C12 - 11.6 - President vs. Committee Notes

How many ways can you organize 3 people?

$$\underset{1,2,3}{3} \times \underset{1,2}{2} \times \underset{1}{1} = 3! = 6$$

President Example:

A class is voting on a president, secretary and treasurer out of the 10 people running. How many different choices are there?

A president, secretary, and treasurer are all different positions. Particular Order matters.

$$\underset{1\text{-}10}{10} \times \underset{1\text{-}9}{9} \times \underset{1\text{-}8}{8} = 720$$

$$_{n}P_{r} = \frac{n!}{(n-r)!}$$
$$_{10}P_{3} = \frac{10!}{(10-3)!}$$
$$_{10}P_{3} = \frac{10!}{7!}$$
$$_{10}P_{3} = \frac{10 \times 9 \times 8 \times 7!}{7!}$$
$$_{10}P_{3} = 10 \times 9 \times 8$$
$$_{10}P_{3} = 720$$

Committee Example:

A class is voting on a committee of 3 people out of the 10 people running. How many different choices are there?

All people on a committee are equal. Order doesn't matter.

$$\frac{10 \times 9 \times 8}{3!} = \frac{720}{6} = 120$$

$$_{n}C_{r} = \frac{n!}{r!\,(n-r)!}$$
$$_{10}C_{3} = \frac{10!}{3!\,(10-3)!}$$
$$_{10}C_{3} = \frac{10!}{3!\,(7)!}$$
$$_{10}C_{3} = \frac{10!}{3!\,7!}$$
$$_{10}C_{3} = \frac{10 \times 9 \times 8 \times 7!}{3 \times 2 \times 1!\,7!}$$
$$_{10}C_{3} = \frac{720}{6}$$
$$_{10}C_{3} = 120$$

$$_{n}C_{r} = \frac{_{n}P_{r}}{r!}$$
$$_{10}C_{3} = \frac{_{10}P_{3}}{3!}$$
$$_{10}C_{3} = \frac{720}{6}$$
$$_{10}C_{3} = 120$$

The number of ways you can choose a committee is the number of ways you can choose Pres, Vice, and Sec, divided by the number of ways you can organize 3 people.

$$_{n}P_{r} = {_{n}C_{r}} \times r!$$
$$_{10}P_{3} = {_{10}C_{3}} \times 3!$$
$$_{10}P_{3} = 120 \times 6$$
$$_{10}P_{3} = 720$$

The number of ways you can choose Pres, Vice, and Sec, is the number of ways you can choose 3 people from 10 multiplied by the number of ways you can organize 3 people

C12 - 11.6 - All Minus None Notes

We have three boys and four girls. 3 b's 4 g's

How many different ways can we make a group of three, with no restrictions?

$${}_7C_3 = 35 \qquad \text{Or} \qquad \frac{(3+4)!}{3!\,4!} = 35$$

How many different ways can we make a group of three, with exactly two boys and one girl?

${}_3C_2 \times {}_4C_1$ Choose two boys from 3 boys, and 1 girl from 4 girls.

How many different ways can we make a group of three, with at least one boy?

Three cases:

Case 1: 1 b, 2 g **Case 2:** 2 b, 1 g **Case 3:** 3 b, 0 g

$${}_3C_1 \times {}_4C_2 \quad + \quad {}_3C_2 \times {}_4C_1 \quad + \quad {}_3C_3 \times {}_4C_0$$

$$3 \times 6 \quad + \quad 3 \times 4 \quad + \quad 1 \times 1$$

$$18 \quad + \quad 12 \quad + \quad 1 \qquad = 31$$

OR

$All - None$

$${}_7C_3 - {}_3C_0 \times {}_4C_3$$

$$35 - 1 \times 4$$

$$35 - 4 = 31$$

(The total number of ways we can choose three people from seven minus a case with no boys)

Note: ${}_7C_3 = (Case{:}\,0\ boys) + (Case{:}\,1\ boy) + (Case{:}\,2\ boys) + (Case{:}\,3\ boys)$
$35 = 4 + 18 + 12 + 1$
$35 = 35$

We have 10 boys and 11 girls. How many different ways can we make a group of 10 with at least one boy?

$All - None$
${}_{21}C_{10} - ({}_{10}C_0 \times {}_{11}C_{10}) = 352705$

We did this instead of adding the cases 1 boys, 2 boys, 3 boys, 4 boys, 5 boys, 6 boys, 7 boys, 8 boys, 9 boys, and 10 boys.

A lot of the time it is easier to figure out the number of ways something can't be done, rather than be done, and then subtract this from the total number of possible outcomes.

A family of 5 takes a family photo. How many ways can the parents not sit together? Answer. The total number of ways the family can sit with no restrictions, Minus the number of ways they can sit Together. Think about it! Very Useful!