

C12 - 1.0 - VHT VHCE VHR Function/Point Transformations Review

$$y - k = af(b(x - h)) + k$$

$+k$: Vertical Translation $VT = k$ (up)
 $-k$: Vertical Translation $VT = k$ (down)
 $(x - h)$: Horizontal Translation $HT = h$ (right) (h is positive)
 $(x + h)$: Horizontal Translation $HT = h$ (left) (h is negative)

$a < 0$: Vertical Reflection (VR) over the x -axis
 $|a| > 1$: Vertical Expansion $VE = a$
 $0 < |a| < 1$: Vertical Compression $VC = a$

$b < 0$: Horizontal Reflection (HR) over the y -axis
 $|b| > 1$: Horizontal Compression $HC = \frac{1}{b}$
 $0 < |b| < 1$: Horizontal Expansion $HE = \frac{1}{b}$

Mapping

$$(x, y); \text{ old point} \rightarrow \text{new point}; \left(\frac{x}{b} + h, ay + k\right)$$

$ a = 2$	\rightarrow	$VE = 2$	\rightarrow	$(2,3) \rightarrow (2,6)$	multiply y-value by two	$(x, 2y)$	(x, ay)
$ a = \frac{1}{2}$	\rightarrow	$VC = \frac{1}{2}$	\rightarrow	$(3,4) \rightarrow (3,2)$	multiply y-value by a half	$(x, \frac{1}{2}y)$	
$ b = \frac{1}{2}$	\rightarrow	$HE = 2$	\rightarrow	$(2,3) \rightarrow (4,3)$	multiply x-value by two	$(2x, y)$	$\left(\frac{1}{b}x, y\right)$
$ b = 2$	\rightarrow	$HC = \frac{1}{2}$	\rightarrow	$(4,1) \rightarrow (2,1)$	multiply x-value by a half	$\left(\frac{1}{2}x, y\right)$	
$a < 0$	\rightarrow	VR	\rightarrow	$(3,5) \rightarrow (3,-5)$	Multiply y-value by -1	$(x, -y)$	
$b < 0$	\rightarrow	HR	\rightarrow	$(-4,3) \rightarrow (4,3)$	Multiply x-value by -1	$(-x, y)$	
$k = 3$	\rightarrow	$VT = +3$	\rightarrow	$(2,1) \rightarrow (2,4)$	add 3 to y-value	$(x, y + 3)$	$(x, y \pm k)$
$k = -2$	\rightarrow	$VT = -2$	\rightarrow	$(1,4) \rightarrow (1,2)$	Subtract 2 from y-value	$(x, y - 2)$	
$h = -2$	\rightarrow	$HT = -2$	\rightarrow	$(4,1) \rightarrow (2,1)$	Subtract 2 from x-value	$(x - 2, y)$	$(x \pm h, y)$
$h = 4$	\rightarrow	$HT = +4$	\rightarrow	$(2,3) \rightarrow (6,3)$	Add 4 to x-value	$(x + 4, y)$	

Inverse $f^{-1}(x)$: switch x and y

$$y = f(x) \rightarrow x = f(y)$$

A reflection over the xy axis ($y = x$)

Inverse Check:

$$f(f^{-1}(x)) = x \quad f^{-1}(f(x)) = x$$

Switch Domain and Range*.

Invariant Points:

Horizontal Ref/Exp: y -intercepts $(0, y)$

Vertical Ref/Exp: x -intercepts $(x, 0)$

Inverses: (a, a) $(2, 2)$ (any points on the line $y = x$)

Roots: $(x, 0)$ $(x, 1)$

Rationals $(x, \pm 1^*)$

Inverse 1st
Functions Operations 1st.
DMAS, Inside Out

C12 - 1.0 - VHT VHCE VHR Function Sub Review

$$y = x^2$$

<p>$VT = +3$ Vertical Translation up 3</p>	$y = x^2$ $y - 3 = x^2$ $y = x^2 + 3$	<p>Put 'y - 3' in for 'y'</p> $y \rightarrow y - 3$	$f(x) = x^2$ $f(x) - 3 = x^2$ $f(x) = x^2 + 3$	<p><u>Substitute the Opposite Operation for the Variable</u></p>
<p>$HT = +3$ Horizontal Translation right 3</p>	$y = x^2$ $y = (x - 3)^2$	<p>Put 'x - 3' in for 'x'</p> $x \rightarrow x - 3$	$f(x) = x^2$ $f(x - 3) = (x - 3)^2$	<p><u>Substitute the Opposite Operation for the Variable</u></p>
<p>$HT = -3$ Horizontal Translation left 3</p>	$y = x^2$ $y = (x + 3)^2$	<p>Put 'x + 3' in for 'x'</p> $x \rightarrow x + 3$		<p><u>Substitute the Opposite Operation for the Variable</u></p>

<p>$VE = 2$ Vertical Expansion of 2</p>	$y = x^2$ $\frac{1}{2}y = x^2$ $y = 2x^2$	<p>Put $\frac{1}{2}y$ in for 'y'</p> $y \rightarrow \frac{1}{2}y$		<p><u>Substitute the Opposite Operation for the Variable</u></p>
<p>$HE = 2$ Horizontal Expansion of 2</p>	$y = x^2$ $y = \left(\frac{1}{2}x\right)^2$ $y = \frac{1}{4}x^2$	<p>Put $\frac{1}{2}x$ in for 'x'</p> $x \rightarrow \frac{1}{2}x$	$f(x) = x^2$ $f\left(\frac{1}{2}x\right) = \left(\frac{1}{2}x\right)^2$	<p><u>Substitute the Opposite Operation for the Variable</u></p>
<p>$VC = \frac{1}{2}$ Vertical Compression of $\frac{1}{2}$</p>	$y = x^2$ $2y = x^2$ $y = \frac{1}{2}x^2$	<p>Put 2y in for 'y'</p> $y \rightarrow 2y$		<p><u>Substitute the Opposite Operation for the Variable</u></p>

<p>HR Horizontal Reflection</p>	$y = x^2$ $y = (-x)^2$ $y = x^2$	<p>Put $-x$ in for x</p> $x \rightarrow -x$		<p><u>Substitute the Opposite Operation for the Variable</u></p>
<p>VR Vertical Reflection</p>	$y = x^2$ $-y = x^2$ $y = -x^2$	<p>Put $-y$ in for y</p> $y \rightarrow -y$		<p><u>Substitute the Opposite Operation for the Variable</u></p>

$x^2 + y^2 = 4$ $x^2 + (y - 3)^2 = 4$	<p>Vertical Translation up 3</p> $y \rightarrow y - 3$
$x^2 + y^2 = 4$ $(x - 3)^2 + y^2 = 4$	<p>Horizontal Translation right 3</p> $x \rightarrow x - 3$

OR

$$x^2 + y^2 = 4$$

$$y = \pm\sqrt{4 - x^2}$$

$$y = \pm\sqrt{4 - x^2} + 3$$