C11 - 1.3 - Geometric Means Notes

Write the first terms 5 of the sequence

$t_1 = 2, r = 3$

$$\times 3$$

$$\underline{\quad 2 \quad}_{t_1}, \underline{\quad\quad}_{t_2}, \underline{\quad\quad}_{t_3}, \underline{\quad\quad}_{t_4}, \underline{\quad\quad}_{t_5}$$

$$\times 3$$
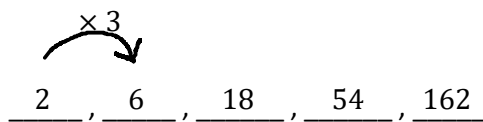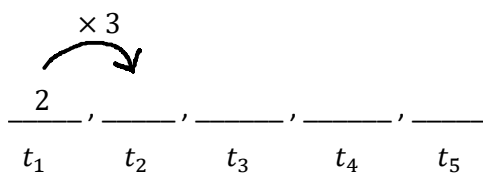
$$\underline{\quad 2 \quad}, \underline{\quad 6 \quad}, \underline{\quad 18 \quad}, \underline{\quad 54 \quad}, \underline{\quad 162 \quad}$$

$$\boxed{2,6,18,54,162}$$

$t_1 = 1st\ term\ (aka: "a\ or\ u_1")$
$r = common\ ratio$
$t_n = term\ n, every\ term$
$n = Term\ \#, or\ \#\ of\ terms$

$t_2 = 4, t_4 = 16$

$$\div r \quad \times r \quad \times r \quad \times r$$

$$\underline{\quad\quad}_{t_1}, \underline{\quad 4 \quad}_{t_2}, \underline{\quad\quad}_{t_3}, \underline{\quad 16 \quad}_{t_4}, \underline{\quad\quad}_{t_5}$$

$$4r^2 = 16 \qquad 4 - 2 = 2$$
$$r^2 = 4$$
$$\sqrt{r^2} = \sqrt{4}$$

$$\boxed{r = \pm 2}$$

$$\div +2 \quad \times +2 \quad \times +2 \quad \times +2$$

$$\underline{\quad 2 \quad}, \underline{\quad 4 \quad}, \underline{\quad 8 \quad}, \underline{\quad 16 \quad}, \underline{\quad 32 \quad}$$

$$\div -2 \quad \times -2 \quad \times -2 \quad \times -2$$

$$\underline{\quad -2 \quad}, \underline{\quad 4 \quad}, \underline{\quad -8 \quad}, \underline{\quad 16 \quad}, \underline{\quad -32 \quad}$$

$$\boxed{2,4,8,16,32} \qquad \boxed{-2,4,-8,16,-32}$$

$t_2 = 9, t_5 = 243$

$$9r^3 = 243 \qquad 5 - 2 = 3$$
$$r^3 = 27$$
$$\sqrt{r^3} = \sqrt{27}$$

$$\boxed{r = 3} \qquad \boxed{3, 9, 27, 81, 243}$$

$t_1 = 2, t_5 = 162$

$$2r^4 = 162 \qquad 5 - 1 = 4$$
$$r^4 = 81$$

$$\boxed{r = \pm 3}$$

$$\boxed{2, 6, 18, 54, 162} \qquad \boxed{2, -6, 18, -54, 162}$$

C11 - 1.3 - Geometric Sequences Notes

$$3,6,12 \ldots \qquad r = ? \qquad t_n = ? \qquad t_5 = ? \qquad t_n = 768, n = ?$$

$\times 2 \searrow \times 2 \searrow$

$$\underset{\substack{t_1 \\ n=1}}{3}, \underset{\substack{t_2 \\ n=2}}{6}, \underset{\substack{t_3 \\ n=3}}{12}, \underset{\substack{t_4 \\ n=4}}{\frac{?}{\,\,}} \cdots \underset{\substack{t_{10} \\ n=5}}{\frac{?}{\,\,}} \ldots \ldots \underset{\substack{t_n \\ n=?}}{\frac{768}{\,\,}}$$

$$\underline{\quad\quad}, \underline{\quad\quad}, \underline{\quad\quad} \cdots \underline{\quad\quad} \cdots \underline{\quad\quad}$$

$t_1 = 3$

Ratio

A term divided by the term before it

$$r = \frac{t_n}{t_{n-1}} \qquad\qquad r = \frac{t_n}{t_{n-1}} \qquad\qquad r = \frac{t_n}{t_{n-1}} \qquad\qquad t_{n-1} = term\ before\ t_n$$

$$r = \frac{6}{3} \qquad\qquad r = \frac{12}{6}$$

$\boxed{r = 2}$ (circled) $\qquad\qquad \boxed{r = 2}$ (circled)

Geometric: r must always be the same

Find the General term t_n =?   General term formula

$$t_n = t_1 r^{n-1}$$
$$t_n = 3(2)^{n-1}$$

$$\boxed{t_n = t_1 r^{n-1}}$$

The first term
times $'r - 1'$ differences

What is the fifth term t_5? $t_5 = ?, n = 5$.

$$t_n = 3(2)^{n-1}$$
$$t_5 = 3(2)^{n-1}$$
$$t_5 = 3(2)^{5-1}$$
$$t_5 = 3(2)^4$$
$$\boxed{t_5 = 48}$$ (circled)

Check your answer: 3,6,12,24,48 ✓

Or, Start from beginning

$$t_n = t_1 r^{n-1}$$
$$t_5 = 3(2)^{5-1}$$
$$\boxed{t_5 = 48}$$ (circled)

Remember: You could have also multiplied by the common ratio repeatedly

The number 768 is what term? $t_n = 768, n = ?$

$$t_n = 3(2)^{n-1}$$
$$768 = 3(2)^{n-1} \qquad\qquad divide\ both\ sides\ by\ 3$$
$$256 = 2^{n-1}$$
$$2^8 = 2^{n-1} \qquad\qquad Change\ of\ base:\ 256 = 2^8$$
$$8 = n - 1 \qquad\qquad Same\ Base, exponents\ are\ equal$$

$$\boxed{n = 9}$$ (circled)

Check your answer: 3,6,12,24,48,96,192,384,768 ✓