1 & (0, 1) \\ 2^1 = 2 & (1, 2) \\ 2^2 = 4 & (2, 4) \end{array}$$



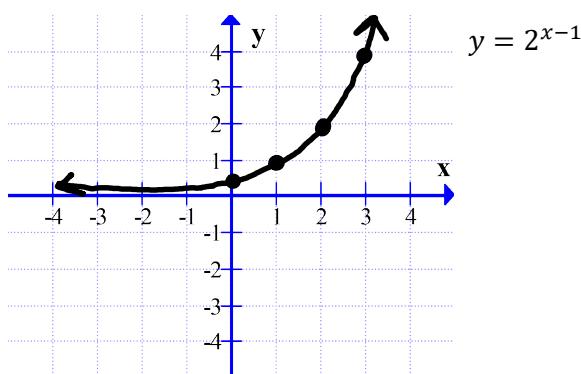
$$y = 2^x - 2$$

Down 2
 $y - 2$

$$y = C^x \pm HA$$

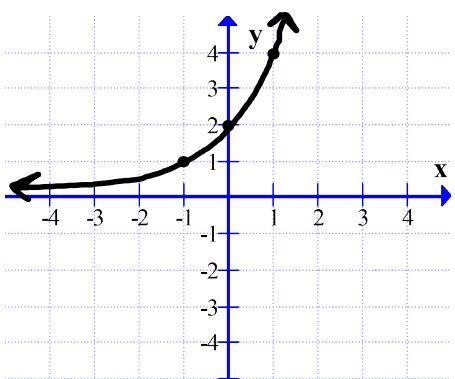
HA:

$$y = -2$$



Right One

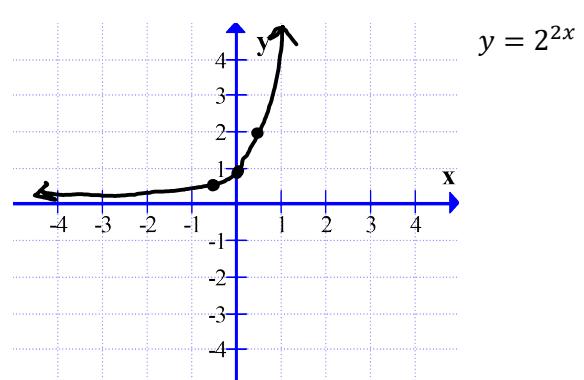
$$x + 1$$



$$y = 2(2^x)$$

Vertical Expansion = 2

$$y \times 2$$



Horizontal Compression = 1/2

$$x \div 2$$