C11 - 7.1 - Absolute Value: $|x|$ Notes

$|2| =$ $|2| = 2$

$|-3| = 3$

$|2-4| =$ $|-2| = 2$

$|3| - |-5| =$ $3 - 5 = -2$

$-|3| = -3$

$-|-5| =$ $-(5) = -5$

Do whatever is inside the absolute value, then make it positive.

Solve algebraically.

$|x| = 4$

"+" case:

$+(x) = 4$

$x = 4$

Distribute a positive into the absolute value

$|x| = 4$
$|4| = 4$
$4 = 4$ ✓

"−" case:

$-(x) = 4$

$x = -4$

Distribute a negative into the absolute value

$|x| = 4$
$|-4| = 4$
$4 = 4$ ✓

$|x| = -6$

Impossible.

Check your answer. (Left Hand Side LHS = RHS Right Hand Side)

$|x-2| = 2$

"+" case:

$+(x-2) = 2$
$x - 2 = 2$
$x = 4$

$|x-2| = 2$
$|4-2| = 2$
$|2| = 2$ ✓

"−" case:

$-(x-2) = 2$
$-x + 2 = 2$
$-x = 0$
$x = 0$

$|x-2| = 2$
$|0-2| = 2$
$|-2| = 2$ ✓

$2|x-2| = 6$

"+" case:

$+2(x-2) = 6$
$2x - 4 = 6$
$2x = 10$
$x = 5$

$2|x-2| = 6$
$2|5-2| = 6$
$2|3| = 6$ ✓

"−" case:

$-2(x-2) = 6$
$-2x + 4 = 6$
$-2x = 2$
$x = -1$

$2|x-2| = 6$
$2|-1-2| = 6$
$2|-3| = 6$ ✓

$|x^2 - 1| = x - 1$

"+" case:

$+(x^2 - 1) = x - 1$
$x^2 - x = 0$
$x(x-1) = 0$

$x = 0$

~~$x - 1 = 0$~~
~~$x = 1$~~

"−" case:

$-(x^2-1) = x - 1$
$-x^2 + 1 = x - 1$
$x^2 + x - 2 = 0$
$(x+2)(x-1) = 0$

~~$x - 1 = 0$~~
~~$x = 1$~~

~~$x + 2 = 0$~~
~~$x = -2$~~

$|x^2 - 1| = x - 1$
$|1^2 - 1| = 1 - 1$
$|0| = -0$ ✓

$|x^2 - 1| = x - 1$
$|0^2 - 1| = 0 - 1$
$|-1| = -1$ ✗

$|x^2 - 1| = x - 1$
$|(-2)^2 - 1| = -2 - 1$
$|4 - 1| = -2 - 1$
$|3| = -3$ ✗

C11 - 7.1 - Absolute Value Inequalities: $|x|$ Notes

$|x| \geq 2$

"+" case:

"−" case:

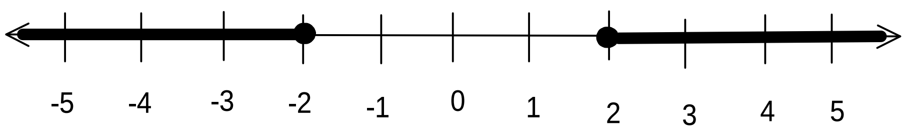
$+(x) \geq 2$

$x \geq 2$

$-(x) \geq 2$

$x \leq -2$

Divide by a negative, change direction of sign.



$\geq, \leq$ = ●

Shade greater than two, and less than negative two.

Check your answer. Test values in shaded region.

$|3| \geq$

$|3| \geq 3$

$3 \geq 2$

$|-3| \geq$

$|-3| \geq$

$3 \geq 2$

$|x - 3| < 2$

"+" case:

"−" case:

$+(x - 3) < 2$

$x - 3 < 2$

$x < 5$

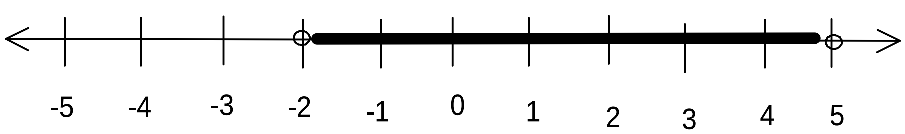
$-(x - 3) < 2$

$-x + 3 < 2$

$-x < 2$

$x > -2$

Divide by a negative, change direction of sign.



$>, <$ = ○

Shade less than five, and greater than negative two.

Check your answer. Test values in shaded region.

$|3| \geq$

$|3| \geq 3$

$3 \geq 2$

$|-3| \geq$

$|-3| \geq$

$3 \geq 2$

Graphing Absolute Values

$y = |x + 2|$

"+" case:

$y_1 = +(x + 2)$
$y_1 = x + 2$

"−" case:

$y_2 = -(x + 2)$
$y_2 = -x - 2$

Distribute a positive into the absolute value

Distribute a negative into the absolute value

If already negative combine

$y = |x + 2|$

Table of Values

x	y
-5	-3
-3	-1
-2	0
-1	1
0	2

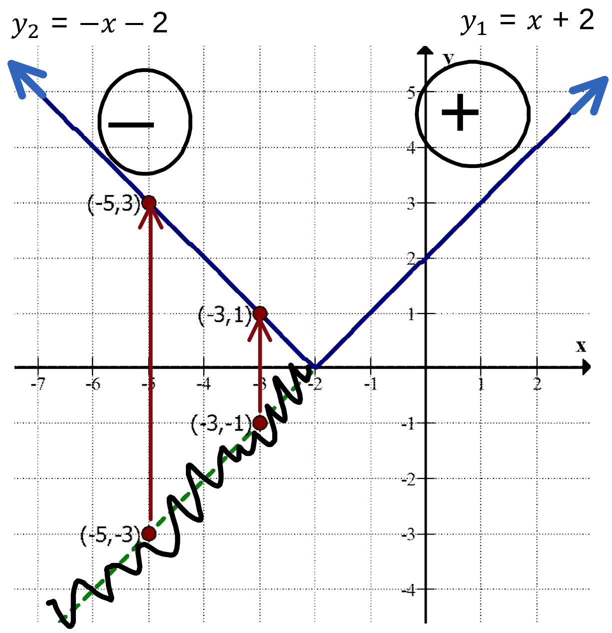
$y = x + 2$

→

x	y
-5	3
-3	1
-2	0
-1	1
0	2

$y = |x + 2|$

Pt.
(-5,2)
(-3,1)
(-2,0)
(-1,1)
(0,2)



Set inside absolute value = 0 and solve
TOV

Vertex: $(-2,0)$

Notice the graph of $y = |x + 2|$ is the graph of $y = x + 2$ and $y = -x - 2$ without any negative y values. Transfer any negative y value to a positive y value.

Piecewise function: $y = \begin{cases} x + 2, if & x \geq -2 \\ -x - 2, if & x < -2 \end{cases}$

$y = \begin{cases} \text{"+" case,} & \text{Domain of "+" case} \\ \text{"−" case,} & \text{Domain of "−" case} \end{cases}$

Notice: The domain of the negative case is not equal to.

Domain of positive case:

$x + 2 \geq 0$
$-2 \quad -2$
$x \geq -2$

Set what is inside the absolute value greater than or equal to zero.

Domain of negative case:

$x + 2 < 0$
$-2 \quad -2$
$x < -2$

Set what is inside the absolute value less than zero.

C11 - 7.3 - $|x| = c$ Equations Absolute Value Notes

Solve algebraically

$|x+2| = 4$

"+" case:

$+(x+2) = 4$
$x + 2 = 4$
$x = 2$

"−" case:

$-(x+2) = 4$
$-x - 2 = 4$
$-x = 6$
$x = -6$

Check your answer.

$|x+2| = 4$
$|2+2| = 4$
$|4| = 4$ ✓

$|-6+2| = 4$
$|-4| = 4$
$|-4| = 4$ ✓

Solve graphically.

$|x+2| = 4$ Left hand side (LHS) = Right hand side (RHS)

$y = |x+2|$

$y = 4$

y=Left hand side (LHS)

y=Right hand side (RHS)

"+" case:

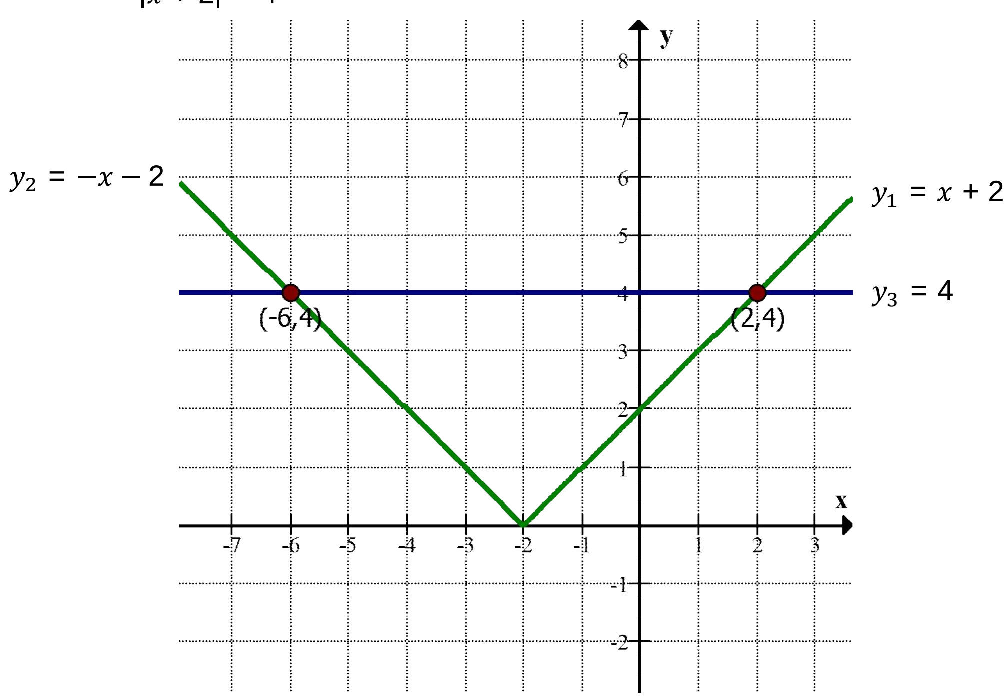
$y_1 = +(x+2)$
$y_1 = x + 2$

"−" case:

$y_2 = -(x+2)$
$y_2 = -x - 2$

$y_3 = 4$

$|x+2| = 4$



C11 - 7.4 - Quadratic Absolute Value Notes

$y = |x^2 - 4|$

"+" case:

$y_1 = +(x^2 - 4)$

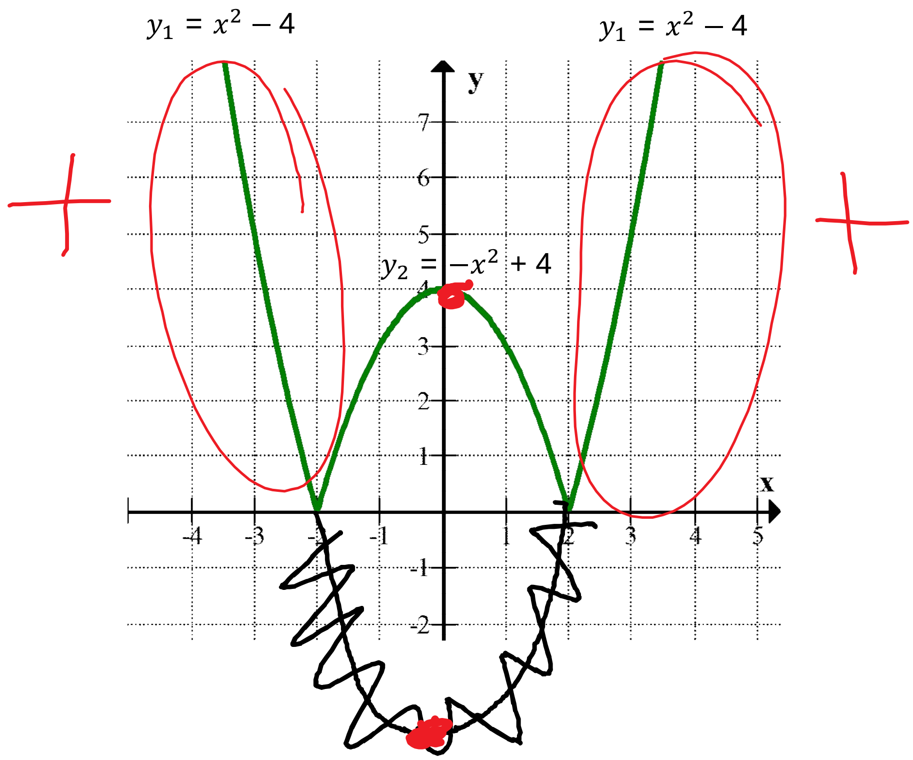
$y_1 = x^2 - 4$

"−" case:

$y_2 = -(x^2 - 4)$

$y_2 = -x^2 + 4$

$y = |x^2 - 4|$



Notice the graph of $y = |x^2 - 4|$ is the graph of $y_1 = x^2 - 4$ less than two and greater than two and is the graph of $y_2 = -x^2 + 4$ less than two and greater than negative two.

Piecewise function:

$$y = \begin{cases} x^2 - 4, & if\ x \geq 2, x \leq -2 \\ -x^2 + 4, & if -2 < x < 2 \end{cases}$$

C11 - 7.5 - Quadratic Absolute Value Equations Notes

Solve algebraically.

$|x^2 - 4| = x + 2$

"+" case:

$+(x^2 - 4) = x + 2$
$x^2 - 4 = x + 2$
$x^2 - x - 6 = 0$
$(x - 3)(x + 2) = 0$
$x = 3, -2$

"−" case:

$-(x^2 - 4) = x + 2$
$-x^2 + 4 = x + 2$
$0 = x^2 + x - 2$
$0 = (x + 2)(x - 1)$
$x = -2, 1$

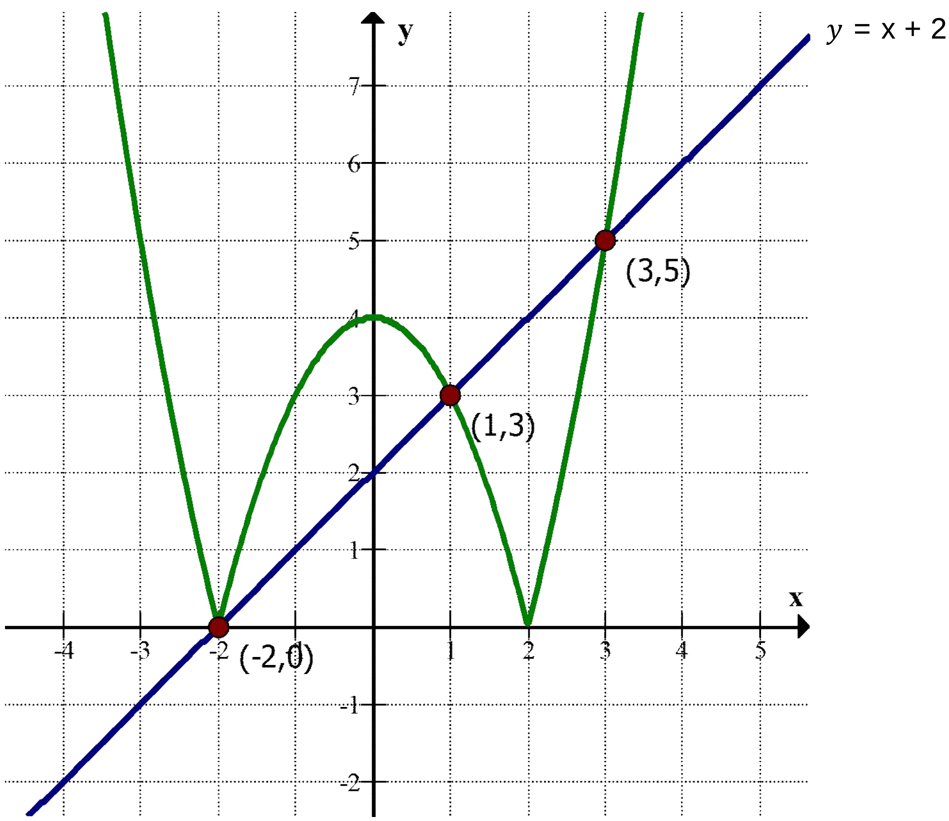
Check Answers!

$x = 3, -2$

$x = -2, 1$

Solve Graphically

$y = |x^2 - 4|$



C11 - 7.6 - Reciprocal Restrictions Notes

Find the restrictions

$$\frac{1}{x-2}$$

$Set\ denominator = 0, and\ solve.$

$x - 2 = 0$

$x = 2$

$$\frac{1}{(x+2)(2x-1)}$$

$Set\ denominator = 0, and\ solve.$

$2x^2 + 3x - 2 = (x + 2)(2x - 1)$

$x + 2 = 0$

$x = -2$

$2x - 1 = 0$

$x = \frac{1}{2}$

C11 - 7.7 - Linear Reciprocals Notes

$y = x + 4$ Line

$y = \dfrac{1}{x + 4}$ Reciprocal line

Pick a y value, What's one divided by that y value. Put a point on the graph. X value is same as it was.

Solve algebraically: set denominator = 0, 1, -1.

Vertical asymptote (VA):
Denominator = 0

$x + 4 = 0$
$x = -4$

VA: $x = -4$

$D: x \neq -4$

Invariant points (IP):
Denominator = 1

$x + 4 = 1$
$x = -3$

$(-3, 1)$

Invariant points (IP):
Denominator = -1

$x + 4 = -1$
$x = -5$

$(-5, -1)$

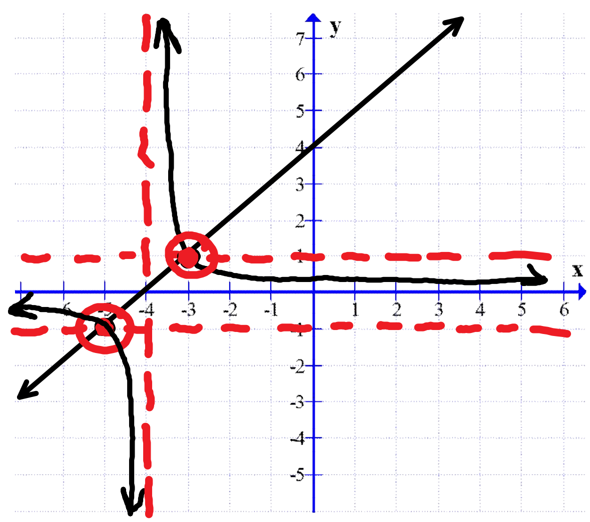
1. Graph original
2. Graph VA: Dotted line
3. Graph IP's
4. Graph reciprocal

x	y
-5	-1
-4	0
-3	1

x	$\dfrac{1}{x+4}$
-100	$-.01$
-5	-1
-4.1	-10
-4.01	-100
-4	UND
-3.99	100
-3.9	10
-3	1
100	.01

$D: x \neq -4$

$y = \dfrac{1}{x + 4}$ $y = x + 4$

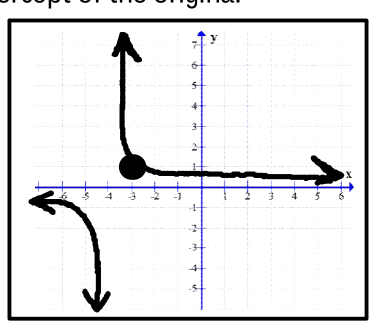


$y \neq 0$

Close to the vertical asymptote, through the point, close the x-axis/vertical asymptote

Notice: The invariant points are the intersection of the original and the lines $y = 1, y = -1$

Notice: The vertical asymptote(s) of the reciprocal is the X intercept of the original



C11 - 7.8 - Quadratic Reciprocals Notes

$y = x^2 - 4$ Parabola

$y = \frac{1}{x^2 - 4}$ Reciprocal Parabola

Solve algebraically: set denominator = 0, 1, -1.

Vertical asymptote (VA): Denominator = 0	Invariant points (IP): Denominator = 1	Invariant points (IP): Denominator = -1
$x^2 - 4 = 0$	$x^2 - 4 = 1$	$x^2 - 4 = -1$
$(x + 2)(x - 2) = 0$	$x^2 = 5$	$x^2 = 3$
$x = 2, -2$	$x = \sqrt{5}, -\sqrt{5}$	$x = \sqrt{3}, -\sqrt{3}$
VA's: $x = 2$, $x = -2$	$(\sqrt{5}, 1)$, $(-\sqrt{5}, 1)$	$(\sqrt{3}, -1)$, $(-\sqrt{3}, -1)$

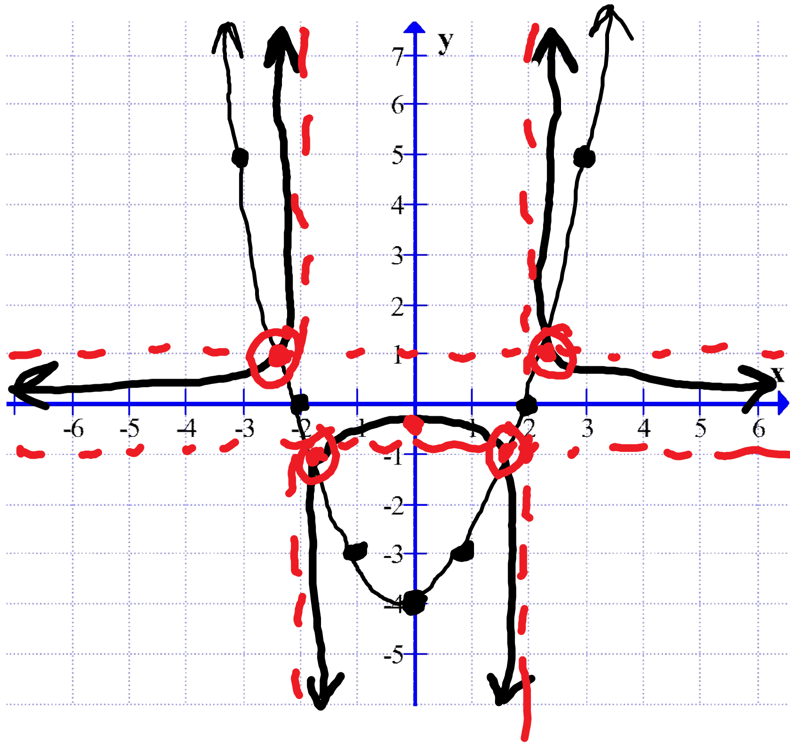
Solve graphically.

$y = x^2 - 4$

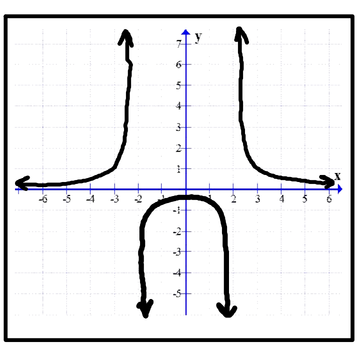
$y = \frac{1}{x^2 - 4}$

$D: x \neq \pm 2$

1. Graph original
2. Graph VA's: Dotted lines
3. Graph IP's
4. Graph reciprocal
5. y-int



$y \neq 0$



$(0, -4) \longrightarrow (0, -\frac{1}{4})$ $\frac{1}{y}$ $\frac{1}{-4}$