C11 - 3.1 - Quadratics Graphing x^2 TOV Notes

Graphing: $y = x^2$

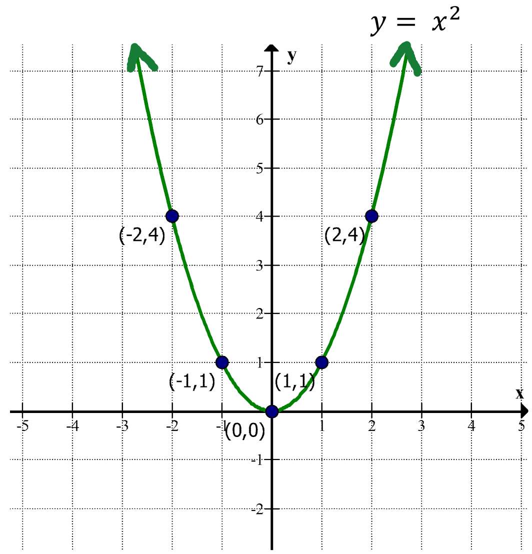


Table of Values

Vertex:

x	y	Pt.
-2	4	(-2,4)
-1	1	(-1,1)
0	0	(0,0)
1	1	(1,1)
2	4	(2,4)

$y = x^2$
$y = (-2)^2$
$y = 4$

$y = x^2$
$y = (-1)^2$
$y = 1$

$y = x^2$
$y = (0)^2$
$y = 0$

$y = x^2$
$y = (1)^2$
$y = 1$

$y = x^2$
$y = (2)^2$
$y = 4$

Notice: the pattern from the vertex (0,0) is **symmetrical** on both sides.

Over 1, 1 squared = 1, up 1. Back to the vertex. Over 2, 2 squared = 4, up 4.

$V: (0,0)$

$AOS: x = 0$ $\quad$ $R: y \geq 0$ $\quad$ Domain xER

Range

C11 - 3.1 - Quadratic Vertical Translation Notes $y = x^2 + q$

Graphing: $y = x^2 + c$

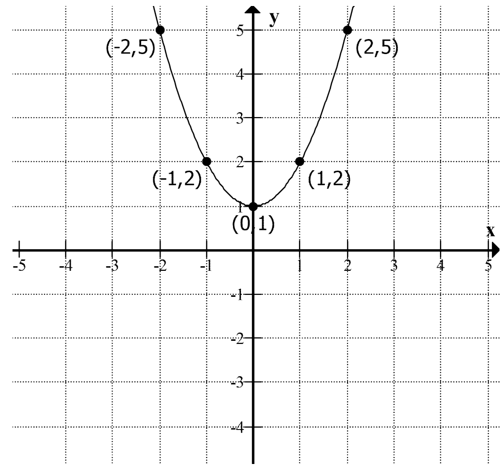
$y = x^2 + 1$

Table of Values

x	y
-2	5
-1	2
0	1
1	2
2	5

Pt.
(-2,5)
(-1,2)
(0,1)
(1,2)
(2,5)

$y = x^2 + 1$



$y = x^2 + 1$
$y = (-2)^2+1$
$y = 4 + 1$
$y = 5$

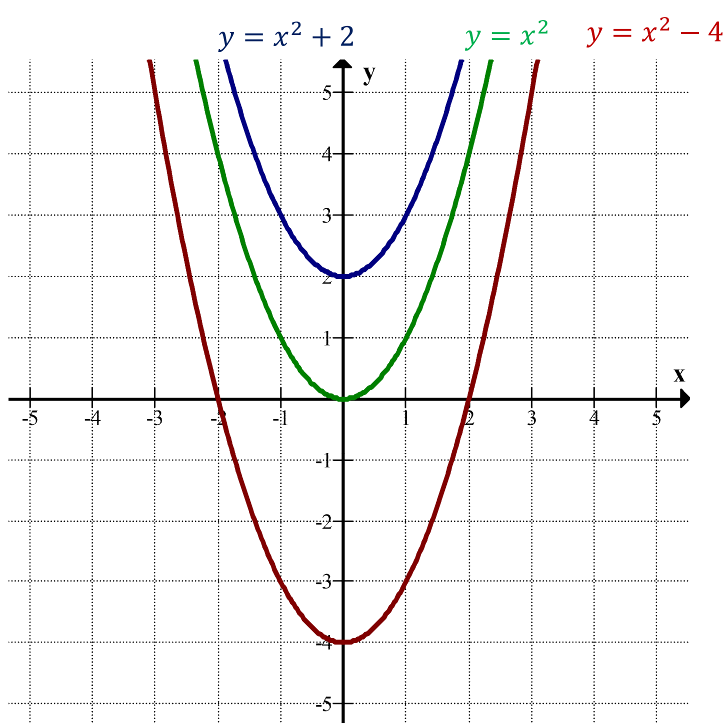
$y = x^2 + 1$
$y = (-1)^2+1$
$y = 1 + 1$
$y = 2$

$y = x^2 + 1$
$y = (0)^2+1$
$y = 0 + 1$
$y = 5$

$y = x^2 + 1$
$y = (1)^2+1$
$y = 1 + 1$
$y = 2$

$y = x^2 + 1$
$y = (2)^2+1$
$y = 4 + 1$
$y = 5$

Notice: the graph of $y = x^2 + 1$ is the graph $y = x^2$ shifted up 1. "c" is the y intercept. "c" is only the vertex if there is no "b".



C11 - 3.1 - Quadratics Horizontal Translation Notes $(x-p)^2$

Graphing: $y = (x-p)^2$

$y = (x-2)^2$

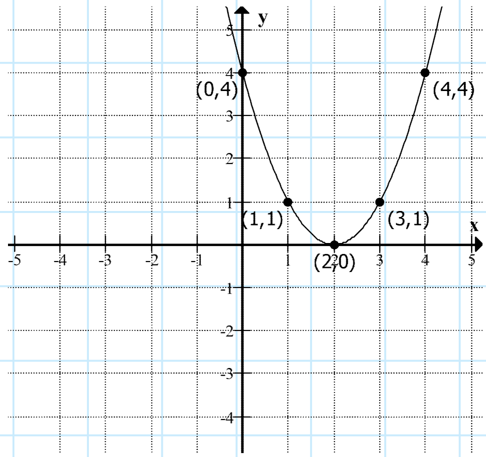


Table of Values

x	y
0	4
1	1
2	0
3	1
4	4

Pt.
(0,4)
(1,1)
(2,0)
(3,1)
(4,4)

$y = (x-2)^2$
$y = ((0)-2)^2$
$y = (0-2)^2$
$y = (-2)^2$
$y = 4$

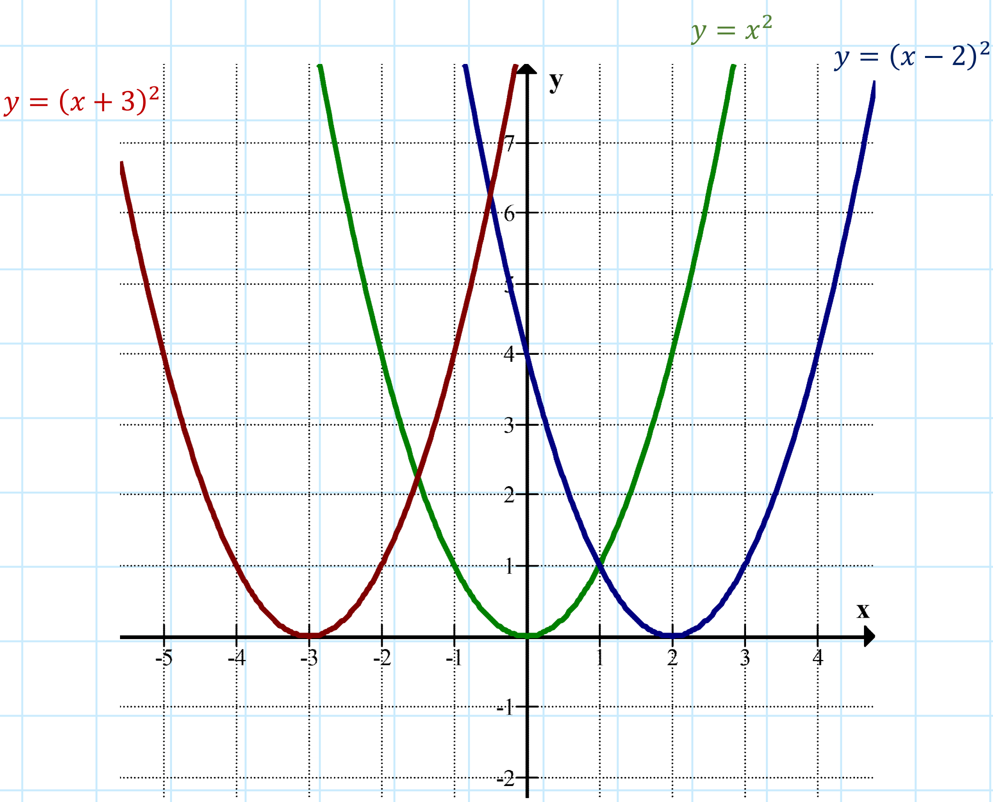
$y = (x-2)^2$
$y = ((1)-2)^2$
$y = (1-2)^2$
$y = (-1)^2$
$y = 1$

$y = (x-2)^2$
$y = ((2)-2)^2$
$y = (2-2)^2$
$y = (0)^2$
$y = 0$

$y = (x-2)^2$
$y = ((3)-2)^2$
$y = (3-2)^2$
$y = (-1)^2$
$y = 1$

$y = (x-2)^2$
$y = ((4)-2)^2$
$y = (4-2)^2$
$y = (2)^2$
$y = 4$

Notice: the graph of $y = (x-p)^2$ is the graph $y = x^2$ shifted right 2.
Notice we shift the opposite of "p".



C11 - 3.1 - Quadratics Reflection Notes $-x^2$

Graphing: $y = -x^2$

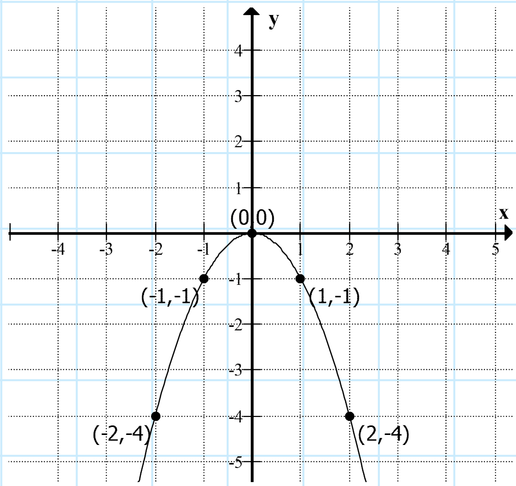
$y = -x^2$

Table of Values

x	y
-2	-4
-1	-1
0	0
1	-1
2	-4

Pt.
(-2,-4)
(-1,-1)
(0,0)
(1,-1)
(2,-4)

$y = -x^2$



$y = -x^2$
$y = -(-2)^2$
$y = -4$

$y = -x^2$
$y = -(-1)^2$
$y = -1$

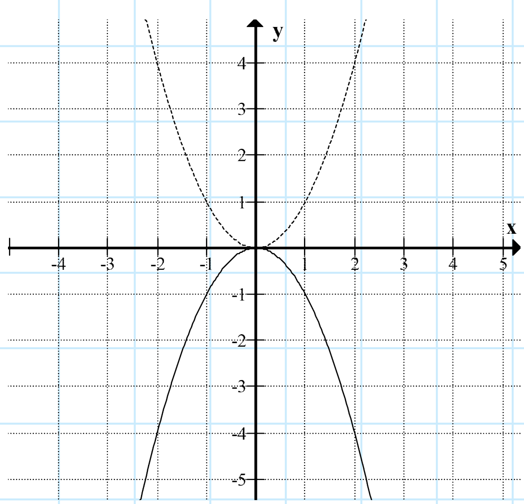
$y = -x^2$
$y = -(0)^2$
$y = -4$

$y = -x^2$
$y = -(1)^2$
$y = -1$

$y = -x^2$
$y = -(2)^2$
$y = -4$

Notice: The graph of $y = -x^2$ is the graph of $y = x^2$ opening downwards.
Over 1, 1 squared = 1, down 1. Back to the vertex. Over 2, 2 squared = 4, down 4.

$y = x^2$



$y = -x^2$

C11 - 3.2 - Quadratics Vertical Exp Notes ($2x^2, \frac{1}{2}x^2$)

Graphing: $y = ax^2$

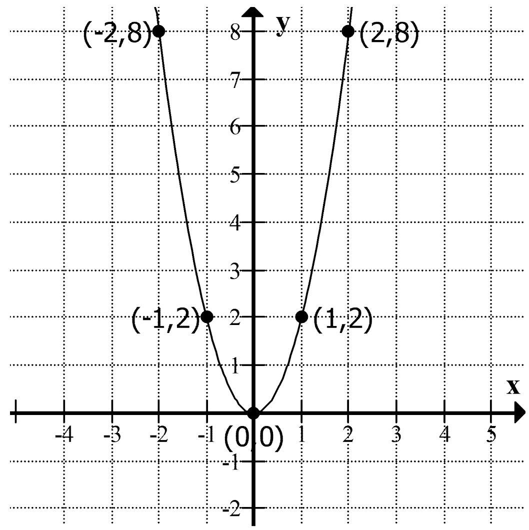
$\boldsymbol{y = 2x^2}$

Table of Values

x	y
-2	8
-1	2
0	0
1	2
2	8

Pt.
$(-2,8)$
$(-1,2)$
$(0,0)$
$(1,2)$
$(2,8)$

$y = 2x^2$



$y = 2x^2$
$y = 2(-2)^2$
$y = 2(4)$
$y = 8$

$y = 2x^2$
$y = 2(-1)^2$
$y = 2(1)$
$y = 2$

$y = 2x^2$
$y = 2(0)^2$
$y = 2(0)$
$y = 0$

$y = 2x^2$
$y = 2(1)^2$
$y = 2(1)$
$y = 2$

$y = 2x^2$
$y = 2(2)^2$
$y = 2(4)$
$y = 8$

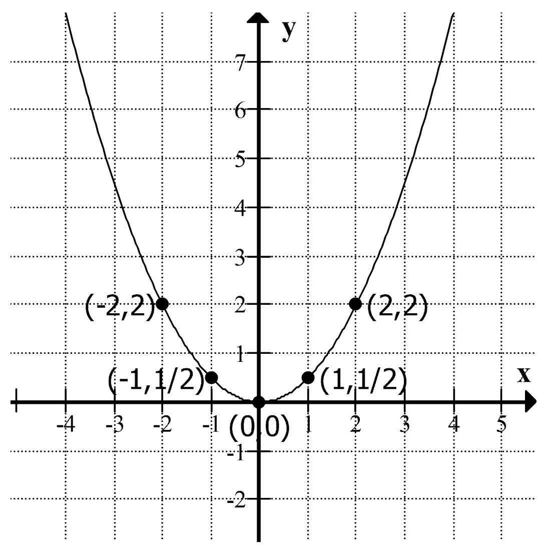
Notice: the pattern from the vertex (0,0) is symmetrical on both sides.
Over 1, 1 squared = 1, 1 times 2 = 2, up 2. Back to the vertex. Over 2, 2 squared = 4, 4 times 2 = 8, up 8.
In the last two steps, we are multiplying by 2 because $a = 2$.

$\boldsymbol{y = \frac{1}{2}x^2}$

Table of Values

x	y
-2	2
-1	$\frac{1}{2}$
0	0
1	$\frac{1}{2}$
2	2

Pt.
$(-2,2)$
$(-1,\frac{1}{2})$
$(0,0)$
$(1,\frac{1}{2})$
$(2,2)$



$y = \frac{1}{2}x^2$
$y = \frac{1}{2}(-2)^2$
$y = \frac{1}{2}(4)$
$y = 2$

$y = \frac{1}{2}x^2$
$y = \frac{1}{2}(-1)^2$
$y = \frac{1}{2}(1)$
$y = \frac{1}{2}$

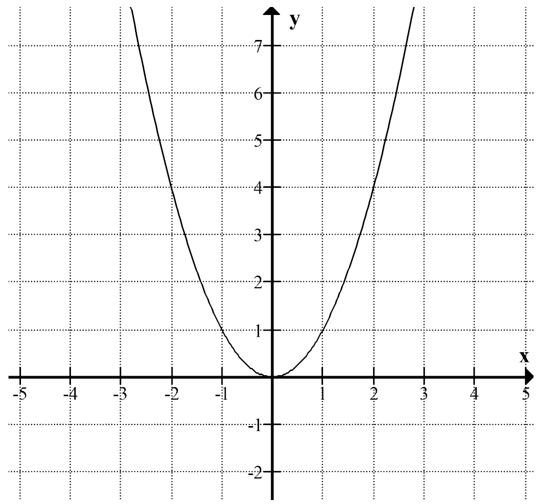
$y = \frac{1}{2}x^2$
$y = \frac{1}{2}(0)^2$
$y = \frac{1}{2}(0)$
$y = 0$

$y = \frac{1}{2}x^2$
$y = \frac{1}{2}(1)^2$
$y = \frac{1}{2}(1)$
$y = \frac{1}{2}$

$y = \frac{1}{2}x^2$
$y = \frac{1}{2}(2)^2$
$y = \frac{1}{2}(4)$
$y = 2$

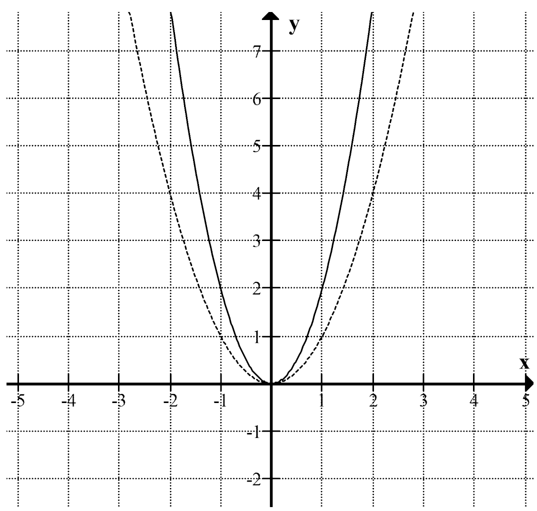
C11 - 3.2 - Quadratics Compression/Expansion Summary

$y = x^2$



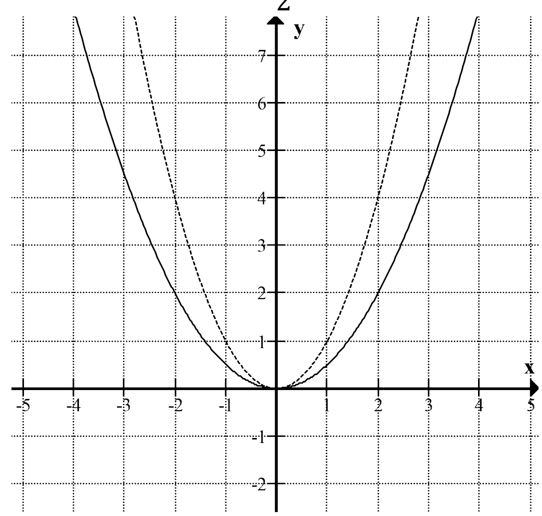
Expand

$y = 2x^2$

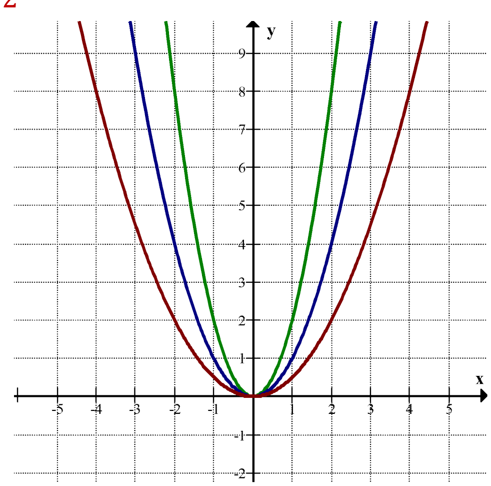


Compress

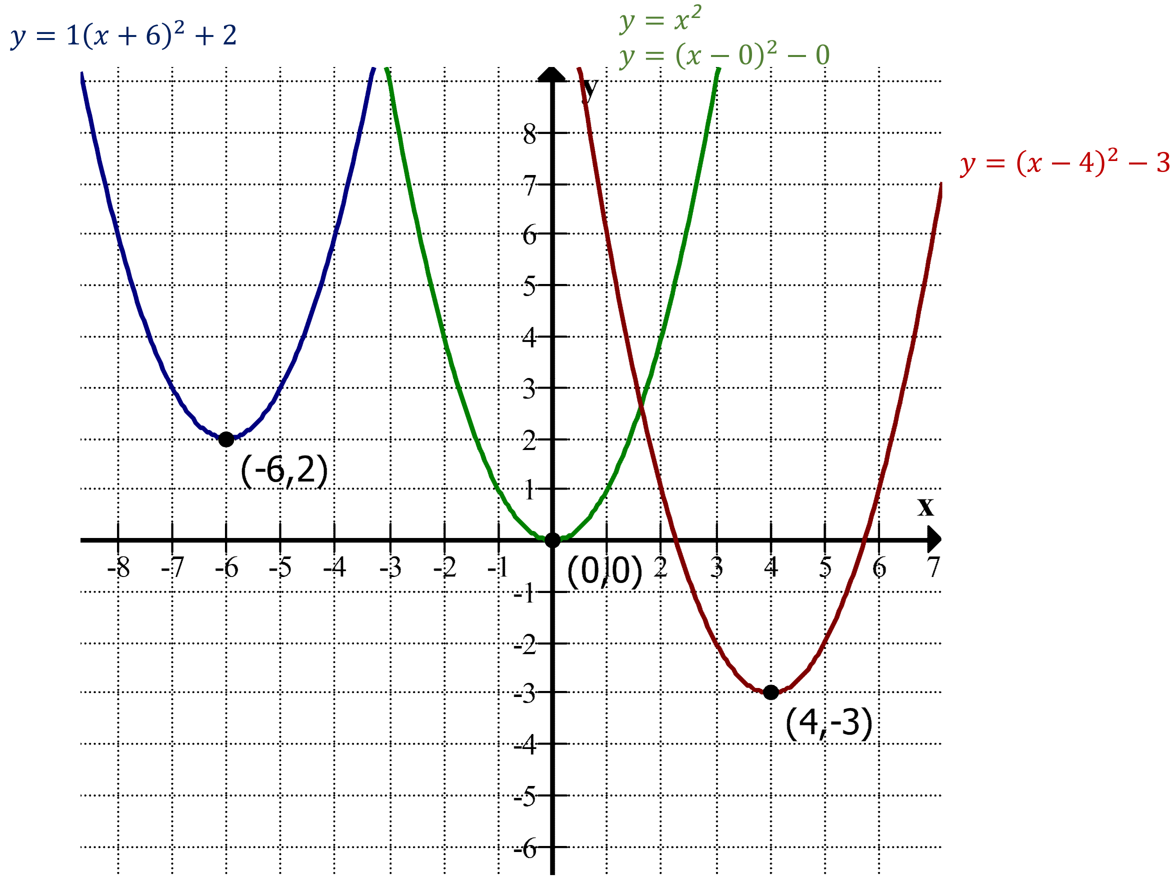
$y = \frac{1}{2}x^2$

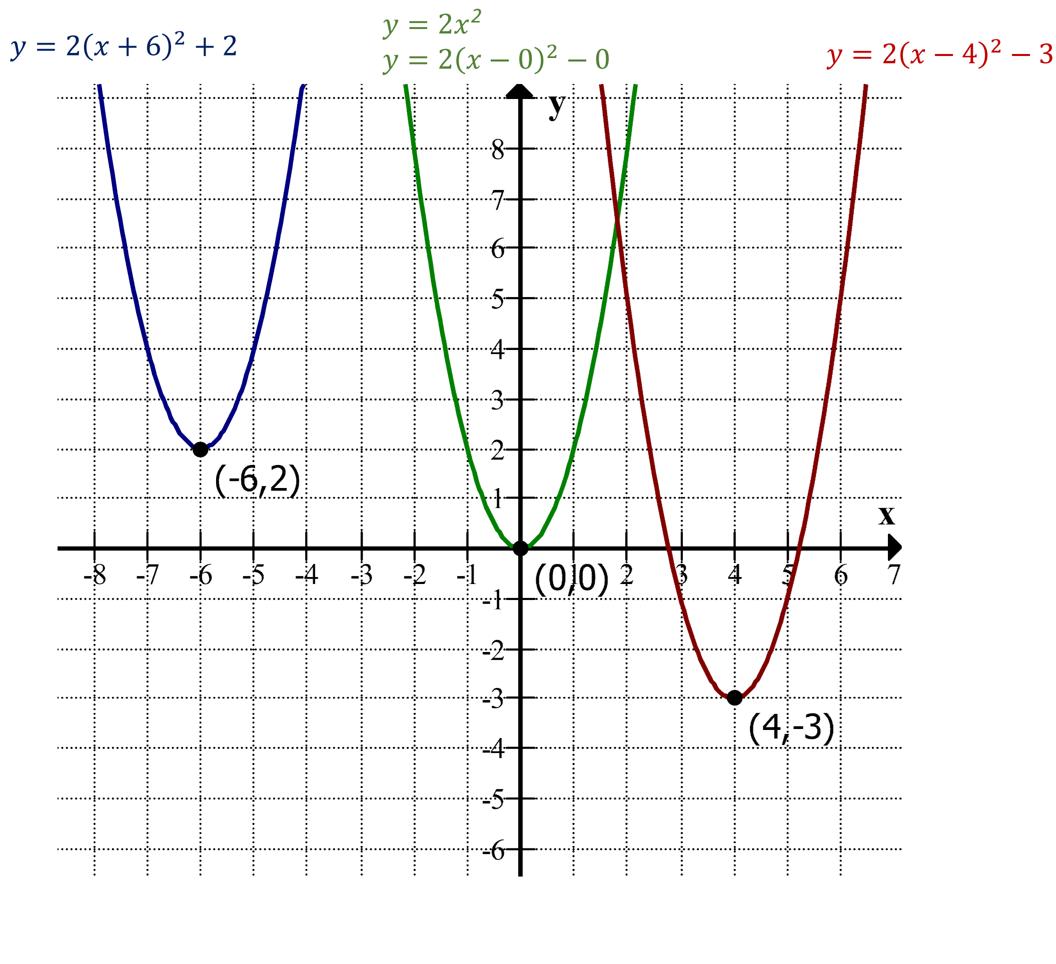


$y = \frac{1}{2}x^2$ $y = 2x^2$ $y = x^2$



C11 - 3.2 - Quadratics Vertical/Horizontal Combo Notes





C11 - 3.3 - Completing the Square Notes

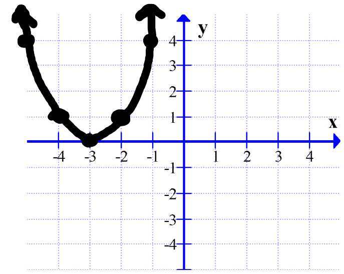
Standard form → *Vertex form*

$y = ax^2 + bx + c$ → $y = a(x-p)^2 + q$ $Vertex = (p, q)$

$y = x^2 + 6x + c$

$$\boxed{\left(\frac{b}{2}\right)^2 = \left(\frac{6}{2}\right)^2 = (3)^2 = 9}$$

"b" divided by 2 all squared:



$y = x^2 + 6x + 9$

Factor

$y = (x+3)(x+3)$
$y = (x+3)^2$

Vertex form: Vertex = $(-3, 0)$

---

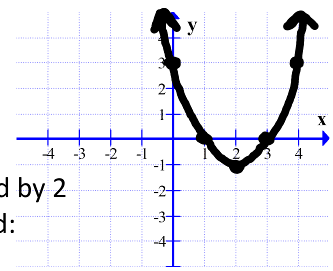
a = 1

$y = x^2 - 4x + 3$
$y = (x^2 - 4x) + 3$

Group x terms

$$\boxed{\left(\frac{b}{2}\right)^2 = \left(\frac{-4}{2}\right)^2 = (-2)^2 = 4}$$

"b" divided by 2 all squared:



$y = (x^2 - 4x + 4 - 4) + 3$ Add and subtract inside brackets

$y = (x^2 - 4x + 4) - 4 + 3$ Remove number not contributing to perfect square (-ve)

$y = (x-2)(x-2) - 1$ Factor brackets, simplify outside

$y = (x-2)^2 - 1$ Vertex form: Vertex = $(2, -1)$

$a \neq 1$

$y = 2x^2 - 8x + 3$
$y = (2x^2 - 8x) + 3$ *Group x terms*

$y = 2(x^2 - 4x) + 3$ Factor out coefficient of x^2

$$\boxed{\left(\frac{b}{2}\right)^2 = \left(\frac{-4}{2}\right)^2 = (-2)^2 = 4}$$

New "x" coefficient divided by 2 all squared:

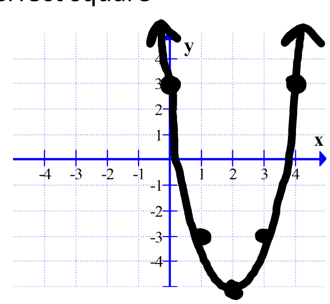
x2

$y = 2(x^2 - 4x + 4 - 4) + 3$ Add and subtract inside brackets

$y = 2(x^2 - 4x + 4) - 8 + 3$ Remove number not contributing to perfect square
Don't forget to multiply by "a"

$y = 2(x-2)(x-2) - 5$ Factor brackets, simplify outside

$y = 2(x-2)^2 - 5$ Vertex form: Vertex = $(2, -5)$



Remember: $\frac{b}{2a}$ or $\frac{\text{"new } b\text{"}}{2}$ is the number that goes inside the brackets with x

C11 - 3.4 - Find Vertex Form Vertex Point Notes

Using the vertex and a point on the parabola, find the equation in Vertex Form.

$\boldsymbol{Vertex}: (\mathbf{-1}, \mathbf{-4})$ $\boldsymbol{and}$ $\boldsymbol{Point}: (\mathbf{2}, \mathbf{-3})$

$y = a(x-p)^2 + q$

$y = a\left(x-(-1)\right)^2 - 4$

$y = a(x+1)^2 - 4$

Write Vertex Form

Substitute Vertex for (p, q)

$(\mathbf{-1}, \mathbf{-4})$

$\boxed{y = a(x-p)^2 + q}$

$-3 = a(-2+1)^2 - 4$

$-3 = a(1)^2 - 4$

$-3 = 1a - 4$

$+4 \qquad +4$

$1 = 1a$

$\frac{1}{1} = \frac{1a}{1}$

$1 = a$

$a = 1$

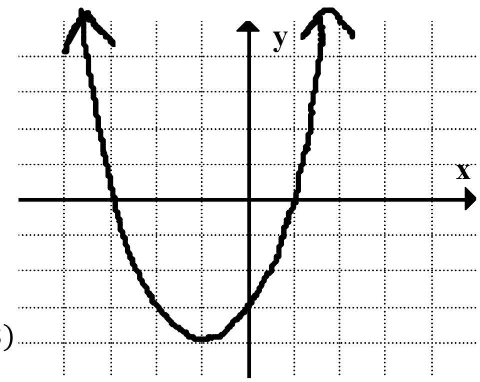
Substitute (x, y)

$(\mathbf{-2}, \mathbf{-3})$

Draw on Graph

(x, y)

$(\mathbf{-2}, \mathbf{-3})$



(p, q)

$(\mathbf{-1}, \mathbf{-4})$

Solve for a.

$y = 1(x+1)^2 - 4$

Substitute 'a' and Vertex into Vertex Form

$\boldsymbol{Vertex}: (\mathbf{3}, \mathbf{-2})$ $\boldsymbol{and}$ $\boldsymbol{x - intercept} = \mathbf{4}$ $(4,0)$

$y = a(x-p)^2 + q$

$y = a\left(x-(3)\right)^2 - 2$

$y = a(x-3)^2 - 2$

$0 = a(4-3)^2 - 2$

$0 = a(1)^2 - 2$

$0 = 1a - 2$

$+2 \qquad +2$

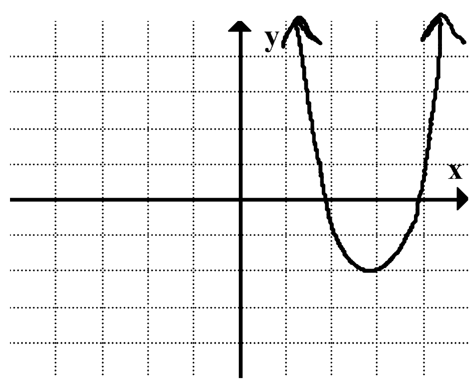
$2 = a$

$a = 2$

$y = 2(x-3)^2 - 2$

Draw on Graph

Check on Graphing Calculator Table of Values



(x, y)

$(\mathbf{0}, \mathbf{0})$

(p, q)

$(\mathbf{3}, \mathbf{-2})$

C11 - 3.5 - Vertex: $\left(-\frac{b}{2a}, y\right)$ Quadratics in Standard Form Notes

$y = x^2 - 6x + 5$

$$Vertex = \left(\frac{-b}{2a}, y\right)$$
$$Vertex = \left(\frac{-(-6)}{2(1)}, y\right)$$
$$Vertex = \left(\frac{6}{2}, y\right)$$
$$Vertex = (3, y)$$

$$\boxed{Vertex = \left(\frac{-b}{2a}, y\right)}$$

$$\boxed{\left(\frac{-b}{2a}, c - \frac{b^2}{4a}\right)}$$

$y = x^2 - 6x + 5$
$y = (3)^2 - 6(3) + 5$
$y = 9 - 18 + 5$
$y = -4$

Substitute 3 in for x and solve for y

$Vertex = (3, -4)$

$y = x^2 - 6x + 5$

$Vertex = (3, -4)$

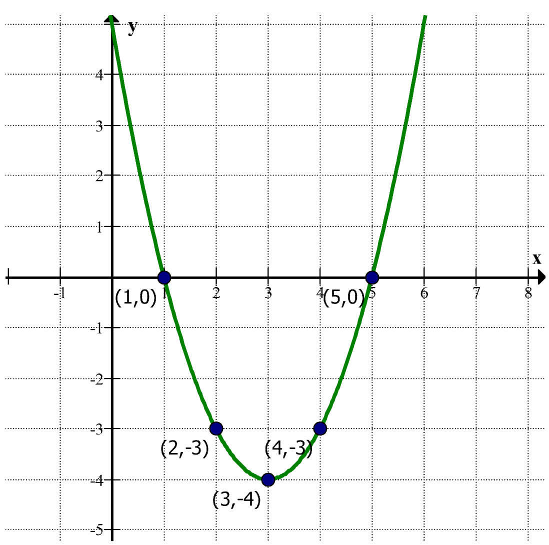
	x	y
	1	0
	2	-3
Vertex:	3	-4
	4	-3
	5	0

$y = x^2 - 6x + 5$
$y = (1)^2 - 6(1) + 5$
$y = 1 - 6 + 5$
$y = 0$

$y = x^2 - 6x + 5$
$y = (2)^2 - 6(2) + 5$
$y = 4 - 12 + 5$
$y = -3$

$y = x^2 - 6x + 5$
$y = (4)^2 - 6(4) + 5$
$y = 16 - 24 + 5$
$y = -3$

$y = x^2 - 6x + 5$
$y = (5)^2 - 6(5) + 5$
$y = 25 - 30 + 5$
$y = 0$



AOS: Average Two Horizontal Points ($x - int's$)

$$x = \frac{1+5}{2}$$
$$x = 3$$

C11 - 3.6 - Product of Numbers is a Min Notes

The difference between two numbers is 10. Their product is a minimum.

Let $a = 1st\ \#$
Let $b = 2nd\ \#$

Let statements: get used to using variables other than x and y

1 $a - b = 10$

2 $a \times b = minimum$
$a \times b = \cancel{minimum}\quad y$
$y = a \times b$

Equation 1, equation 2. The minimum or maximum will be y.

$a - b = 10$
$+b \qquad +b$
$a = (10 + b)$

Equation #1
Isolate a variable

$y = a \times b$
$y = (10 + b) \times b$
$y = 10b + b^2$
$y = b^2 + 10b$

Equation #2
Substitute the isolated variable

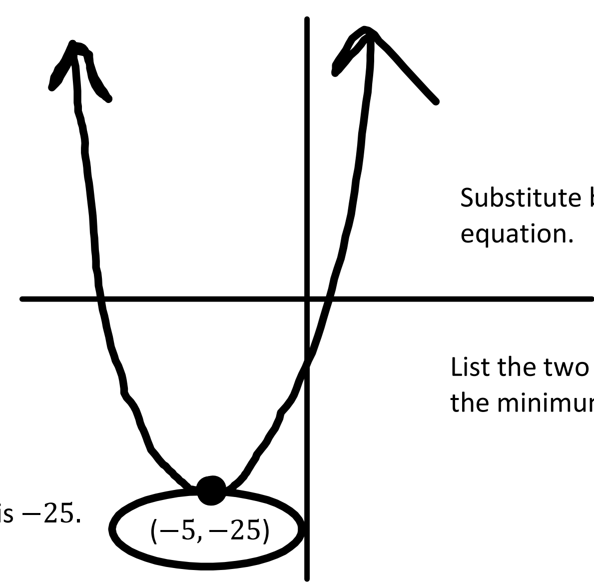
Complete the square.
$\left(\frac{b}{2}\right)^2 = \left(\frac{10}{2}\right)^2 = (5)^2 = 25$

$y = b^2 + 10b$
$y = (b^2 + 10b + 25 - 25)$
$y = (b^2 + 10b + 25) - 25$
$y = (b + 5)^2 - 25$

Vertex = $(-5, -25)$

b

Minimum



(x, y)
(b, min)

$a = 10 + b$
$a = 10 - 5$
$a = 5$

Substitute b into the other equation.

$a = 5$
$b = -5$

List the two numbers and the minimum.

The minimum product is -25.

C11 - 3.6 - Product of Numbers is a Min Notes

Two numbers differ by 10. The product of the larger number and twice the smaller number is a minimum. What are the numbers?

Let $a = 1st\ \#$
Let $b = 2nd\ \#$

Let statements:

1 $a - b = 10$

2 $a \times 2b = minimum$
$a \times 2b = \cancel{minimum}\ y$
$y = a \times 2b$

Equation 1, equation 2.
The minimum or maximum will be y.

$a - b = 10$
$a = 10 + b$

Equation #1
Isolate a variable

$y = a \times 2b$
$y = (10 + b) \times 2b$
$y = 20b + 2b^2$
$y = 2b^2 + 20b$

Equation #2
Substitute the isolated variable

×2

$y = 2b^2 + 20b$
$y = 2(b^2+10b + 25 - 25)$
$y = 2(b^2+10b + 25) - 50$
$y = 2(b + 5)^2-50$

Complete the square.
$\left(\frac{b}{2}\right)^2 = \left(\frac{10}{2}\right)^2 = (5)^2 = 25$

Vertex = (-5, -50)

b Minimum

$a = 10 + b$
$a = 10 - 5$
$a = 5$

Substitute b into the other equation.

$a = 5$
$b = -5$

List the two numbers and the minimum.

The minimum product is -50.

C11 - 3.6 - Sum of Squares is a Min Notes

Two numbers sum to 8. The sum of their squares is a minimum.

Let statements:

Let $a = 1st\ \#$
Let $b = 2nd\ \#$

Equation 1, equation 2.
The minimum or maximum will be y.

1 $a + b = 8$

2 $a^2 + b^2 = minimum$
$a^2 + b^2 = \cancel{minimum}\ y$
$y = a^2 + b^2$

Equation #1
Isolate a variable

$$
\begin{aligned}
a + b &= 8 \\
-b \quad & \; -b \\
a &= 8 - b \\
a &= (8 - b)
\end{aligned}
$$

Equation #2
Substitute the isolated variable

$$
\begin{aligned}
y &= a^2 + b^2 \\
y &= (8 - b)^2 + b^2 \\
y &= 64 - 16b + b^2 + b^2 \\
y &= 2b^2 - 16b + 64
\end{aligned}
$$

Complete the square.
$\left(\frac{b}{2}\right)^2 = \left(\frac{8}{2}\right)^2 = (4)^2 = 16$

$$
\begin{aligned}
y &= 2b^2 - 16b + 64 \\
y &= 2(b^2 - 8b) + 64 \\
y &= 2(b^2 - 8b + 16 - 16) + 64 \\
y &= 2(b^2 - 8b + 16) + 64 - 32 \\
y &= 2(b - 4)^2 + 32
\end{aligned}
$$

Vertex = (4, 32)

b

Minimum

Substitute b into the other equation.

$$
\begin{aligned}
a &= 8 - b \\
a &= 8 - (4) \\
a &= 4
\end{aligned}
$$

List the two numbers and the maximum.

$a = 4$
$b = 4$

The minimum product is 32.

C11 - 3.6 - Product of Numbers is a Max Notes

The sum of two times one number and six times another is sixty. Find the numbers if their product is a maximum.

Let $a = 1st\ \#$
Let $b = 2nd\ \#$

Let statements:

1 $2a + 6b = 60$

2
$$a \times b = maximum$$
$$a \times b = \cancel{maximum}\ y$$
$$y = a \times b$$

Equation 1, equation 2.
The minimum or maximum will be y.

$$\frac{2a}{2} + \frac{6b}{2} = \frac{60}{2}$$
$$a + 3b = 30$$
$$a = 30 - 3b$$

Equation #1
Isolate a variable

$$y = a \times b$$
$$y = (30 - 3b) \times b$$
$$y = 30b - 3b^2$$
$$y = -3b^2 + 30b$$

Equation #2
Substitute the isolated variable

$$y = -3b^2 + 30b$$
$$y = -3(b^2 - 10b + 25 - 25)$$
$$y = -3(b^2 - 10b + 25) + 75$$
$$y = -3(b - 5)^2 + 75$$

Complete the square.
$\left(\frac{b}{2}\right)^2 = \left(\frac{10}{2}\right)^2 = (5)^2 = 25$

Vertex = (5, 75)

b Maximum

$$a = 30 - 3b$$
$$a = 30 - 3(5)$$
$$a = 15$$

Substitute b into the other equation.

$a = 15$
$b = 5$

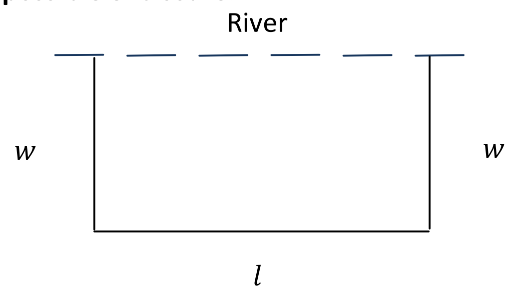
List the two numbers and the maximum.

The maximum product is 75

C11 - 3.7 - Fence w/ River Notes (p = 8m)

A rectangular enclosure is bounded on the side of a river. 3 sides total 8m of fencing. Find the dimensions of the largest possible enclosure.

Let $w = width$
Let $l = length$



Let statements:

① $2w + l = P$
$2w + l = 8$

② $A = l \times w$

Equation 1, equation 2.
The minimum or maximum will be y.

$$\begin{aligned} 2w + l &= 8 \\ -2w \quad &\quad -2w \\ l &= 8 - 2w \end{aligned}$$

Equation #1
Isolate a variable

$$\begin{aligned} A &= l \times w \\ A &= (8 - 2w) \times w \\ A &= 8w - 2w^2 \\ A &= -2w^2 + 8w \end{aligned}$$

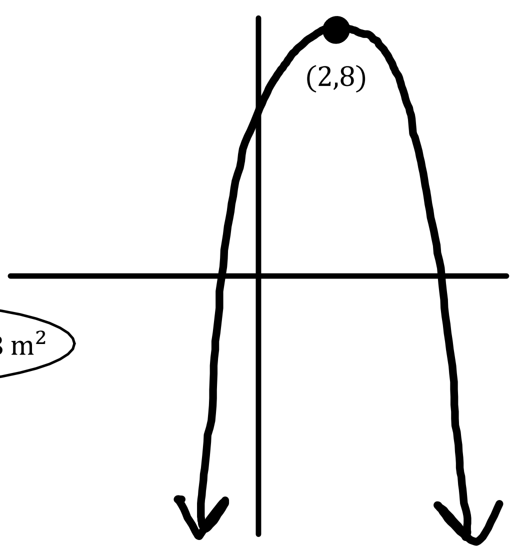
Equation #2
Substitute the isolated variable

$$\begin{aligned} A &= -2w^2 + 8w \\ A &= -2(w^2 - 8w) \\ A &= -2(w^2 - 4w + 4 - 4) \\ A &= -2(w^2 - 4w + 4) + 8 \\ A &= -2(w - 2)^2 + 8 \end{aligned}$$

Complete the square.
$\left(\frac{b}{2}\right)^2 = \left(\frac{-4}{2}\right)^2 = (-2)^2 = 4$

$Vertex = (2,8)$

w $\quad$ $Maximum\ Area$



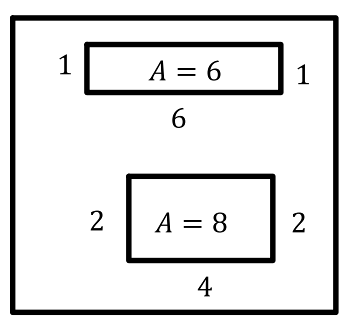
Substitute w into the other equation.

$$\begin{aligned} l &= 8 - 2w \\ l &= 8 - 2(2) \\ l &= 4 \end{aligned}$$

$width = 2$ m
$length = 4$ m

The maximum area is 8 m^2

List the length and width and the maximum area.

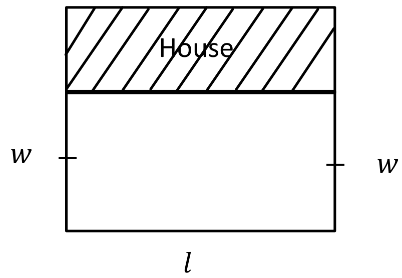


Or, factor, solve, average solutions, substitute.

C11 - 3.7 - Fence w/ River Notes (p = 60m)

Jack has 60m of fencing to build a three sided fence on the side of his house. Determine the maximum possible area of the fenced area, and the dimensions of the fence.

Let $w = width$
Let $l = length$



Let statements:

① $P = 2w + \cancel{2}l$
$60 = 2w + l$

② $\cancel{a} = l \times w$
$\cancel{max} = l \times w$
$y = l \times w$

Equation 1, equation 2. The minimum or maximum will be y.

$$
\begin{aligned}
60 &= 2w + l \\
-2w &\quad -2w \\
60 - 2w &= l \\
l &= 60 - 2w
\end{aligned}
$$

Equation #1
Isolate a variable

$$
\begin{aligned}
y &= l \times w \\
y &= (60 - 2w)w \\
y &= 60w - 2w^2 \\
y &= -2w^2 + 60w
\end{aligned}
$$

Equation #2
Substitute the isolated variable

$$
\begin{aligned}
y &= -2(w^2 + 30w) \\
y &= -2(w^2 - 30w + 225 - 225) \\
y &= -2(w^2 - 30w + 225) + 450 \\
y &= -2(w - 15)^2 + 450
\end{aligned}
$$

Complete the square.
$\left(\frac{b}{2}\right)^2 = \left(\frac{10}{2}\right)^2 = (5)^2 = 25$

$Vertex = (15{,}450)$

w $\quad$ $Maximum$

$$
\begin{aligned}
l &= 60 - 2w \\
l &= 60 - 2(15) \\
l &= 60 - 30 \\
l &= 30
\end{aligned}
$$

Substitute w into the other equation.

$width = 15\text{m}$
$length = 30\text{ m}$

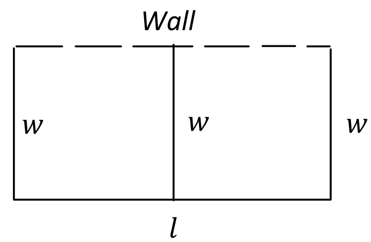
The maximum area is 450 m^2

List the length and width and the maximum area.

C11 - 3.7 - Fence w/ wall Split in Two

A rectangular fence that is split in half is against a wall. The total fencing length is 42 m. What is the max area of the fence?

Let $w = width$
Let $l = length$



Let statements:

$$F = l + 3w$$

$$A = l \times w$$
$$max = l \times w$$
$$y = l \times w$$

Equation 1, equation 2.
The minimum or maximum will be y.

$$P = l + 3w$$
$$42 = l + 3w$$
$$-3w \quad -3w$$
$$42 - 3w = l$$
$$l = 42 - 3w$$

Equation #1
Isolate a variable

$$A = l \times w$$
$$y = (42 - 3w) \times w$$
$$y = 42w - 3w^2$$
$$y = -3w^2 + 42w$$
$$y = -3(w^2 - 14w)$$
$$y = -3(w^2 - 14w + 49 - 49)$$
$$y = -3(w^2 - 14w + 49) + 147$$
$$y = -3(w - 7)^2 + 147$$

Equation #2
Substitute the isolated variable

Complete the square.
$\left(\frac{b}{2}\right)^2 = \left(\frac{-14}{2}\right)^2 = (7)^2 = 49$

$Vertex: (7,147)$

w $\quad$ $Maximum$

The maximum is the y value.

$$l = 42 - 3w$$
$$l = 42 - 3(7)$$
$$l = 21$$

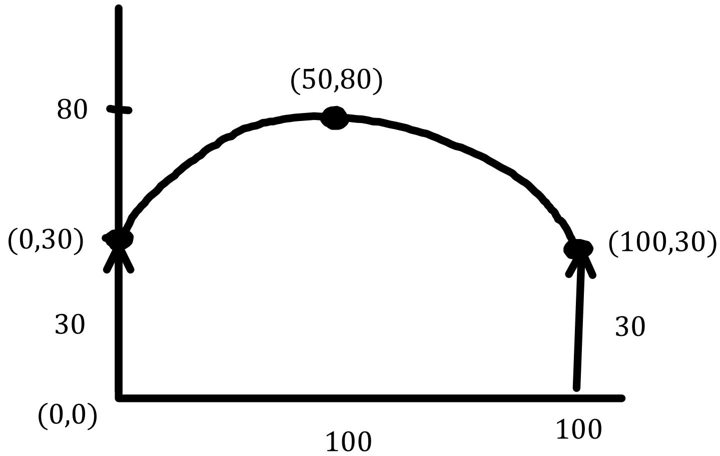
$length = 21m$
$width = 7m$

List the length and width and the maximum area.

$Max\ area = 147\ m^2$

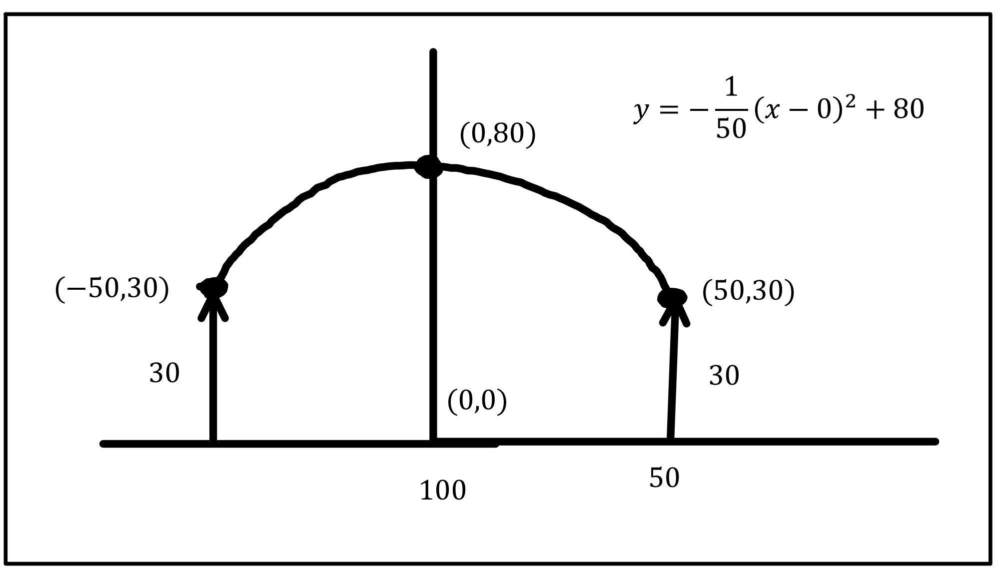
C11 - 3.8 - Bridge Find Equation Notes

A bridge has pillars 30 m tall and are 100 m apart. The maximum at the center of the bridge is 80 m tall. Find the equation of the parabolic bridge. What is the height 5 m away from each pillar.



$$y = a(x-p)^2 + q$$
$$y = a(x-50)^2 + 80$$
$$30 = a(0-50)^2 + 80$$
$$30 = a(50)^2 + 80$$
$$-80 \qquad\qquad -80$$
$$-\frac{50}{2500} = \frac{2500a}{-2500}$$
$$a = -\frac{1}{50}$$

$$y = -\frac{1}{50}(x-50)^2 + 80$$



$$y = -\frac{1}{50}(x-0)^2 + 80$$

C11 - 3.9 - Set Up Maximize Candy Sales Notes

A student sells candy to all of his friends. Each candy costs 6 dollars, and he has 10 friends who buy the candy each day. Every time he increases the cost by 1 dollar, 1 of his friends decides not to buy the candy. Set up how this question will look.

$Let\ p = price$
$Let\ q = quantity$
$Let\ r = revenue$
$\boxed{Let\ x = \#\ of\ price\ increases}$

$Revenue = price \times quantity$
If $p = 6, \quad q = 10 \quad r = 6 \times 10$
$r = 60$

$p = 6 + 1x \longrightarrow$ Raising the price by 1 dollar x times.

$q = 10 - 1x \longrightarrow$ Each x times he raises the price, 1 less friend will buy the candy.

$r = p \times q$
$r = (6 + 1x) \times (10 - 1x)$

Price

x	p
-2	4
-1	5
0	6
1	7
2	8

Starting Price and Quantity (zero price increase)

Quantity

x	q
-2	12
-1	11
0	10
1	9
2	8

C11 - 3.9 - Maximize Candy Sales Notes

A student sells candy to all of his friends. Each candy costs 6 dollars, and he has 10 friends who buy the candy each day. Every time he increases the cost by 1 dollar, 1 of his friends decides not to buy the candy. What is the price that will maximize revenue?

$Let\ p = price$
$Let\ q = quantity$
$Let\ r = revenue$
$\boxed{Let\ x = \#\ of\ price\ increases}$

$Revenue = price \times quantity$	$r = p \times q$
If $p = 6, \quad q = 10$	$r = 6 \times 10$
$r = 60$	$r = \$60$

$p = 6 + 1x$ → If he decides to raise the price by 1 dollar x times.

$q = 10 - 1x$ → One less friend will buy the candy each time he increases the price.

$$r = p \times q$$
$$r = (6 + x)(10 - x)$$
$$r = 60 - 6x + 10x - x^2$$
$$r = 60 + 4x - x^2$$
$$r = -x^2 + 4x + 60$$
$$r = -(x^2 - 4x) + 60 \quad \times(-1)$$
$$r = -(x^2 - 4x + 4 - 4) + 60$$
$$r = -(x^2 - 4x + 4) + 60 + 4$$
$$r = -(x - 2)^2 + 64$$

Complete the square.

$$\left(\frac{b}{2}\right)^2 = \left(-\frac{4}{2}\right)^2 = (-2)^2 = 4$$

$y = \max\ revenue = \$64$

Vertex: $(2, 64)$

$x = 2\ price\ increases$

$$p = 6 + 1x$$
$$p = 6 + 1(2)$$
$$p = 6 + 2$$
$$p = 8$$
$$\boxed{price = 8}$$

$$q = 10 - 1x$$
$$q = 10 - 1(2)$$
$$q = 10 - 2$$
$$q = 8$$
$$\boxed{quantity = 8}$$

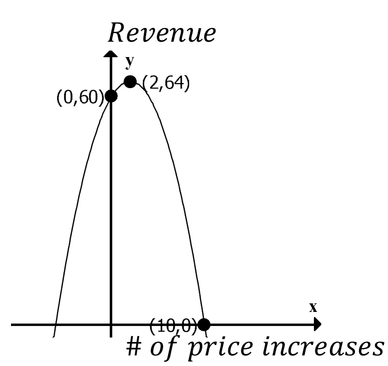
Check with Table of Values

Price	Quantity	(x)	Revenue (y)
6	10	0	60
7	9	1	63
8	8	2	64
9	7	3	63
10	6	4	60
11	5	5	55

1st increase

2nd increase

Max revenue



C11 - 3.9 - Maximize Car Sales Notes

A car salesman sells a car for $4000, with 20 people buying the car. For every $200 he takes off the price, 2 more people buy a car. What is the price that will maximize revenue?

$Let\ p = price$
$Let\ q = quantity$
$Let\ r = revenue$
$\boxed{Let\ x = \#\ of\ price\ decreases}$

$Revenue = price \times quantity$
$If\ p = \$4000, \quad q = 20$
$r = \$80{,}000$

If they sell 20 cars at $4000, revenue is $80,000.

$p = 4000 - 200x$ → If he decides to decrease the price by $200 x times.

$q = 20 + 2x$ → Two more people will buy the car each time he decreases the price.

$$r = p \times q$$
$$r = (4000 - 200x)(20 + 2x)$$
$$r = 80000 + 8000x - 4000x - 400x^2$$
$$r = -400x^2 + 4000x + 80000$$
$$r = -400(x^2 - 10x) + 80000$$
$$r = -400(x^2 - 10x + 25 - 25) + 80000$$
$$r = -400(x^2 - 10x - 25) + 80000 + 10000$$
$$r = -400(x - 5)^2 + 90000$$

Complete the square.
$$\left(\frac{b}{2}\right)^2 = \left(-\frac{10}{2}\right)^2 = (-5)^2 = 25$$

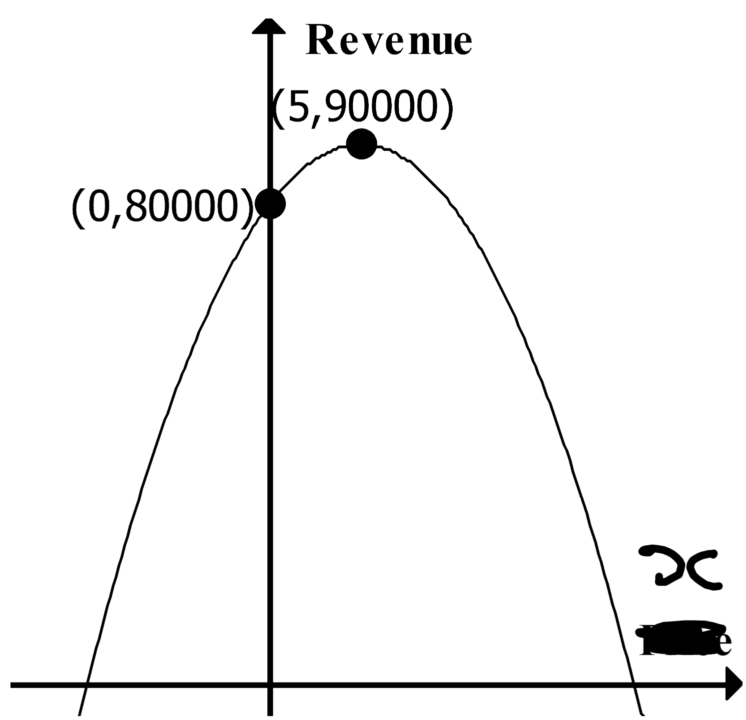
Vertex: $(5, 90000)$

$x = 5\ price\ decreases$

$\mathrm{y = max}\ \ revenue = \$90000$

$$p = 4000 - 200x$$
$$p = 4000 - 200(5)$$
$$p = 4000 - 1000$$
$$p = 3000$$
$$\boxed{price = \$3000}$$

$$q = 20 + 2x$$
$$q = 20 + 2(5)$$
$$q = 20 + 10$$
$$q = 30$$
$$\boxed{quantity = 30\ people}$$



C11 - 3.9 - Maximize Car Sales Notes (No Price Increases)

A car salesman sells a car for \$2000, with 20 people buying the car. For every \$200 he takes off the price, 2 more people buy a car. What is the price that will maximize revenue?

$Let\ p = price$
$Let\ q = quantity$
$Let\ r = revenue$
$\boxed{Let\ x = \#\ of\ price\ decreases}$

$Revenue = price \times quantity$
$If\ p = \$2000, \quad q = 20$
$r = \$40{,}000$

If they sell 20 cars at \$8000, revenue is \$40,000.

$p = 2000$
$p = 2000 - 200x$

If he decides to decrease the price by \$200 x times.

$q = 20$
$q = 20 + 2x$

Two more people will buy the car each time he decreases the price.

$r = p \times q$
$r = (2000 - 200x)(20 + 2x)$
$r = 40000 + 4000x - 4000x - 400x^2$
$r = -400x^2 + 40000$
$r = -400(x + 0)^2 + 40000$

Vertex: $(0, 40000)$

$x = 0\ price\ decreases$

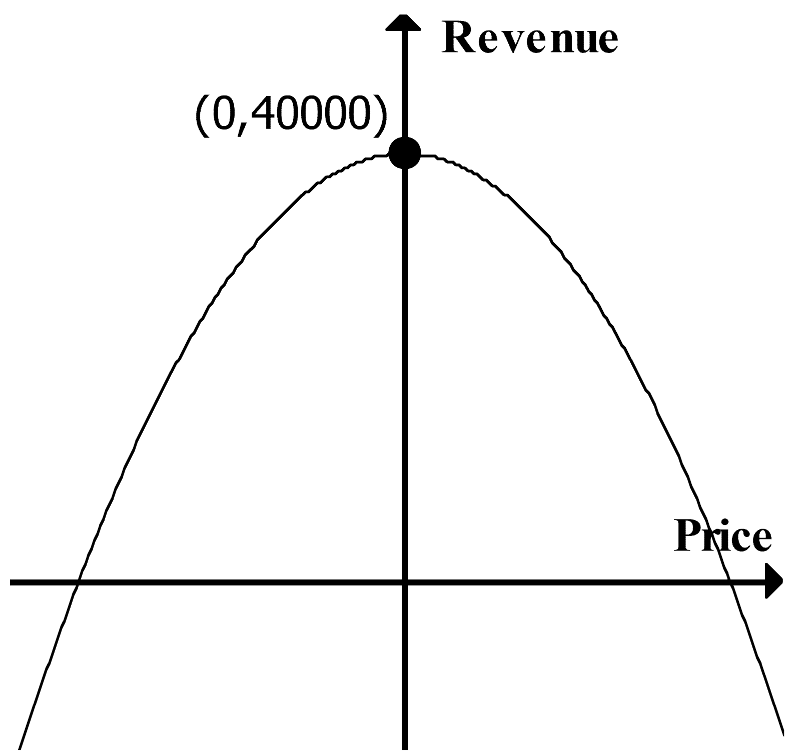
$\mathrm{y = max}\ \ revenue = \$30000$

$p = 2000 - 200x$
$p = 2000 - 200(0)$
$p = 2000$

$\boxed{price = \$2000}$

$q = 20 + 2x$
$q = 20 + 2(0)$
$q = 20 - 0$
$q = 20$

$\boxed{quantity = 20\ people}$



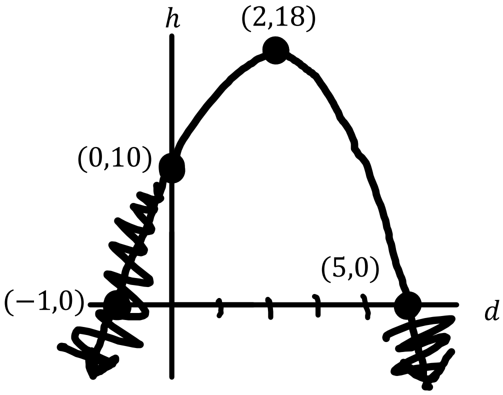
C11 - 3.10 - Max Height/Total Distance

Or 2nd Calc

The height vs distance of a bow and arrow shot off a cliff is represented by following equation:

$$h = -2d^2 + 8d + 10$$

What is the maximum height and the distance it took to get there? Draw on a graph.



$h = -2d^2 + 8d + 10$

Complete the Square

$$h = -2d^2 + 8d + 10$$
$$h = (-2d^2 + 8d) + 10$$
$$h = -2(d^2 - 4d) + 10$$
$$h = -2(d^2 - 4d + 4 - 4) + 10$$
$$h = -2(d^2 - 4d + 4) + 8 + 10$$
$$h = -2(d-2)^2 + 18$$

$$\boxed{\begin{array}{c} \left(\frac{b}{2}\right)^2 \\ \left(-\frac{4}{2}\right)^2 \\ (-2)^2 \\ 4 \end{array}}$$

$V: (2,18)$

(d, h)

$d = 2 \quad h = 18$

What was the height of the cliff?

$h - int$

$d = 0$

$$h = -2d^2 + 8d + 10$$
$$h = -2(0)^2 + 8(0) + 10$$

$h = 10$

How far did the arrow go before it hit the ground?

$h = 0$

$$h = -2(d^2 - 4d - 5)$$
$$0 = -2(d-5)(d+4)$$

Factor

$\cancel{d + 1 = 0}$ $\quad d - 5 = 0$

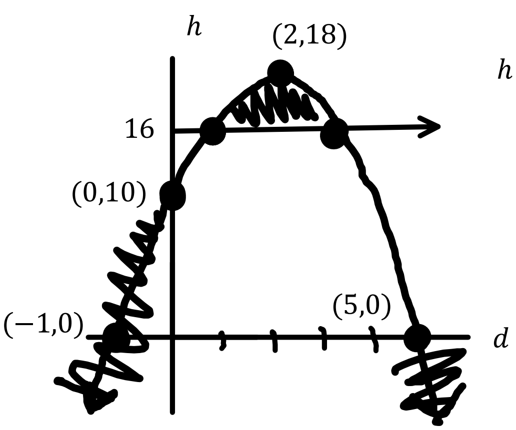
$\cancel{d = -1}$ $\quad d = 5$

Reject

Find Domain and Range

$D: [0,5]\ or\ 0 \le x \le 5 \qquad R: [0,18]\ or\ 0 \le y \le 18$

At what distance is the height 16 m (CH8)? At what distance is the height greater than 16m (CH9)?

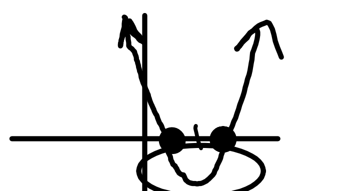


$h = 16 \qquad h = -2d^2 + 8d + 10$

$$h = -2d^2 + 8d + 10$$
$$16 = -2d^2 + 8d + 10$$
$$-16 \qquad\qquad -16$$
$$0 = -2d^2 + 8d - 6$$
$$\frac{0}{-2} = \frac{-2d^2 + 8d - 6}{-2}$$
$$0 = d^2 - 4d + 3$$
$$0 = (d-3)(d-1)$$

$d = 3 \quad d = 1$

$$-2d^2 + 8d + 10 \ge 16$$
$$-16 \quad -16$$
$$-2d^2 + 8d - 6 \ge 0$$
$$\frac{-2d^2 + 8d - 6}{-2} \ge \frac{0}{-2}$$
$$d^2 - 4d + 3 \le 0$$
$$(d-3)(d-1) \le 0$$



$0 \le d \le 3$