C11 - 4.1 - Solving $x - intercepts$ Notes

Solve for $x - intercepts$.

$y = x^2 - 4x + 3$

$y = (x - 1)(x - 3)$

$0 = (x - 1)(x - 3)$

$\underline{\quad 1 \quad} \times \underline{\quad 5 \quad} = 5$

$\underline{\quad 1 \quad} + \underline{\quad 5 \quad} = 6$

Factor

$x - int$: *Set y equal to zero*, $(y = 0)$

Set the brackets equal to zero seperately

$x - 1 = 0 \qquad x - 3 = 0$

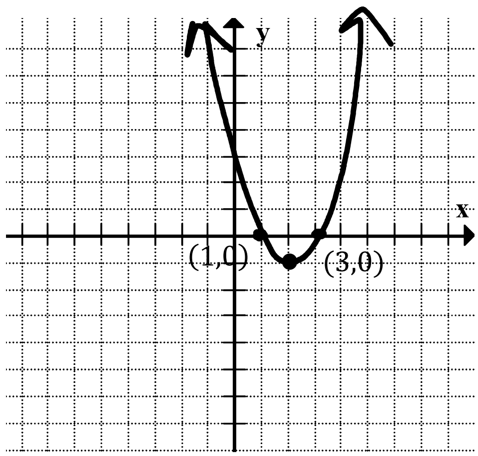
$+1 \quad +1 \qquad +3 \quad +3$

$x = +1 \qquad x = +3$

Solve

$(1,0) \quad x - int: \quad (3,0)$

State $x - intercepts$ $(x, 0)$



Draw a graph and label $x - intercepts$.

$(a)(b) = 0$
$a = 0 \qquad b = 0$

$y = 2x^2 - 3x - 2$

$y = 2x^2 - 4x + 1x - 2$

$y = (2x^2 - 4x)(+1x - 2)$

$y = 2x(x - 2) + 1(x - 2)$

$y = (x - 2)(2x + 1)$

$0 = (x - 2)(2x + 1)$

$\underline{\quad -4 \quad} \times \underline{\quad 1 \quad} = -4$

$\underline{\quad -4 \quad} + \underline{\quad 1 \quad} = -3$

Factor
Decompose
Group
GCF
Switch

$x - int$: *Set y equal to zero*, $(y = 0)$

$x - 2 = 0 \qquad 2x + 1 = 0$

$+2 \quad +2 \qquad -1 \quad -1$

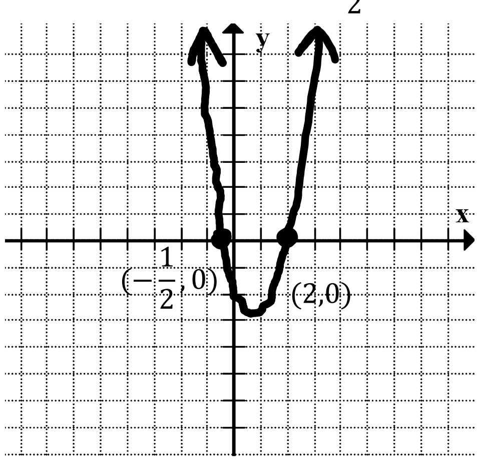
$x = 2 \qquad \frac{2x}{2} = -\frac{1}{2}$

$x = -\frac{1}{2}$

State $x - intercepts$ $(x, 0)$

$x - int$: $\quad (2,0) \quad (-\frac{1}{2}, 0)$

Draw a graph and label $x - intercepts$.



Set the brackets equal to zero seperately

Solve

C11 - 4.1 - Solving $x - intercepts$ Notes

Set $y = 0$ $and\ factor\ to\ find\ x - intercepts.$ $(x, 0)$

$y = x^2 - 6x + 5$
$0 = x^2 - 6x + 5$
$0 = (x - 5)(x - 1)$

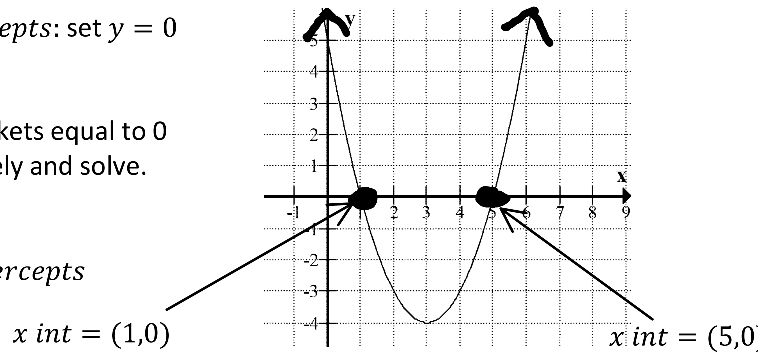
x $intercepts$: set $y = 0$
Factor.

Set brackets equal to 0 separately and solve.

$x - 5 = 0$ | $x - 1 = 0$
$+5 \quad +5$ | $+1 \quad +1$
$x = 5$ | $x = 1$

$(5,0)$ | $(1,0)$

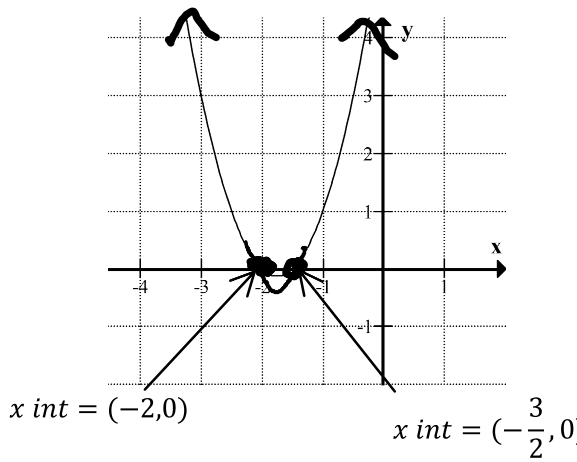
$x - intercepts$



$y = 2x^2 + 7x + 6$
$0 = 2x^2 + 7x + 6$
$0 = 2x^2 + 4x + 3x + 6$
$0 = 2x(x + 2) + 3(x + 2)$
$0 = (2x + 3)(x + 2)$

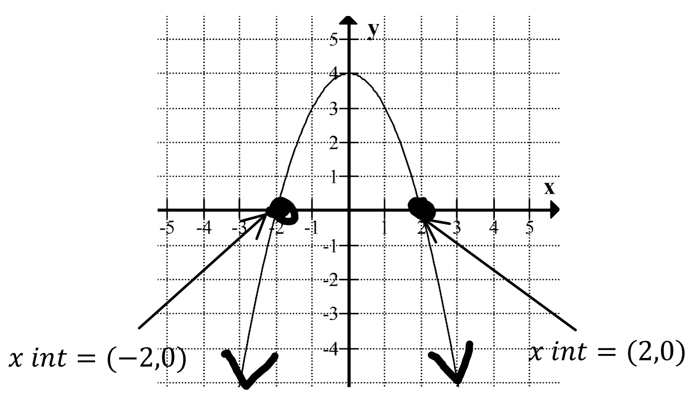
$2x + 3 = 0$
$-3 \quad -3$
$2x = -3$
$\frac{2x}{2} = \frac{-3}{2}$
$x = -\frac{3}{2}$

$x + 2 = 0$
$-2 \quad -2$
$x = -2$



$y = -x^2 + 4$
$0 = -x^2 + 4$
$0 = -(x^2 - 4)$ GCF: -1
$0 = -(x + 2)(x - 2)$ Factor.

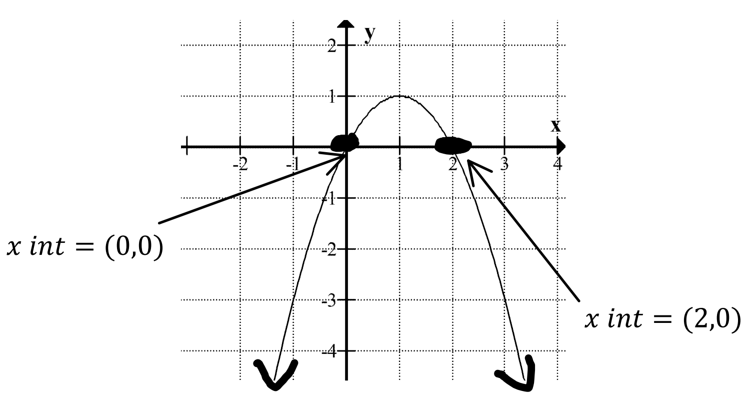
$x + 2 = 0$ | $x - 2 = 0$
$-2 \quad -2$ | $+2 \quad +2$
$x = -2$ | $x = 2$



$y = -x^2 + 2x$
$0 = -x^2 + 2x$
$0 = -x(x - 2)$

$x = 0$

$x - 2 = 0$
$+2 \quad +2$
$x = 2$



C11 - 4.2 - $x - int$/Standard Form Notes

$x\ int = (2,0)\ ,(6,0)$

$$x = 2$$
$$-2 \quad -2$$
$$x - 2 = 0$$

$$x = 6$$
$$-6 \quad -6$$
$$x - 6 = 0$$

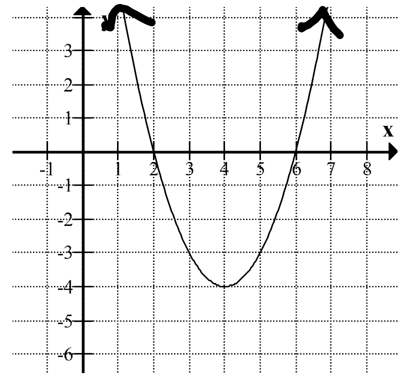
Write down the x values.

Add or subtract to both sides to make = 0

$$y = (x-2)(x-6)$$

$$y = x^2 - 8x + 12$$

Factored Form
Standard Form



$x\ int = \left(\frac{1}{2}, 0\right), (4, 0)$

$$x = \frac{1}{2}$$
$$2 \times x = \frac{1}{2} \times 2$$
$$2x = 1$$
$$-1 \quad -1$$
$$2x - 1 = 0$$

$$x = 4$$
$$-4 \quad -4$$
$$x - 4 = 0$$

Multiply and Add or subtract to both sides to make = 0

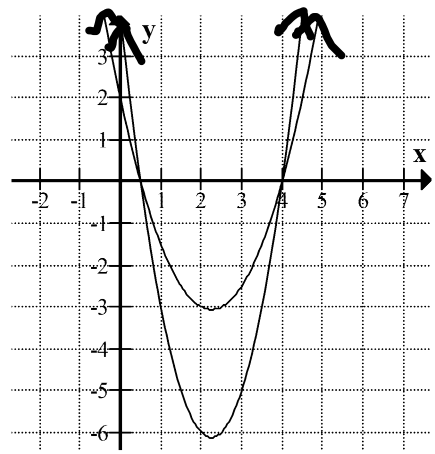
$$y = x^2 - \frac{9}{2}x + 2$$
$$0 = x^2 - \frac{9}{2}x + 2$$

$$y = (2x-1)(x-4)$$
$$y = 2x^2 - 9x + 4$$

$x\ int = \left(\frac{1}{2}, 0\right), (4, 0)$

$$x = \frac{1}{2}$$
$$-\frac{1}{2} \quad -\frac{1}{2}$$
$$x - \frac{1}{2} = 0$$

$$x = 4$$
$$-4 \quad -4$$
$$x - 4 = 0$$



$$y = 2x^2 - 9x + 4$$
$$0 = 2x^2 - 9x + 4$$

$$y = \left(x - \frac{1}{2}\right)(x-4)$$
$$y = x^2 - 4x - \frac{1}{2}x + 2$$
$$y = x^2 - \frac{9}{2}x + 2$$

Notice: two different graphs in standard form can have the same x-intercepts.

C11 - 4.2 - Find Standard Form x-int "a" and a Point Notes

Find equation in Standard Form using $x - intercepts$ *and* "a"

$$y = a(x + \#)(x + \#)$$

$x - int = 2$ *and* 6
$a = 1$

$$x = 2$$
$$-2 \quad -2$$
$$x - 2 = 0$$

$$x = 6$$
$$-6 \quad -6$$
$$x - 6 = 0$$

Set $x - int = \#$ *and make equal to zero*

$y = a(x + \#)(x + \#)$ — *Write Factored Form*
$y = 1(x - 2)(x - 6)$ — *Substitute Factors*
$y = (x - 2)(x - 6)$
$y = x^2 - 8x + 12$ — *Foil*

$x - int = 2$ *and* -2
$a = 2$

$$x = 2$$
$$-2 \quad -2$$
$$x - 2 = 0$$

$$x = -2$$
$$+2 \quad +2$$
$$x + 2 = 0$$

$$y = a(x + \#)(x + \#)$$
$$y = 2(x - 2)(x + 2)$$
$$y = 2(x^2 + 2x - 2x - 4)$$
$$y = 2(x^2 - 4)$$
$$y = 2x^2 - 8$$

$x - int = \frac{3}{2}$ *and* $-\frac{7}{2}$

$$x = \frac{3}{2}$$
$$2 \times x = \frac{3}{2} \times 2$$
$$2x = 3$$
$$-3 \quad -3$$
$$2x - 3 = 0$$

$$x = -\frac{7}{2}$$
$$2 \times x = \frac{3}{2} \times 2$$
$$2x = -7$$
$$+7 \quad +7$$
$$2x + 7 = 0$$

$$y = a(x + \#)(x + \#)$$
$$y = (2x - 3)(2x + 7)$$
$$y = 4x^2 + 14x - 6x - 21$$
$$y = 4x^2 + 8x - 21$$

$x - int = -1$ *and* 3
$(2, -6)$

$$y = a(x + 1)(x - 3)$$
$$-6 = a(2 + 1)(2 - 3)$$
$$-6 = a(3)(-1)$$
$$-6 = -3a$$
$$a = 2$$

$$y = 2(x + 1)(x - 3)$$

C11 - 4.3 - x −Intercepts/Vertex/AOS Form

$y = x^2 - 2x - 8$
$y = (x - 2)(x + 4)$

$x - 2 = 0$ $\quad$ $+2 \quad +2$ $\quad$ $x = 2$

$x + 4 = 0$ $\quad$ $-4 \quad -4$ $\quad$ $x = -4$

$x - int$: $(2,0)$ $\quad$ $(-4,0)$

The x coordinate of the vertex is always halfway between the two x-intercepts.

$$x = \frac{(2) + (-4)}{2} = \frac{-2}{2}$$

$x = -1$

Find the average between the two x-intercept values.
(Or any two horizontal x-values)

Vertex: $(-1, y)$

$Axis\ of\ Symmetry: x = -1$

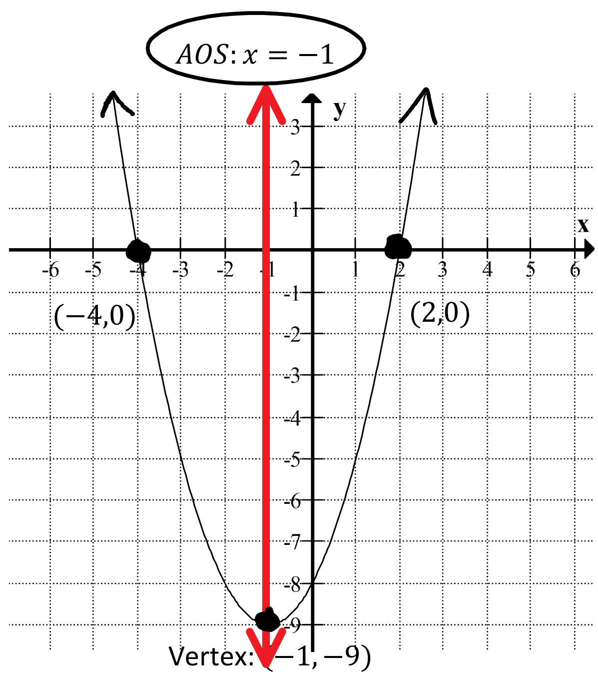
$y = (x - 2)(x + 4)$
$y = ((-1) - 2)((-1) + 4)$
$y = (-3)(3)$
$y = -9$

Find the y value of the vertex by putting in the x value of the vertex

Vertex: $(-1, -9)$

x	y
-3	-5
-2	-8
-1	-9
0	-8
1	-5

Vertex:



C11 - 4.3 - Solving by Square Root Method Notes

$$x^2 - 4 = 0$$
$$+4 \quad +4$$
$$x^2 = 4$$
$$\sqrt{x^2} = \pm\sqrt{4}$$
$$x = \pm 2$$
$$x = 2 \qquad x = -2$$

$$x^2 - 4 = 0$$
$$(x+2)(x-2) = 0$$
$$x + 2 = 0 \qquad x - 2 = 0$$
$$x = -2 \qquad x = 2$$

$$(x-2)^2 - 1 = 0$$
$$+1 \quad +1$$
$$(x-2)^2 = 1$$
$$\sqrt{(x-2)^2} = \pm\sqrt{1}$$
$$x - 2 = \pm 1$$
$$x - 2 = 1 \qquad x - 2 = -1$$
$$x = 3 \qquad x = 1$$

$$(x-2)^2 - 1 = 0$$
$$(x-2)(x-2) - 1 = 0$$
$$x^2 - 4x + 4 - 1 = 0$$
$$x^2 - 4x + 3 = 0$$
$$(x-1)(x-3) = 0$$
$$x - 1 = 0 \qquad x - 3 = 0$$
$$x = 1 \qquad x = 3$$

$$x^2 + 16 = 0$$
$$-16 \quad -16$$
$$x^2 = -16$$
$$\sqrt{x^2} = \pm\sqrt{-16}$$
$$DNE$$

Can't square root a negative.

$$(x+2)^2 + 2 = 0$$
$$-2 \quad -2$$
$$(x+2)^2 = -2$$
$$\sqrt{(x+2)^2} = \pm\sqrt{-2}$$
$$DNE$$

$$2(x+1)^2 - 8 = 0$$
$$+8 \quad +8$$
$$2(x+1)^2 = 8$$
$$\frac{2(x+1)^2}{2} = \frac{8}{2}$$
$$(x+1)^2 = 4$$
$$\sqrt{(x+1)^2} = \pm\sqrt{4}$$
$$x + 1 = \pm 2$$
$$x + 1 = 2 \qquad x + 1 = -2$$
$$-1 \quad -1 \qquad -1 \quad -1$$
$$x = 1 \qquad x = -3$$

$$(x-2)^2 - 7 = 0$$
$$+7 \quad +7$$
$$(x-2)^2 = 7$$
$$\sqrt{(x-2)^2} = \pm\sqrt{7}$$
$$x - 2 = \pm\sqrt{7}$$
$$x = \pm\sqrt{7} + 2$$
$$x = \sqrt{7} + 2 \qquad x = -\sqrt{7} + 2$$

$$\left(x - \frac{1}{2}\right)^2 - 7 = 0$$
$$\left(x - \frac{1}{2}\right)^2 = 7$$
$$x - \frac{1}{2} = \pm\sqrt{7}$$
$$x = \pm\sqrt{7} + \frac{1}{2}$$
$$x = \pm\sqrt{7} \times \frac{2}{2} + \frac{1}{2}$$
$$x = \frac{\pm 2\sqrt{7}}{2} + \frac{1}{2}$$
$$x = \frac{\pm 2\sqrt{7} + 1}{2}$$

$$2\left(x + \frac{1}{2}\right)^2 - 8 = 0$$
$$2\left(x + \frac{1}{2}\right)^2 = 8$$
$$\left(x + \frac{1}{2}\right)^2 = 4$$
$$\sqrt{\left(x + \frac{1}{2}\right)^2} = \pm\sqrt{4}$$
$$x + \frac{1}{2} = \pm 2$$
$$x = \pm 2 - \frac{1}{2}$$
$$x = 1.5 \qquad x = -2.5$$

$$2(x-2)^2 - 7 = 0$$
$$2(x-2)^2 = 7$$
$$\sqrt{(x-2)^2} = \pm\sqrt{\frac{7}{2}}$$
$$x - 2 = \pm\sqrt{\frac{7}{2}}$$
$$x = \pm\sqrt{\frac{7}{2}} + 2$$
$$x = \pm\frac{\sqrt{7}}{\sqrt{2}} + \frac{2\sqrt{2}}{\sqrt{2}}$$
$$x = \frac{\pm\sqrt{7} + 2\sqrt{2}}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$$
$$x = \frac{\pm\sqrt{14} + 4}{2}$$

$$2\left(x - \frac{2}{3}\right)^2 - 7 = 0$$
$$2\left(x - \frac{2}{3}\right)^2 = 7$$
$$\sqrt{\left(x - \frac{2}{3}\right)^2} = \pm\sqrt{\frac{7}{2}}$$
$$x - \frac{2}{3} = \pm\sqrt{\frac{7}{2}}$$
$$x = \pm\sqrt{\frac{7}{2}} + \frac{2}{3}$$
$$x = \pm\frac{\sqrt{7}}{\sqrt{2}} \times \frac{3}{3} + \frac{2}{3} \times \frac{\sqrt{2}}{\sqrt{2}}$$
$$x = \frac{\pm 3\sqrt{7} + 2\sqrt{3}}{3\sqrt{2}}$$
$$x = \frac{\pm 3\sqrt{14} + 2\sqrt{6}}{6}$$

Rationalize

C11 - 4.4 - Quadratic Equation Notes

Solve

1 -4 3

$1x^2 - 4x + 3 = 0$

$a = 1$
$b = -4$
$c = 3$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Quadratic Equation

Substitute With Brackets

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(3)}}{2(1)}$$

$$x = \frac{+4 \pm \sqrt{4}}{2}$$

$(-4)^2 - 4(1)(3) = 4$

Type underneath Square Root into Calculator

$$x = \frac{4 \pm 2}{2}$$

$$x = \frac{4+2}{2} \qquad x = \frac{4-2}{2}$$

$x = 3$ $\qquad x = 1$

2 Rational Roots.

2 $+5$ 1

$2x^2 + 5x + 1 = 0$

$a = 2$
$b = -5$
$c = 1$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(+5) \pm \sqrt{(5)^2 - 4(2)(1)}}{2(2)}$$

$$x = \frac{-5 \pm \sqrt{17}}{4}$$

$x = \frac{-5 + \sqrt{17}}{4}$ $\qquad x = \frac{-5 - \sqrt{17}}{4}$ Exact Value

$x = -0.21$ $\qquad x = -2.28$ Decimal

2 Irrational Roots.

$b^2 - 4ac > 0$
Discriminant > 0
2 Real Roots.

---

2 -6 -7

$2x^2 - 6x - 7 = 0$

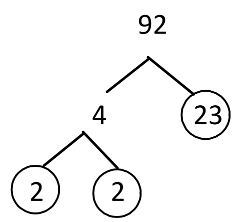
$a = 2$
$b = -6$
$c = -7$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(2)(-7)}}{2(2)}$$

$$x = \frac{6 \pm \sqrt{92}}{4}$$

$\sqrt{92} = \sqrt{2 \times 2 \times 23}$
$\sqrt{92} = 2\sqrt{23}$



$$x = \frac{6 \pm 2\sqrt{23}}{4}$$

Divide top and bottom by 2 $\quad \frac{6}{2} = 3 \quad \frac{2}{2} = 1 \quad \frac{4}{2} = 2$

$$x = \frac{3 \pm \sqrt{23}}{2}$$

$x = \frac{3 + \sqrt{23}}{2}$ $\qquad x = \frac{3 - \sqrt{23}}{2}$

1 6 11

$x^2 + 6x + 11 = 0$

$a = 1$
$b = 6$
$c = 11$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(6) \pm \sqrt{(6)^2 - 4(1)(11)}}{2(1)}$$

$$x = \frac{-6 \pm \sqrt{-8}}{2}$$

Cant Square Root Negative

$$x = \frac{-6 \pm \sqrt{-8}}{2}$$ ✗

$b^2 - 4ac < 0$
Discriminant < 0
No Real Roots.

3 -6 3

$3x^2 - 6x + 3 = 0$

$a = 3$
$b = -6$
$c = 3$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(3)(3)}}{2(3)}$$

$$x = \frac{6 \pm \sqrt{0}}{6}$$

$$x = \frac{6 \pm 0}{6}$$

$x = 1$

$b^2 - 4ac = 0$
Discriminant $= 0$
One Roots.

$3x^2 - 6x + 3 = 0$

$$\frac{3x^2}{3} - \frac{6x}{3} + \frac{3}{3} = \frac{0}{3}$$

$x^2 - 2x + 1 = 0$

1 -2 1

$a = 1$
$b = -2$
$c = 1$

Simplify 1st!

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(1)}}{2(1)}$$

$$x = \frac{2 \pm \sqrt{0}}{2}$$

$$x = \frac{2 \pm 0}{2}$$

$x = 1$

C11 - 4.5 - Discriminant Notes

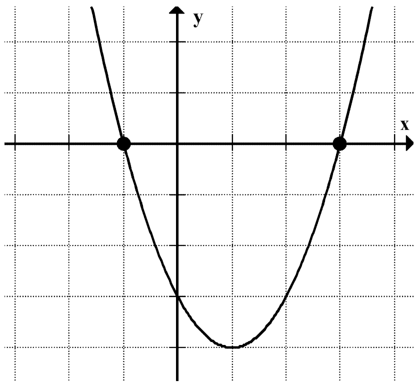
Discriminant: $b^2 - 4ac$

Quadratic Formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-b \pm \sqrt{DISCRIMINANT}}{2a}$$

Case 1: $b^2 - 4ac > 0$ *Inside the root is positive*



$x^2 - 2x - 3$

$b^2 - 4ac$
$(-2)^2 - 4(1)(-3)$
$4 + 12$
$+16$

$+$

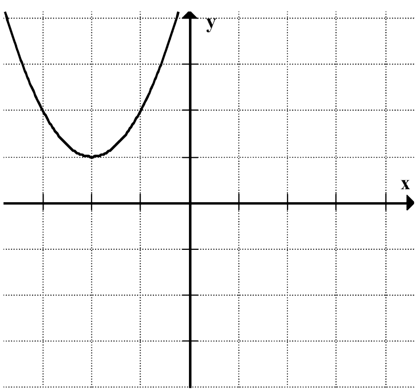
$$x = \frac{2 \pm \sqrt{16}}{2}$$

$x = 3$ $x = -1$

Two x-intercepts
Two Real Roots
Two Solutions

If we add and subtract a positive number we get two answers

Case 2: $b^2 - 4ac < 0$ *Inside the root is negative*



$x^2 + 4x + 5$

$b^2 - 4ac$
$(4)^2 - 4(1)(5)$
$16 - 20$
-4

$-$

$$x = \frac{-4 \pm \sqrt{-4}}{2}$$

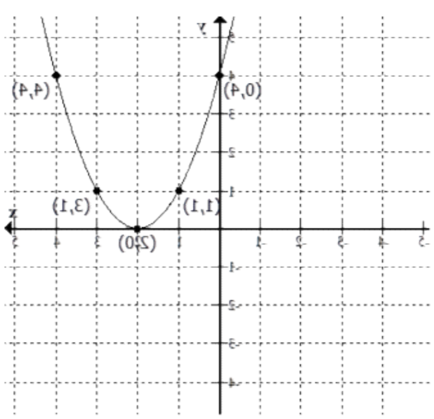
No Solution

Zero x-intercepts
No Real Roots
No Solutions
Imaginary Roots

Cant Square Root Negatives

Case 3: $b^2 - 4ac = 0$ *Inside the root is zero*

$b^2 - 4ac = 0, Perfect\ Square$



$x^2 + 4x + 4$

$b^2 - 4ac$
$(4)^2 - 4(1)(4)$
$16 - 16$
0

0

$$x = \frac{-4 \pm \sqrt{0}}{2}$$

$x = -2$

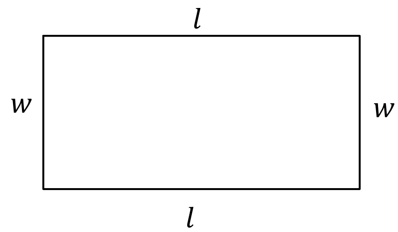
One x-intercepts
Two equal/real roots
One Solution

If we add and subtract zero we get one answer

C11 - 4.6 - Rectangular Garden

A rectangular garden has an Area of 36 and a Perimeter of 30. What are the lengths and widths?

Let $w = width$
Let $l = length$



Let statements:

$P = 2l + 2w$

$A = l \times w$

Equation 1, equation 2.

$$P = 2l + 2w$$
$$30 = 2l + 2w$$
$$\frac{30}{2} = \frac{2l}{2} + \frac{2w}{2}$$
$$15 = l + w$$
$$-w \qquad -w$$
$$15 - w = l$$
$$l = 15 - w$$

Equation #1
Isolate a variable

$$A = l \times w$$
$$36 = l \times w$$
$$36 = (15 - w) \times w$$
$$36 = 15w - w^2$$
$$+w^2 \qquad +w^2$$
$$36 + w^2 = 15w$$
$$-15w \qquad -15w$$
$$w^2 - 15w + 36 = 0$$
$$(w - 12)(w - 3) = 0$$

Equation #2
Substitute the isolated variable

Factor

$$w - 12 = 0 \qquad w - 3 = 0$$
$$w = 12 \qquad w = 3$$

Solve

$$l = 15 - w$$
$$l = 15 - (12)$$
$$l = 3$$

Substitute w into the other equation.

$Length = 12$
$Width = 3$

List the length and width

OR

$$l = 15 - w$$
$$l = 15 - (3)$$
$$l = 12$$

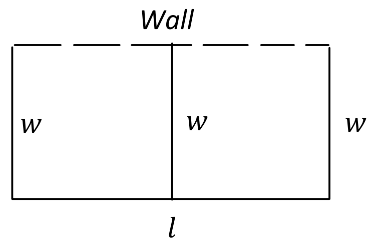
$Length = 3$
$Width = 12$

List the length and width

C11 - 4.6 - Fence Split in Two

A rectangular fence that is split in half is against a wall. The total fencing length is 39, and it has a total area of 66. What are the dimensions of the fence?

Let $w = width$
Let $l = length$



Let statements:

$P = l + 3w$ $\quad$ $A = l \times w$

Equation 1, equation 2.

$$P = l + 3w$$
$$39 = l + 3w$$
$$-3w \quad -3w$$
$$39 - 3w = l$$
$$l = 39 - 3w$$

Equation #1
Isolate a variable

$$A = l \times w$$
$$66 = (39 - 3w) \times w$$
$$66 = 39w - 3w^2$$
$$+3w^2 \quad +3w^2$$
$$66 + 3w^2 = 39w$$
$$-39w \quad -39w$$
$$3w^2 - 39w + 66 = 0$$
$$3(w^2 - 13w + 22) = 0$$
$$3(w - 2)(w - 11) = 0$$

Equation #2
Substitute the isolated variable

Factor

$$w - 2 = 0 \qquad w - 11 = 0$$
$$w = 2 \qquad w = 11$$

Solve

$$l = 39 - 3w$$
$$l = 39 - 3(2)$$
$$l = 39 - 6$$
$$l = 33$$

Substitute w into the other equation.

$$Width = 2$$
$$Length = 33$$

List the length and width

or

$$l = 39 - 3w$$
$$l = 39 - 3(11)$$
$$l = 39 - 33$$
$$l = 6$$

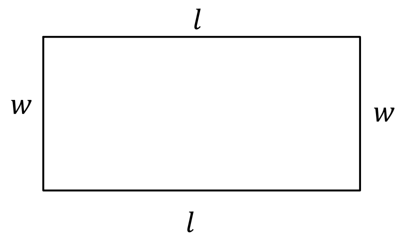
List the length and width

$$Width = 11$$
$$Length = 6$$

C11 - 4.6 - Rectangular Garden Quad

A rectangular garden has an area of 61 and a perimeter of 40. What are the lengths and widths?

Let $w = width$
Let $l = length$



Let statements:

$P = 2l + 2w$ $\qquad$ $A = l \times w$

Equation 1, equation 2.

$$P = 2l + 2w$$
$$40 = 2l + 2w$$
$$\frac{40}{2} = \frac{2l}{2} + \frac{2w}{2}$$
$$20 = l + w$$
$$-w \qquad -w$$
$$20 - w = l$$
$$l = 20 - w$$

Equation #1
Isolate a variable

$$A = l \times w$$
$$91 = l \times w$$
$$61 = (20 - w) \times w$$
$$61 = 20w - w^2$$
$$+w^2 \qquad +w^2$$
$$61 + w^2 = 20w$$
$$-20w \qquad -20w$$
$$w^2 - 20w + 61 = 0$$

Equation #2
Substitute the isolated variable

$$w = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$w = \frac{-(-20) \pm \sqrt{20^2 - 4(1)(61)}}{2(1)}$$

Quadratic Formula

$$w = \frac{20 - \sqrt{156}}{2(1)} \qquad w = \frac{20 + \sqrt{156}}{2(1)}$$
$$w = 3.755 \qquad w = 16.245$$

Solve

$$l = 20 - w$$
$$l = 20 - (16.245)$$
$$l = 3.755$$

Substitute w into the other equation.

$$Length = 16.245$$
$$Width = 3.755$$

List the length and width

OR

$$l = 15 - w$$
$$l = 15 - (3.755)$$
$$l = 16.245$$

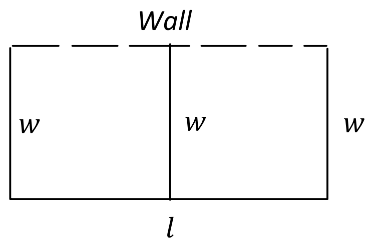
List the length and width

$$Length = 3.755$$
$$Width = 16.245$$

C11 - 4.6 - Fence Split in Two Quad

A rectangular fence that is split in half is against a wall. The total fencing length is 61, and it has a total area of 58. What are the dimensions of the fence?

Let $w = width$
Let $l = length$



Let statements:

$P = l + 3w$ $\quad A = l \times w$

Equation 1, equation 2.

$$P = l + 3w$$
$$61 = l + 3w$$
$$-3w \quad -3w$$
$$61 - 3w = l$$
$$l = 61 - 3w$$

Equation #1
Isolate a variable

$$A = l \times w$$
$$58 = (61 - 3w) \times w$$
$$58 = 61w - 3w^2$$
$$+3w^2 \quad +3w^2$$
$$58 + 3w^2 = 61w$$
$$-61w \quad -61w$$
$$3w^2 - 61w + 58 = 0$$

Equation #2
Substitute the isolated variable

$$w = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$w = \frac{-(-61) \pm \sqrt{61^2 - 4(3)(58)}}{2(3)}$$

Quadratic Formula

$$w = \frac{61 + \sqrt{3025}}{6} \qquad w = \frac{61 - \sqrt{3025}}{6}$$
$$w = 19.\overline{3} \qquad w = 1$$
$$w = \frac{58}{3}$$

Solve

$$l = 61 - 3w$$
$$l = 61 - 3\left(\frac{58}{3}\right)$$
$$l = 61 - 58$$
$$l = 3$$

Substitute w into the other equation.

$$Width = \frac{58}{3}$$
$$Length = 3$$

List the length and width

or

$$l = 61 - 3w$$
$$l = 61 - 3(1)$$
$$l = 61 - 3$$
$$l = 58$$

$$Width = 58$$
$$Length = 1$$

List the length and width