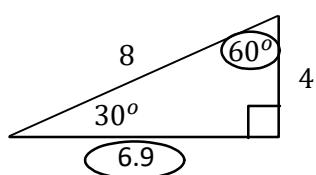
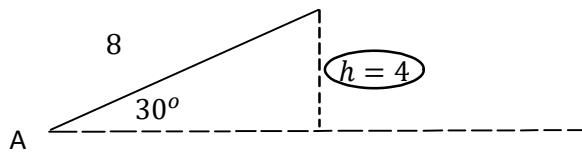


C11 - 2.12 - Ambiguous Case of Sine (ASS) Notes

How many triangles? Solve the triangles.

Remember: Always find the height first.

$$\angle A = 30^\circ, b = 8, a = 4$$

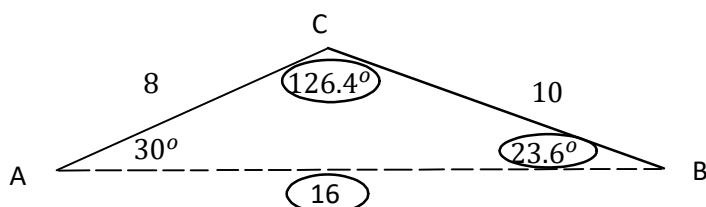


$$\begin{aligned} \sin \theta &= \frac{O}{H} & \cos \theta &= \frac{A}{H} \\ \sin 30^\circ &= \frac{h}{8} & \cos 30^\circ &= \frac{A}{8} \\ 8 \sin 30^\circ &= h & 8 \cos 30^\circ &= A \\ 4 &= h & 6.9 &= A \\ h &= 4 & A &= 6.9 \end{aligned}$$

$a = h$
One triangle

$$\begin{aligned} \theta &= 180^\circ - 30^\circ - 90^\circ \\ \theta &= 60^\circ \end{aligned}$$

$$\angle A = 30^\circ, b = 8, a = 10$$



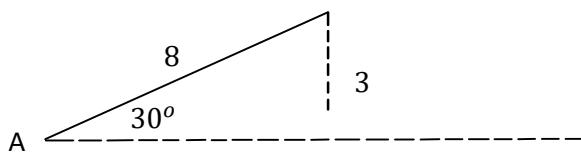
$10 > 8$
 $a > b$
One triangle

$$\begin{aligned} \frac{\sin B}{b} &= \frac{\sin A}{a} \\ \frac{\sin B}{8} &= \frac{\sin 30^\circ}{10} \\ \frac{8}{\sin B} &= \frac{10}{\sin 30^\circ} \\ \frac{8}{\sin B} &= 0.05 \\ 8 \times \frac{\sin B}{8} &= 0.05 \times 8 \\ \sin B &= 0.4 \\ B &= \sin^{-1} 0.4 \\ B &= 23.6^\circ \end{aligned}$$

$$\begin{aligned} \theta &= 180^\circ - 23.6^\circ - 30^\circ \\ \theta &= 126.4^\circ \end{aligned}$$

$$\begin{aligned} \frac{c}{\sin C} &= \frac{a}{\sin A} \\ \frac{c}{\sin 126.4^\circ} &= \frac{10}{\sin 30^\circ} \\ \frac{c}{0.8} &= 20 \\ 0.8 \times \frac{c}{0.8} &= 20 \times 0.8 \\ c &= 16 \end{aligned}$$

$$\angle A = 30^\circ, b = 8, a = 3$$



$3 < 4$
 $a < H$
no triangle

No triangle, can't solve.

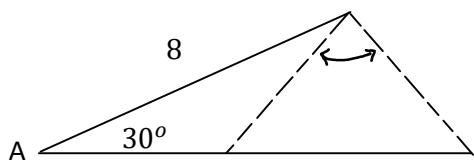
C11 - 2.12 - Ambiguous Case of Sine (ASS) Notes

How many triangles? Solve the triangles.

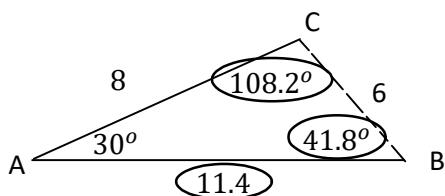
$$\angle A = 30^\circ, b = 8, a = 6$$

Remember: Always find the height first.

$4 < 6 < 8$
$H < a < B$
<u>Two triangles</u>



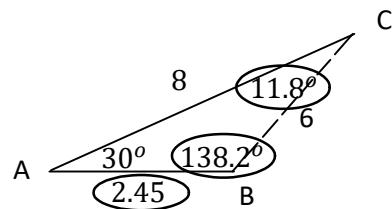
Draw both triangles together and separately.



$$\begin{aligned} \frac{\sin 30^\circ}{6} &= \frac{\sin B}{8} \\ 0.083 &= \frac{\sin B}{8} \\ 8 \times 0.083 &= \frac{\sin B}{8} \times 8 \\ 0.6 &= \sin B \\ \sin B &= 0.6 \\ B &= \sin^{-1} 0.6 \\ B &= 41.8^\circ \end{aligned}$$

$$\begin{aligned} \theta &= 180^\circ - 30^\circ - 41.8^\circ \\ \theta &= 108.2^\circ \end{aligned}$$

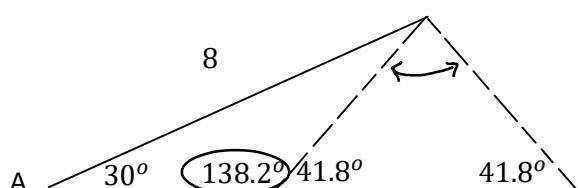
$$\begin{aligned} \frac{c}{\sin C} &= \frac{a}{\sin A} \\ \frac{c}{\sin 108.2^\circ} &= \frac{6}{\sin 30^\circ} \\ \frac{c}{0.95} &= 12 \\ 0.95 \times \frac{c}{0.95} &= 12 \times 0.95 \\ c &= 11.4 \end{aligned}$$



$$\begin{aligned} \theta &= 180^\circ - 41.8^\circ \\ \theta &= 138.2^\circ \end{aligned}$$

$$\begin{aligned} \theta &= 180^\circ - 30^\circ - 138.2^\circ \\ \theta &= 11.8^\circ \end{aligned}$$

$$\begin{aligned} \frac{c}{\sin C} &= \frac{a}{\sin A} \\ \frac{c}{\sin 11.8^\circ} &= \frac{6}{\sin 30^\circ} \\ \frac{c}{0.204} &= 12 \\ 0.204 \times \frac{c}{0.204} &= 12 \times 0.204 \\ c &= 2.45 \end{aligned}$$



Notice: Both triangles have an angle of 30° , a side going up of 8, and a side opposite to 30° of 6.

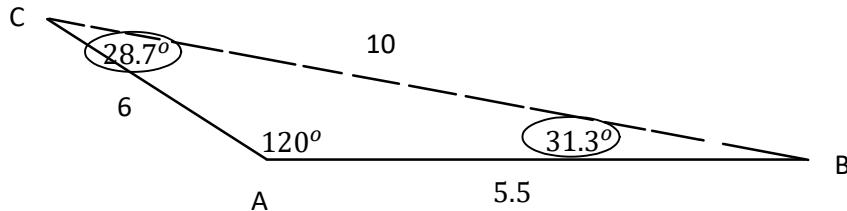
Notice: The isosceles triangle.

C11 - 2.12 - Ambiguous Case of Sine (ASS) Notes

How many triangles? Solve the triangles.

$$\angle A = 120^\circ, b = 6, a = 10$$

$10 > 6$
 $a > b$
One triangle



$$\frac{\sin B}{b} = \frac{\sin A}{a}$$

$$\frac{\sin B}{b} = \frac{\sin 120^\circ}{a}$$

$$\frac{6}{\sin B} = \frac{10}{a}$$

$$\frac{6}{\sin B} = 0.0866$$

$$6 \times \frac{\sin B}{6} = 0.0866 \times 6$$

$$\sin B = 0.52$$

$$B = \sin^{-1} 0.52$$

$$\boxed{B = 31.3^\circ}$$

$$\theta = 180^\circ - 31.3^\circ - 120^\circ$$

$$\boxed{\theta = 28.7^\circ}$$

$$\frac{c}{\sin C} = \frac{a}{\sin A}$$

$$\frac{c}{\sin 28.7^\circ} = \frac{10}{\sin 120^\circ}$$

$$\frac{0.48}{c} = 11.55$$

$$0.48 \times \frac{c}{0.48} = 11.55 \times 0.48$$

$$\boxed{c = 5.5}$$

$$\angle A = 120^\circ, b = 6, a = 4$$

$4 < 6$
 $a < b$
No triangle



No triangle. Can't solve.