

Math 8 Notes



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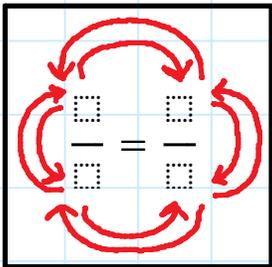
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M8 - 2.1 - Equal Fraction Notes

Over = Over

$$\frac{\square}{\square} = \frac{\square}{\square}$$

Do to Top/Do to Bottom
Do to Left/Do to Right



One over two equals what over 4?

$$\frac{1}{2} = \frac{?}{4}$$

→

$$\frac{1}{2} = \frac{2}{4}$$

Multiply bottom by 2
Multiply top by 2

$? = 2$ $1 \times 2 = 2$

One over three equals what over 6?

Set it up!

$$\frac{1}{3} = \frac{x}{6}$$

$x = 1 \times 2 = 2$

$x = 2$

? or x
Does it matter
No!

Direction Matters

Multiply in one direction
Divide in other direction

$$\frac{1}{2} = \frac{2}{4}$$

$\times 2$ (top to bottom)
 $\div 2$ (bottom to top)

$$\frac{1}{2} = \frac{2}{4}$$

$\times 2$ (left to right)
 $\times 2$ (right to left)

Multiply left by 2
Multiply right by 2

Get an LCD!

$$\frac{1}{2} = \frac{x}{4}$$

$$2 \times \frac{1}{2} = \frac{x}{4}$$

$$\frac{2}{2} = \frac{x}{4}$$

$$\frac{2}{4} = \frac{x}{4}$$

$x = 2$

LCD

Five over twenty equals one over what?

I'm not afraid to work on the bottom of the fraction!

$$\frac{5}{20} = \frac{1}{x}$$

→

$$\frac{5}{20} = \frac{1}{4}$$

→ $x = 4$

Divide top by 5
Divide bottom by 5

$20 \div 5 = 4$

Ratios Too!

$\times 2$ (1:2)
 $\times 2$ (2:4)

Same Rules!

Three over five equals what over twelve?

$$\frac{3}{5} = \frac{x}{12}$$

→

$$\frac{3}{5} = \frac{7.2}{12}$$

→ $x = 7.2$

Multiply bottom by 2.4
Multiply top by 2.4

$3 \times 2.4 = 7.2$

$\frac{12}{5} = 2.4$

If you don't know just divide!
Bigger divided by Smaller!

Bottom goes into Top

Bottom | Top

$$\begin{array}{r} 2.4 \\ 5 \overline{) 12.0} \\ \underline{- 10} \\ 20 \\ \underline{- 20} \\ 0 \end{array}$$

$$\begin{array}{r} 1.66 \dots \\ 3 \overline{) 5.00} \\ \underline{- 3} \\ 20 \\ \underline{- 18} \\ 20 \text{ (Repeat)} \end{array}$$

$x = 7.2$

OR!

Multiply left in down by 2
Divide right in up by 2

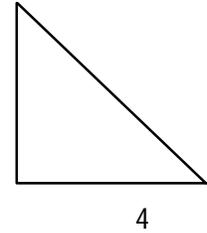
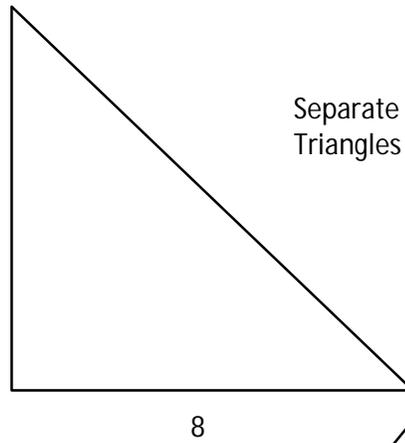
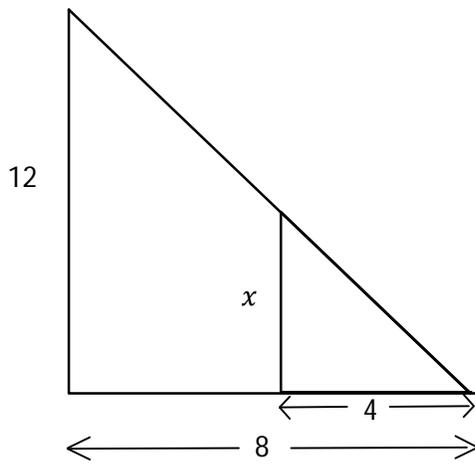
$$\frac{3}{5} = \frac{x}{12}$$

$\times 1.\bar{6}$ (left to right)
 $\div 1.\bar{6}$ (right to left)

$\frac{5}{3} = 1.\bar{6}$

Calculator $1.\bar{6} = 1.666666\dots$

M8 - 2.2 - Similar Shapes Notes



12: x
8: 4
Ratios

Variable on top!

× 1.5

$$\frac{x}{12} = \frac{4}{8}$$

× 1.5

Equal Fractions

$$4 \times 1.5 = 6$$

$x = 6$

$$12 \div 8 = 1.5$$

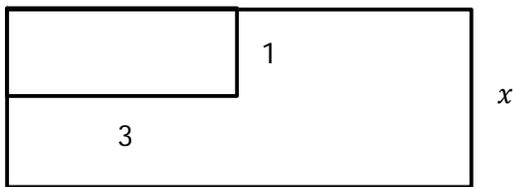
BIGGER = *bigger*
SMALLER = *smaller*

$$\frac{x}{12} = \frac{4}{8}$$

$$12 \times \frac{x}{12} = \frac{4}{8} \times 12$$

$x = 6$

Algebra



6

$$6 \div 3 = 2$$

$x = 2$

$$\times 3 \quad \frac{1}{3} = \frac{x}{6} \quad \div 3$$

smaller = *SMALLER*
bigger = *BIGGER*

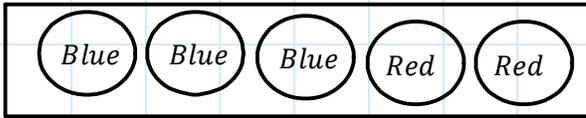
$$\frac{6}{3} = \frac{x}{1}$$

$x = 2$

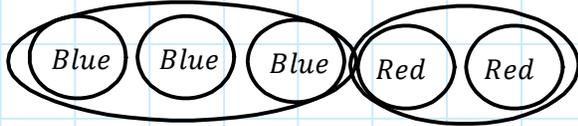
BIGGER = *SMALLER*
bigger = *smaller*

M8 - 2.7 - Ratios Marbles Notes

You have 3 Blue marbles and 2 Red marbles in a box, a total 5 marbles.



What is the ratio of blue to red marbles?



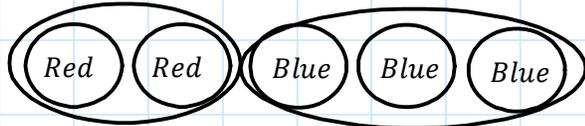
$$\begin{matrix} b : r \\ 3 : 2 \end{matrix}$$

3 Blue : 2 Red

$$3:2$$

Part to Part

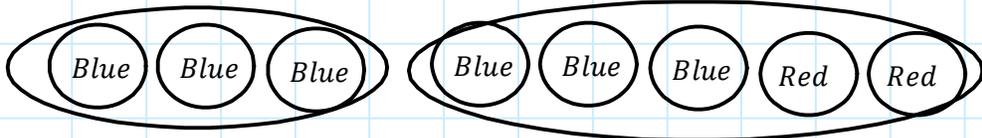
What is the ratio of red to blue marbles?



2 Red : 3 Blue

$$2:3$$

What is the ratio of blue marbles to total marbles?



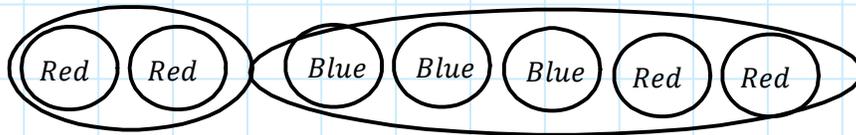
3 Blue : 5 Total

$$3:5$$

Part to Total

$$\begin{matrix} r : b : T \\ 2 : 3 : 5 \end{matrix}$$

What is the ratio of red marbles to total marbles?



2 Red : 5 Total

$$2:5$$

If a larger box has 9 Blue marbles in the same ratio as above how many Red marbles and Total marbles are in the box?

$$\times 3 \begin{matrix} \curvearrowleft 3 \text{ Blue} : 2 \text{ Red} \\ \curvearrowright 9 : \text{Blue} : ? \text{ Red} \end{matrix} \times 3$$

$$9 \text{ Blue} + 6 \text{ Red} = 15 \text{ Marbles}$$

$$2 \times 3 = 6$$

$$6 \text{ Red Marbles}$$

There are 15 marbles in the larger box

If a larger box has 25 marbles in the same ratio as above how many Red marbles are in the box?

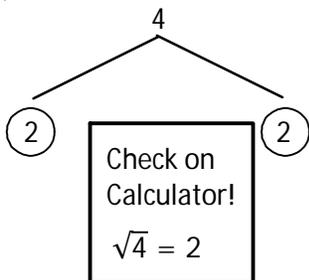
M8 - 3.2 - Solving Square Roots Prime Factorization Notes

Perfect Square: A number that is the product of the same two factors. $9 = 3 \times 3 = 3^2$

3

	3	$\sqrt{9} = 3$	$3^2 = 3 \times 3 = 9$
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$\sqrt{4} = ?$



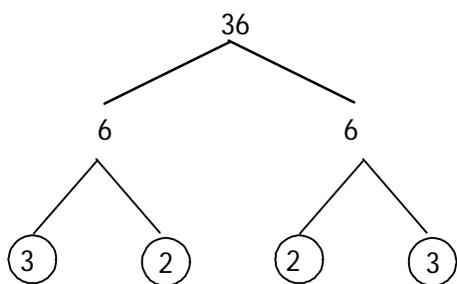
4 is a perfect square because it is a product of the same two factors: 2 and 2.

$$\begin{aligned} \sqrt{4} &= \sqrt{2 \times 2} \\ \sqrt{4} &= \sqrt{2 \times 2} \\ &= 2 \end{aligned}$$

Two identical numbers under a square root: one comes out. Nothing is left.

OR Think about two identical numbers that multiply together to make that number

$\sqrt{36} = ?$



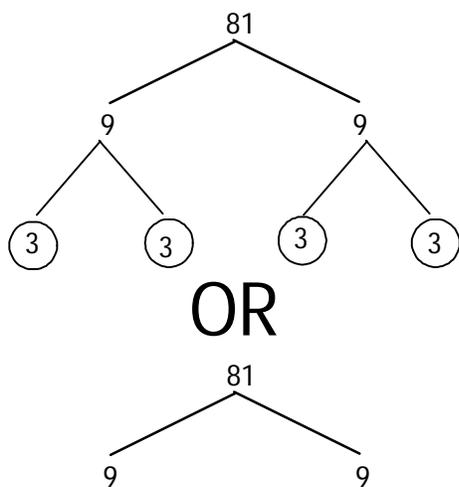
36 is a perfect square because it is a product of even pairs of numbers: 3 and 2, and 3 and 2.

$$\begin{aligned} \sqrt{36} &= \sqrt{2 \times 2 \times 3 \times 3} \\ \sqrt{36} &= \sqrt{(2 \times 2) \times (3 \times 3)} \\ \sqrt{36} &= 2 \times 3 \\ \sqrt{36} &= 6 \end{aligned}$$

Two identical pairs of numbers under a square root: one of each comes out. Nothing is left.

Check on Calculator!
 $\sqrt{36} = 6$

$\sqrt{81} = ?$



81 is a perfect square because it is a product of even pairs of numbers: 3 and 3, and 3 and 3.

$$\begin{aligned} \sqrt{81} &= \sqrt{3 \times 3 \times 3 \times 3} \\ \sqrt{81} &= \sqrt{(3 \times 3) \times (3 \times 3)} \\ \sqrt{81} &= 3 \times 3 \\ \sqrt{81} &= 9 \end{aligned}$$

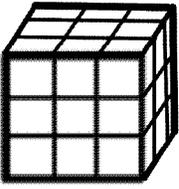
Two identical pairs of numbers under a square root: one of each comes out. Nothing is left.

Check on Calculator!
 $\sqrt{81} = 9$

Notice: when solving square roots using prime factorization either circle a pair of two identical numbers or multiple pairs of identical numbers.

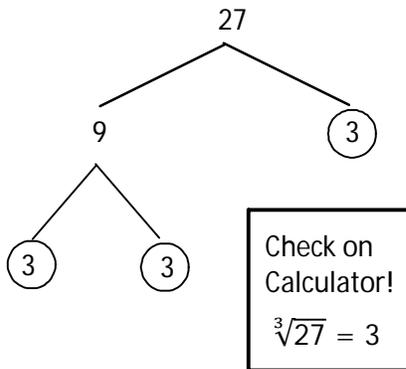
M8 - 3.2 - Solving Cube Roots Prime Factorization Notes

Perfect Cube: a number that is a product of the same three factors. $8 = 2 \times 2 \times 2 = 2^3$



$$\sqrt[3]{27} = 3 \quad 3 \times 3 \times 3 = 3^3 = 27$$

$\sqrt[3]{27} = ?$



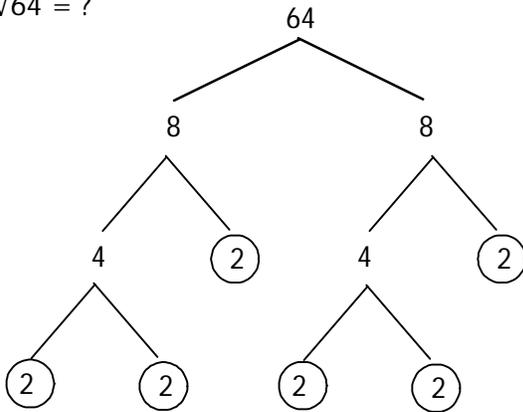
27 is a perfect cube because it is the product of three identical factors:

$$\begin{aligned} \sqrt[3]{27} &= \sqrt[3]{3 \times 3 \times 3} \\ \sqrt[3]{27} &= \sqrt[3]{\textcircled{3} \times \textcircled{3} \times \textcircled{3}} \\ &= \textcircled{3} \end{aligned}$$

Three identical numbers under a cube root: one comes out. Nothing is left.

OR Think about three identical numbers that multiply together to make that number

$\sqrt[3]{64} = ?$

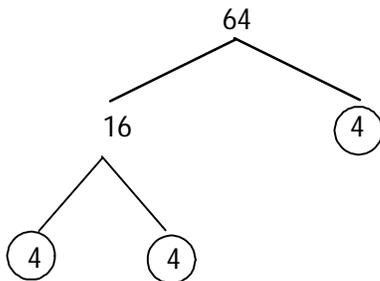


$$\begin{aligned} \sqrt[3]{64} &= \sqrt[3]{2 \times 2 \times 2 \times 2 \times 2 \times 2} \\ \sqrt[3]{64} &= \sqrt[3]{\textcircled{2} \times \textcircled{2} \times \textcircled{2} \times 2 \times 2 \times 2} \\ &= 2 \times 2 \\ &= \textcircled{4} \end{aligned}$$

Three identical numbers under a square root: one of each comes out. Nothing is left.

Check on Calculator!
 $\sqrt[3]{64} = 4$

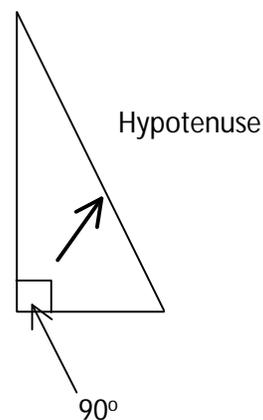
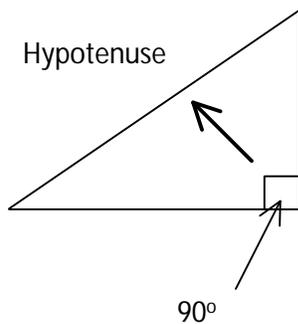
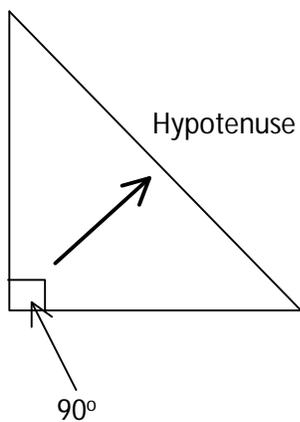
OR



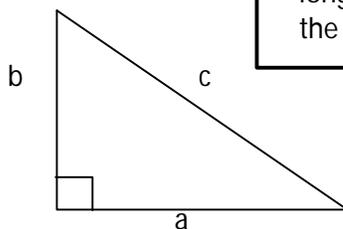
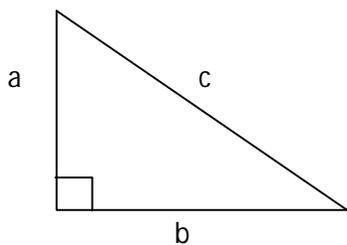
$$\begin{aligned} \sqrt[3]{64} &= \sqrt[3]{\textcircled{4} \times \textcircled{4} \times \textcircled{4}} \\ &= \textcircled{4} \end{aligned}$$

Notice: when solving cube roots using prime factorization either circle a triplet of three identical numbers or multiple triplets of identical numbers.

M8 - 3.3 - Identifying "a, b, c" Notes

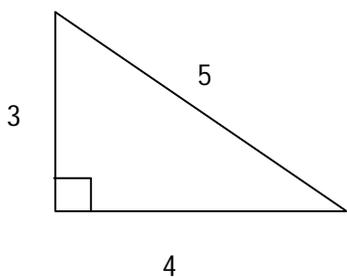


Identifying a, b, and c.

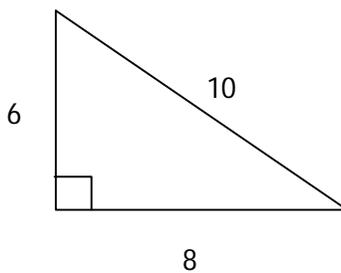


"a" and "b" can switch.
 "c" is always the hypotenuse, the longest side, the side opposite of the 90° angle.

Identifying a, b, and c.



$a = 3$
 $b = 4$
 $c = 5$

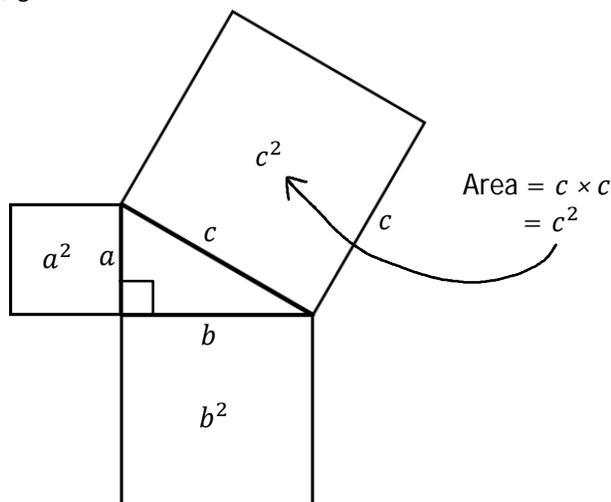
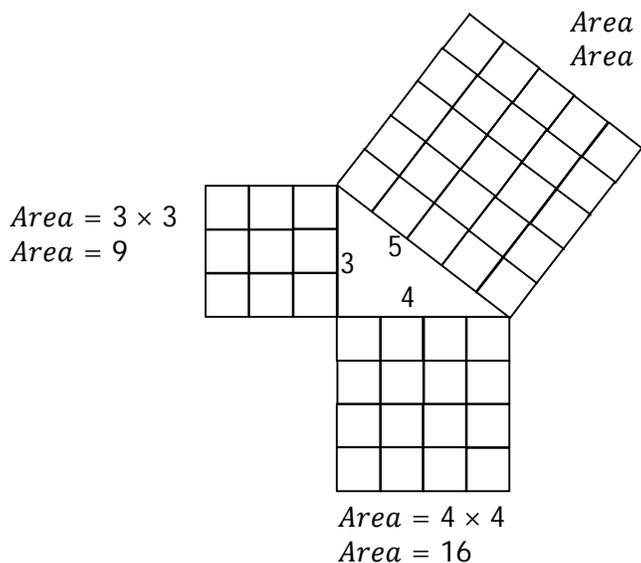


$a = 8$
 $b = 6$
 $c = 10$

M8 - 3.3 - Pythagoras' Theorem Notes

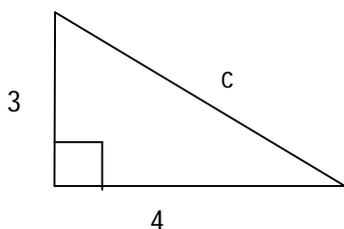
Pythagoras' Theorem: $a^2 + b^2 = c^2$

Remember: "c" is always the Hypotenuse: the longest side



$9 \text{ squares} + 16 \text{ squares} = 25 \text{ squares}$
 $\sqrt{25} = 5$

Solve for "c".

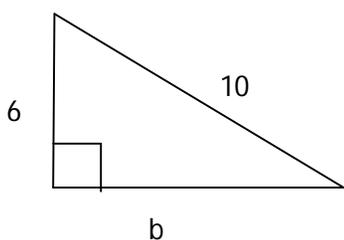


$$\begin{aligned} a^2 + b^2 &= c^2 \\ 3^2 + 4^2 &= c^2 \\ 9 + 16 &= c^2 \\ 25 &= c^2 \\ \sqrt{25} &= \sqrt{c^2} \\ \mathbf{5} &= \mathbf{c} \end{aligned}$$

Remember:
The Area of the two small squares adds to the area of the large square.

$c = \sqrt{a^2 + b^2}$

Solve for "a" or "b".



$$\begin{aligned} a^2 + b^2 &= c^2 \\ 6^2 + b^2 &= 10^2 \\ 36 + b^2 &= 100 \\ -36 & \quad -36 \\ b^2 &= 64 \\ \sqrt{b^2} &= \sqrt{64} \\ \mathbf{b} &= \mathbf{8} \end{aligned}$$

OR

Remember:
Bigger square minus smaller square equals other smaller square.

$$\begin{aligned} c^2 - a^2 &= b^2 \\ 10^2 - 6^2 &= b^2 \\ 100 - 36 &= b^2 \\ 64 &= b^2 \\ \sqrt{64} &= \sqrt{b^2} \\ \mathbf{b} &= \mathbf{8} \end{aligned}$$

$b = \sqrt{c^2 - a^2}$

M8 - 4.2 - Percentage Notes

$\frac{IS}{OF} = \frac{\%}{100}$

$\frac{\square}{\square} = \frac{\square}{\square}$

Calculator!
 Fractions
 Cross Multiply
 Long Division

What is eight out of twenty as a percentage?

①

$\frac{IS}{OF} = \frac{\%}{100}$
 ↑
 Total

$\frac{\square}{\square} = \frac{\square}{\square}$

$\frac{8}{20} = \frac{\%}{100}$
 $\times 5$
 $\frac{8}{20} = \frac{40}{100}$
 $\times 5$

$\% = 40\%$
 ↓
 $\frac{8}{20} = \frac{40}{100}$
 $\frac{8}{20} = 0.4$ Calc!

Test on Calculator!

$0.1 = \frac{1}{10}$	$0.\bar{3} = \frac{3}{9}$
$0.23 = \frac{23}{100}$	$0.\bar{45} = \frac{45}{99}$
$0.045 = \frac{45}{1000}$	

Find 15% of 200.

$x = 30$

$\frac{30}{200} = 0.15$ Check!

$\frac{x}{200} = \frac{15}{100}$
 $\times 2$
 $\frac{x}{100} = \frac{30}{100}$
 $\times 100$
 $x = 30$

Cross Multiply
Divide

 $\frac{x}{200} = \frac{15}{100}$
 $100x = 3000$
 $x = 30$

$\times 100$	→	Move decimal 2 places to the right.
$0.0073 = 0.73\%$		
$0.05 = 5\%$		
$0.1 = 10\%$		$0.1 = 0.10$
$0.12 = 12\%$		
$0.\bar{3} = 33.\bar{3}\%$		One Decimal
$0.5 = 50\%$		
$0.\bar{66} = 66.67\%$		Two Decimal
$1.05 = 105\%$		
$1.35 = 135\%$		
$2.12 = 212\%$		
$\div 100$	←	Move decimal 2 places to the left.

25 is 20% of what?

$x = 125$

$\frac{25}{20} = 1.25$

$\frac{25}{x} = \frac{20}{100}$
 $\times 1.25$
 $\frac{25}{x} = \frac{20}{100}$
 $\times 1.25$

Increase 200 by 15%

$15\% = 0.15$

Logic!

Multiply by the decimal

②

Then Add (Or Subtract*)

OR

$200 \times 0.15 = 30$

$200 + 30 = 230$

Less than 5 Round Down

$5.7\textcircled{4} = 5.7$	Round To
$5.7\textcircled{6} = 5.8$	Tenths

5 or More Round Up

$200 \times 1.15 = 230$

Multiply by the % you want to be!

②a

Check!

 $\frac{230}{200} = 1.15$

$M = 1 + 0.15$	$Multiplier = 1 \pm r$	$Initial \times Multiplier = Final$
$M = 1.15$	$M = 1 \pm r$	

Decrease 200 by 15%

$200 \times 0.15 = 30$

OR

$200 - 30 = 170$

$200 \times 0.85 = 170$

②a

Find the Percent Change increase from 10 to 12.

③

$$\%Change = \frac{Final - Initial}{Initial}$$

$$\%Change = \frac{12 - 10}{10} = \frac{2}{10} = 0.2 = 20\%$$

Find a number decreased by 20% to be 160

④

$x = 200$

$M = 100 - 20 = 80\% = 0.80$

$\frac{160}{x} = \frac{80}{100}$
 $x \times 0.80 = 160$
 $0.80x = 160$
 $\frac{0.80x}{0.80} = \frac{160}{0.80}$
 $x = 200$

⑤

Jack ate 4 more, 40% more than Jill.

How many did Jack eat!

let $j = \#Jill$ ate

$M = 100 + 40 = 140\%$

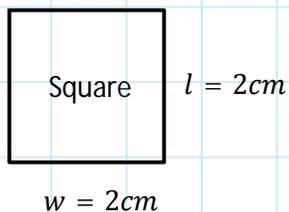
$j + 4 = \frac{140}{100}j$
 $100(j + 4) = 140j$
 $100j + 400 = 140j$
 $400 = 40j$
 $j = 10$

Cross Multiply
Distribute
Algebra

⑥

Jack ate 14! $10 + 4 = 14$

M8 - 5.0 - Area/Perimeter Shapes Notes



$$A = l \times w$$

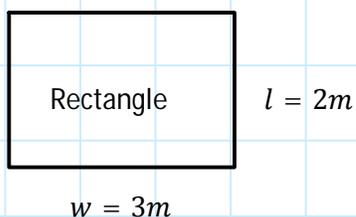
$$A = 2 \times 2$$

$$A = 4cm^2$$

$$p = l + l + w + w$$

$$p = 2 + 2 + 2 + 2$$

$$p = 8cm$$



$$A = l \times w$$

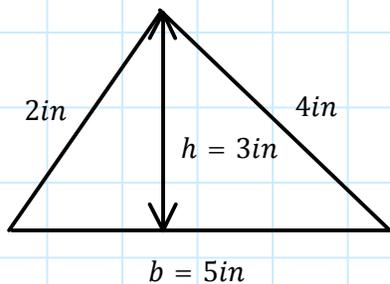
$$A = 2 \times 3$$

$$A = 6m^2$$

$$p = l + l + w + w$$

$$p = 2 + 2 + 3 + 3$$

$$p = 10cm$$



$$A = \frac{bh}{2}$$

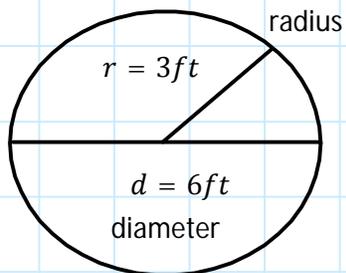
$$A = \frac{5 \times 3}{2}$$

$$A = 7.5in^2$$

$$p = a + b + c$$

$$p = 2 + 4 + 5$$

$$p = 11in$$



Perimeter = Circumference

$$A = \pi r^2$$

$$A = \pi(3)^2$$

$$A = 9\pi ft^2$$

$$A = 28.27 ft^2$$

$$C = 2\pi r$$

$$C = 2\pi(3)$$

$$C = 6\pi ft$$

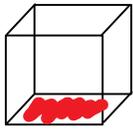
$$C = 18.85 ft$$

Terms of π

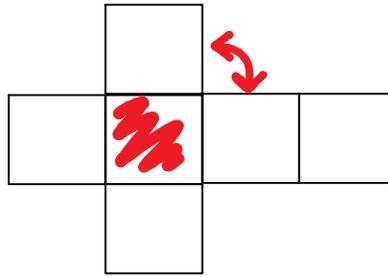
$$r = \frac{d}{2}$$

M8 - 5.1 - Net Surface Area Notes

Cube

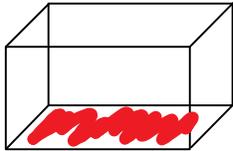


Draw a square
 Draw a square up to the right
 Connect corners

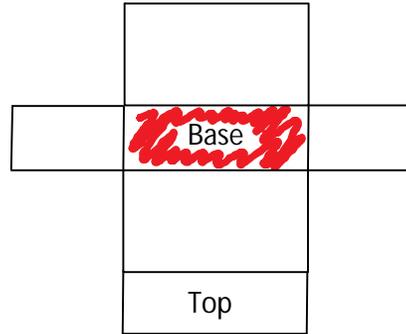


Draw the bottom
 Label Dimensions
 Fold down the sides.
 Fold off the top.

Rectangular Prism

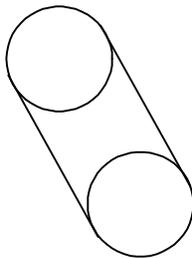


Draw a rectangle
 Draw a rectangle up to the right
 Connect corners

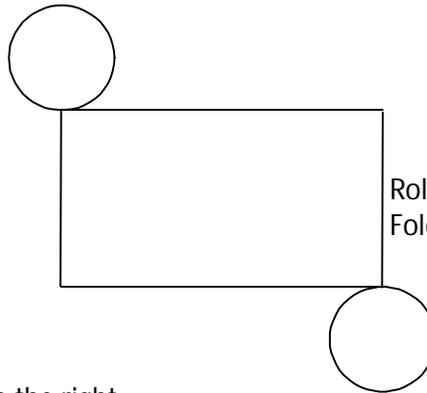


Draw the bottom.
 Fold down the sides.
 Fold off the top.

Cylinder



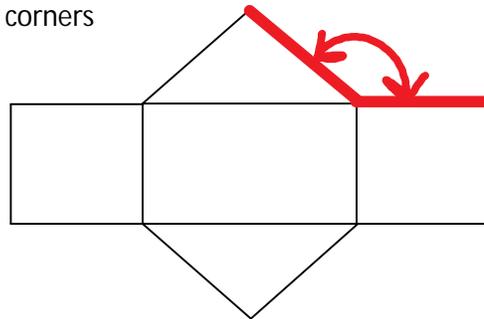
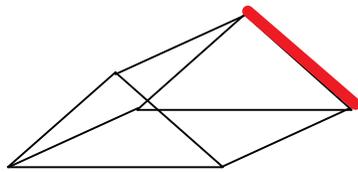
Draw two circles
 not touching
 Connect the circles



Roll it out flat.
 Fold off the top & bottom.

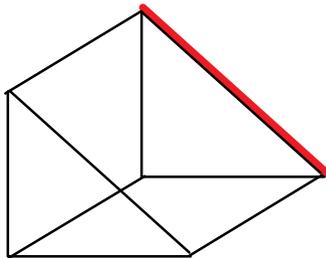
Triangular Prism

Draw a triangle
 Draw a triangle up to the right
 Connect corners

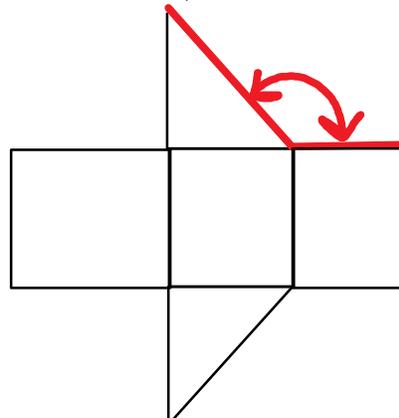


Draw the bottom.
 Fold down the sides.
 Fold down the front and back.

Right Triangular Prism

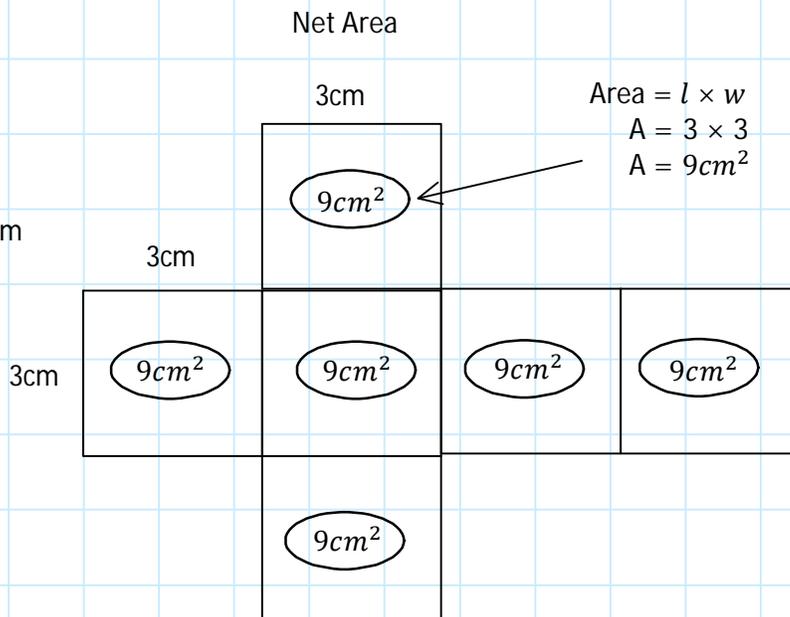
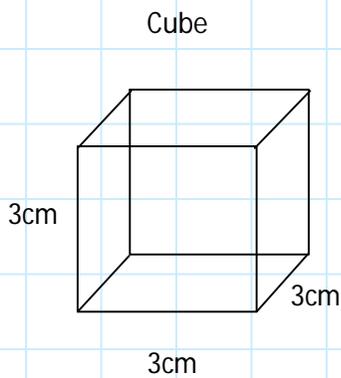


Draw a right triangle
 Draw another up to the right
 Connect corners



Draw the bottom.
 Fold down the sides.
 Fold down the front and back.

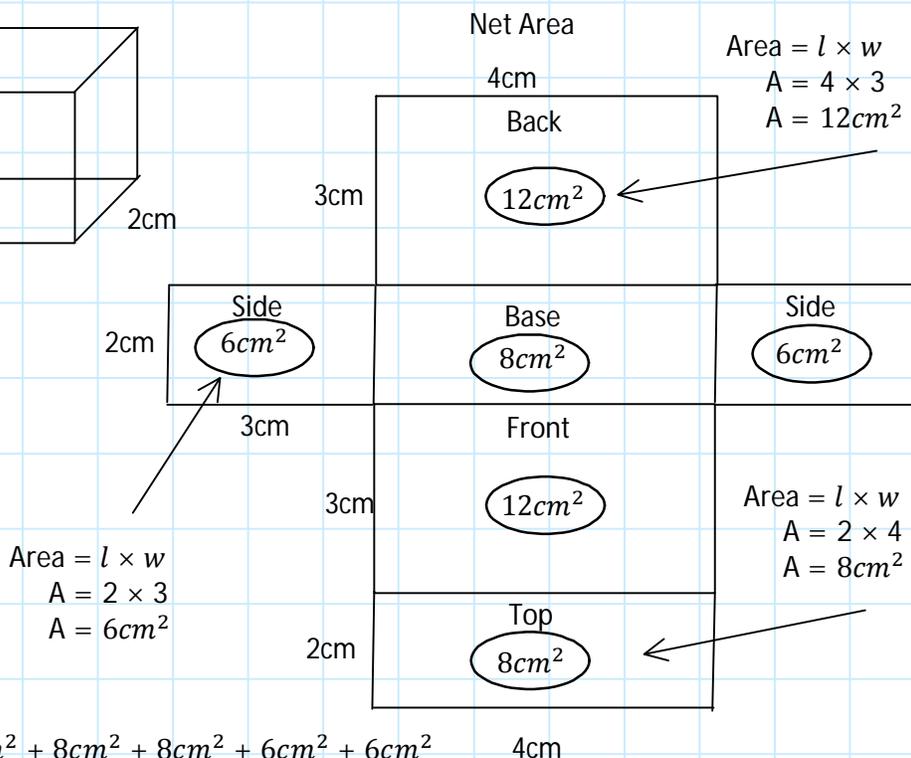
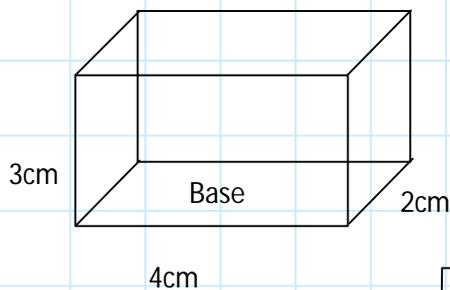
M8 - 5.2 - Cube/Rectangular Prism Surface Area Notes



$$SA = 9cm^2 + 9cm^2 + 9cm^2 + 9cm^2 + 9cm^2 + 9cm^2$$

$SA = 54cm^2$

Rectangular Prism



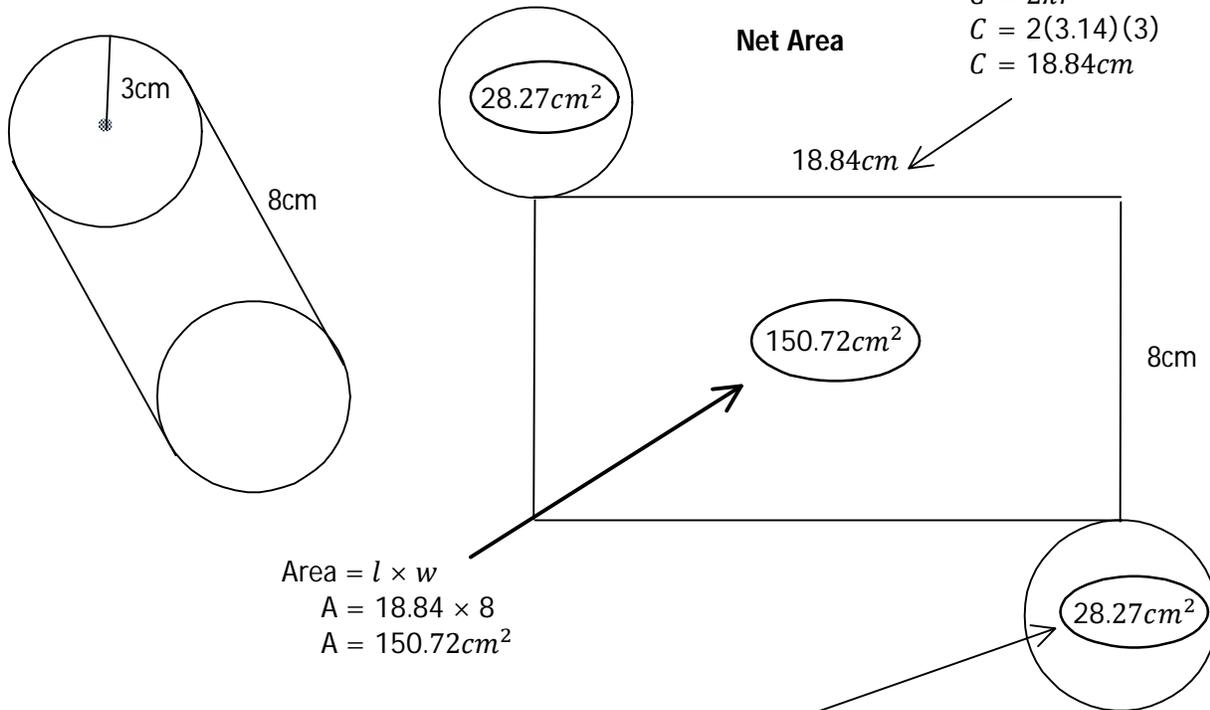
$$SA = 12cm^2 + 12cm^2 + 8cm^2 + 8cm^2 + 6cm^2 + 6cm^2$$

$SA = 52cm^2$

Notice: the top and bottom are the same, the front and back are the same, and both sides are the same.

M8 - 5.3 - Cylinder/Triangular Prism Surface Area Notes

Cylinder



$$SA = 28.27\text{cm}^2 + 28.27\text{cm}^2 + 150.72\text{cm}^2$$

$$SA = 207.26\text{cm}^2$$

$$A = \pi r^2$$

$$A = (3.14)(3)^2$$

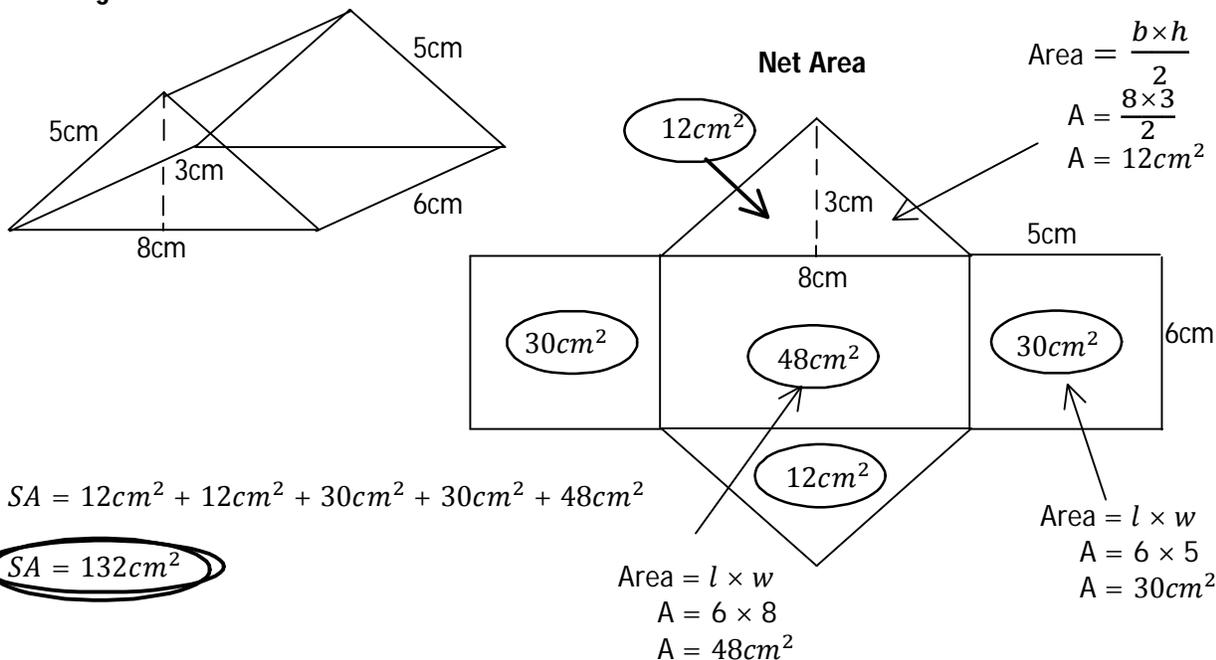
$$A = 28.27\text{cm}^2$$

In terms of π

$$SA = 66\pi\text{cm}^2$$

Notice: the width of the rectangle is the circumference of the circle.

Triangular Prism



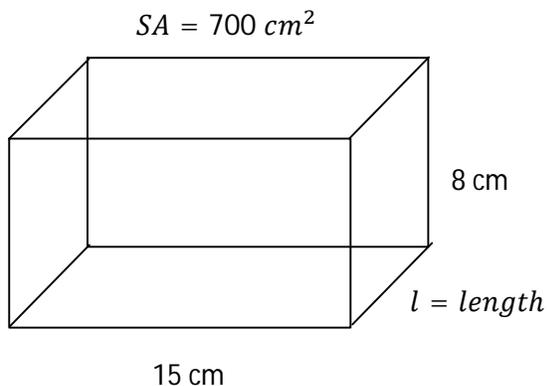
$$SA = 12\text{cm}^2 + 12\text{cm}^2 + 30\text{cm}^2 + 30\text{cm}^2 + 48\text{cm}^2$$

$$SA = 132\text{cm}^2$$

Notice: the front and back are the same, and sides are the same.

M8 - 5.4 - Surface Area Missing Dimension Notes

Find the missing dimension of the following shapes.



$$SA = 2(l \times w) + 2(l \times h) + 2(h \times w)$$

$$700 = 2(15l) + 2(8l) + 2(8 \times 15)$$

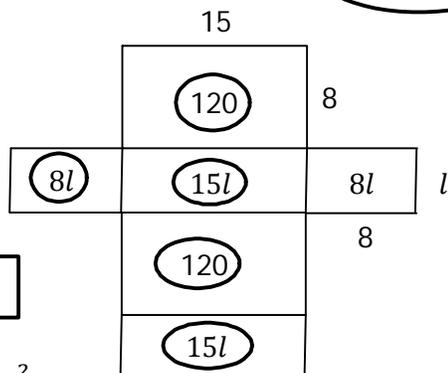
$$700 = 30l + 16l + 240$$

$$\begin{array}{r} 700 = 30l + 16l + 240 \\ -240 -240 \\ \hline 460 = 46l \\ 460 46l \\ \hline 46 46 \\ 10 = l \end{array}$$

$l = 10 \text{ cm}$

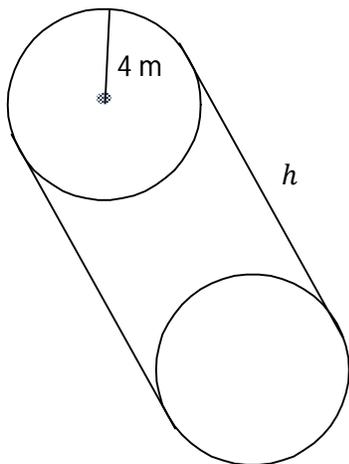
OR

$700 = 240 + 46l$
...



$A = l \times w$

$SA = 326.7 \text{ m}^2 = 104\pi \text{ m}^2$



$$SA = 2\pi r^2 + 2\pi r h$$

$$326.7 = 2\pi(4)^2 + 2\pi(4)h$$

$$326.7 = 100.53 + 25.13h$$

$$\begin{array}{r} 326.7 = 100.53 + 25.13h \\ -100.53 -100.53 \\ \hline 226.17 = 25.13h \\ 226.17 25.13h \\ \hline 25.13 25.13 \\ 9 = h \end{array}$$

$h = 9 \text{ m}$

OR

$SA = 104\pi \text{ m}^2$

$$SA = 2\pi r^2 + 2\pi r h$$

$$104\pi = 2\pi(4)^2 + 2\pi(4)h$$

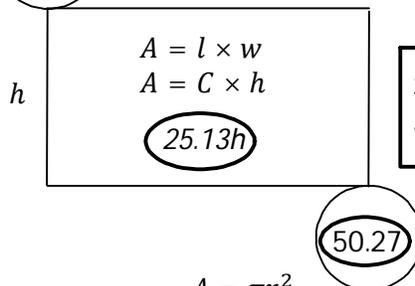
$$\frac{104\pi}{\pi} = \frac{32\pi}{\pi} + \frac{8\pi h}{\pi}$$

$$104 = 32 + 8h$$

$$\begin{array}{r} 104 = 32 + 8h \\ -32 -32 \\ \hline 72 = 8h \\ \frac{72}{8} = \frac{8h}{8} \end{array}$$

$h = 9 \text{ m}$

$C = 2\pi r$
 $C = 2\pi(4)$
 $C = 25.13$



$A = \pi r^2$
 $A = \pi(4)^2$
 $A = 50.27$

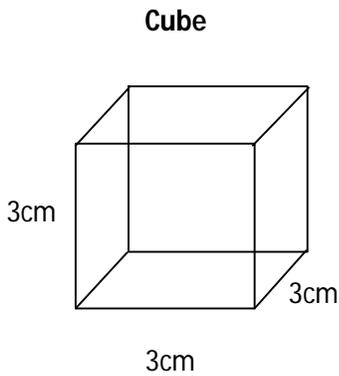
OR

$326.7 = 100.54 + 25.13h$
...

M8 - 7.1 - Quadrilateral Volume Notes

Volume: equal to the area of the base time height: " $V = (\text{area of base}) \times (\text{height})$ ".

The base must be the same as the top.



Volume

$$V = (\text{area of base}) \times (\text{height})$$

$$V = (l \times w) \times (h)$$

$$V = lwh$$

$$V = lwh$$

$$V = 3 \times 3 \times 3$$

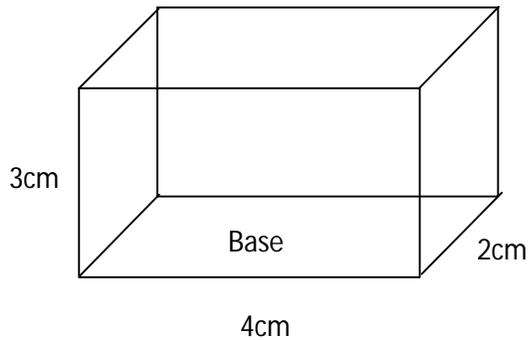
$$V = 27\text{cm}^3$$

If Area of Base Given

A 3D wireframe diagram of a cube. The front-left vertical edge is labeled "3cm". The bottom-left receding edge is labeled " $A_b = 9\text{cm}^2$ ".

$V = (\text{area of base}) \times (\text{height})$
 $V = (9) \times (3)$
 $V = 27\text{cm}^3$

Rectangular Prism



Volume

$$V = (\text{area of base}) \times (\text{height})$$

$$V = (l \times w) \times (h)$$

$$V = lwh$$

$$V = lwh$$

$$V = 4 \times 2 \times 3$$

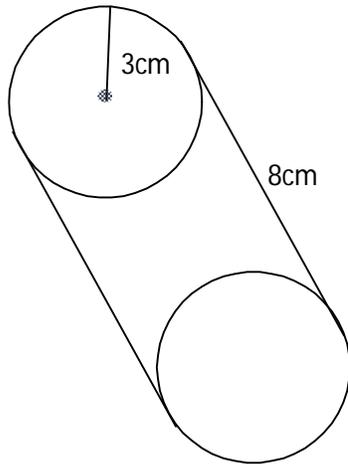
$$V = 24\text{cm}^3$$

Notice: the formula for the volume of a cube and a rectangular prism is just: $V = lwh$.

M8 - 7.2 - Cylinder/Triangular Prism Volume Notes

Volume: equal to the area of the base times the height: " $V = (\text{area of base}) \times (\text{height})$ ". The base must be the same as the top.

Cylinder



Volume

$$V = (\text{area of base}) \times (\text{height})$$

$$V = (\pi r^2) \times (h)$$

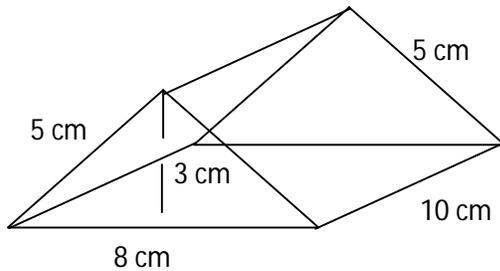
$$V = \pi r^2 h$$

$$V = \pi r^2 h$$

$$V = (3.14)(3)^2(8)$$

$$V = 226.19 \text{ cm}^3$$

Triangular Prism



Volume

$$V = (\text{area of base}) \times (\text{height})$$

$$V = \left(\frac{b \times h}{2}\right) \times (H)$$

$$V = \frac{bh}{2} \times H$$

$$V = \frac{bh}{2} \times H$$

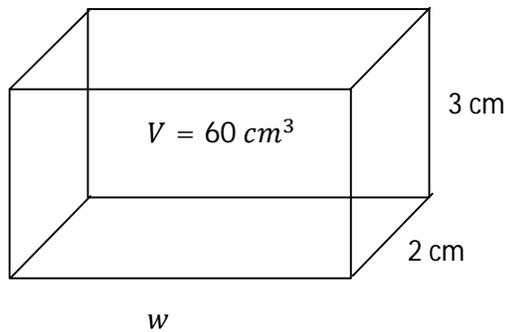
$$V = \frac{(8)(3)}{2} \times (10)$$

$$V = 120 \text{ cm}^3$$

Notice: the volume is calculated by finding the area of the base of the triangular prism using the height of the triangle, h , multiplied by the height of the prism, H .

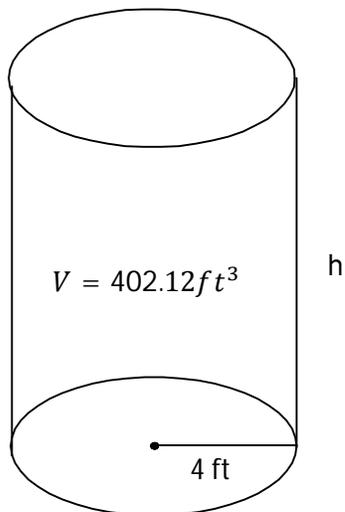
M8 - 7.3 - Rectangular Prism Missing Length Notes

Find the missing length for the shapes below.



$$\begin{aligned}V &= l \times w \times h \\60 &= 2 \times w \times 3 \\60 &= 6 \times w \\ \frac{60}{6} &= \frac{6 \times w}{6} \\10 &= w\end{aligned}$$

$$w = 10 \text{ cm}$$



$$\begin{aligned}V &= \pi r^2 h \\402.12 &= \pi(4)^2 h \\402.12 &= 50.27h \\ \frac{402.12}{50.27} &= \frac{50.27h}{50.27} \\8 &= h\end{aligned}$$

$$h = 8 \text{ ft}$$

M8 - 6.0 - LCM GCF Notes

Lowest common multiple (LCM): the lowest number both numbers go into

Greatest common factor (GCF): the biggest number that goes into two numbers

8 and 12?

Lowest Common Multiple (LCM):

8 and 12 = 24	8 : 8, 16, 24 , 32
	12 : 12, 24 , 36

$8 = 2^3$ Index Form
 $12 = 2^2 \times 3^1$ $LCD = 2^3 \times 3^1 = 24$

LCM: All the numbers to the highest exponent

Greatest Common Factor (GCF):

8 and 12 = 4	8 : 1, 2, 4 , 8
	12 : 1, 2, 3, 4 , 6, 12

$8 = 2^3$ Index form:
 $12 = 2^2 \times 3^1$ $GCF = 2^2 = 4$

GCF: Common numbers to the lowest exponent

72 and 60?

$72 = 2 \times 2 \times 2 \times 3 \times 3$
 $60 = 2 \times 2 \times 3 \times 5$

$LCM = 2 \times 2 \times 2 \times 3 \times 3 \times 5 = 360$
 $LCM = 2^3 \times 3^2 \times 5^1 = 360$

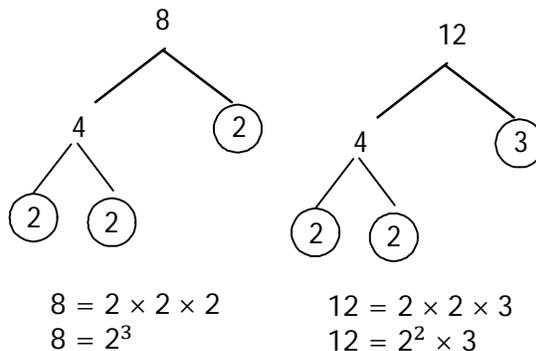
LCM: All the numbers to the highest exponent

$72 = 2 \times 2 \times 2 \times 3 \times 3$
 $60 = 2 \times 2 \times 3 \times 5$
 $GCF = 2 \times 2 \times 3 = 12$
 $GCF = 2^2 \times 3^1 = 12$

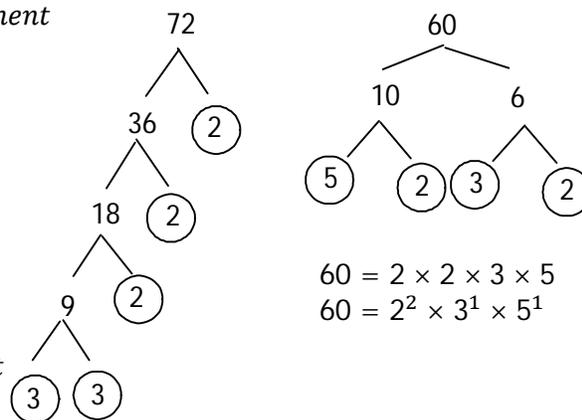
GCF: Common numbers to the lowest exponent

Prime Factorization Tree

8 and 12:



72 and 60:



OR

LCM:
72: 72, 144, 216, 288, 360
60: 60, 120, 180, 240, 300, 360
GCF:
72: 1, 2, 3, 4, 6, 8, 9, 12 , 18, 24, 36, 72
60: 1, 2, 3, 4, 5, 6, 10, 12 , 15, 20, 30, 60

<p>2 goes into even numbers ending in 0, 2, 4, 6, or 8</p> <p>3 goes into numbers whose digits add to multiples of 3. 369? 3+6+9=18. 3 goes into 18! 3 goes into 369.</p> <p>5 goes into numbers ending in 5 or 0</p> <p>Or do Long Division or use calculator</p>
--

M8 - 6.1 - Simplifying Expanding Fractions Notes

Simplification

$$\frac{2}{4} =$$

$$\frac{2 \div 2}{4 \div 2} = \left(\frac{1}{2}\right)$$

Divide the top and bottom by the GCF

$$\frac{6}{9} =$$

$$\frac{6 \div 3}{9 \div 3} = \left(\frac{2}{3}\right)$$

Divide the top and bottom by the GCF

Rule: Do to the top as you did to the bottom.

Expansion

$$\frac{1}{2} =$$

$$\frac{1 \times 2}{2 \times 2} = \left(\frac{2}{4}\right)$$

Multiply the top and bottom by an integer.

$$\frac{1}{2} =$$

$$\frac{1 \times 3}{2 \times 3} = \left(\frac{3}{6}\right)$$

Multiply the top and bottom by an integer.

M8 - 6.2 - Multiplying/Cross Cancelling/Dividing Fractions Notes

$$\frac{2}{3} \times \frac{4}{5} =$$

$$\frac{2 \times 4}{3 \times 5} = \frac{8}{15}$$

Multiply tops: $2 \times 4 = 8$
 Multiply bottoms: $3 \times 5 = 15$

$$\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$$

To multiply fractions just multiply tops and multiply bottoms.

$$2 \times \frac{3}{5} = \frac{2}{1} \times \frac{3}{5} = \frac{6}{5}$$

$$a \times \frac{b}{c} = \frac{a}{1} \times \frac{b}{c} = \frac{ab}{c}$$

Cross Cancelling

$$\frac{1}{2} \times \frac{2}{3} = \frac{2}{6} = \frac{1}{3}$$

OR

$$\frac{1}{\cancel{2}} \times \frac{\cancel{2}}{3} = \frac{1}{3}$$

Cross a 2 off from the top and bottom

$$\frac{1}{4} \times \frac{2}{3} = \frac{2}{12} = \frac{1}{6}$$

$$\frac{1}{4} \times \frac{2}{3} = \frac{1}{\cancel{4}^2} \times \frac{\cancel{2}^1}{3} = \frac{1}{6}$$

$$\frac{2}{4} = \frac{1}{2}$$

$$\frac{1}{2} \div \frac{4}{7} =$$

$$\frac{1}{2} \times \frac{7}{4} =$$

$$\frac{1 \times 7}{2 \times 4} = \frac{7}{8}$$

Flip second fraction, change to multiplication.

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c} = \frac{ad}{bc}$$

To divide fractions just flip the second fraction, and change divided by to multiplication and follow steps above.

$$\frac{\left(\frac{1}{2}\right)}{\left(\frac{4}{7}\right)} = \frac{1}{2} \div \frac{4}{7} = \frac{1}{2} \times \frac{7}{4} = \frac{7}{8}$$

$$\frac{\left(\frac{a}{b}\right)}{\left(\frac{c}{d}\right)} = \frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c} = \frac{ad}{bc}$$

$$\frac{3}{\left(\frac{5}{7}\right)} = 3 \div \frac{5}{7} = 3 \times \frac{7}{5} = \frac{21}{5}$$

$$\frac{a}{\left(\frac{b}{c}\right)} = a \div \frac{b}{c} = a \times \frac{c}{b} = \frac{ac}{b}$$

$$\frac{\left(\frac{2}{3}\right)}{5} = \frac{2}{3} \div 5 = \frac{2}{3} \div \frac{5}{1} = \frac{2}{3} \times \frac{1}{5} = \frac{2}{15}$$

$$\frac{\left(\frac{a}{b}\right)}{c} = \frac{a}{b} \div c = \frac{a}{b} \times \frac{1}{c} = \frac{a}{bc}$$

M8 - 6.3 - Mixed Numbers Improper Fractions Notes

Mixed fraction

Fraction

$$2\frac{3}{5} \longleftrightarrow \frac{13}{5}$$

Mixed fraction \longrightarrow Fraction

$$\begin{aligned} 2\frac{3}{5} &= \frac{\text{bottom} \times \text{left} + \text{top}}{\text{bottom}} \\ &= \frac{5 \times 2 + 3}{5} \\ &= \frac{13}{5} \end{aligned}$$

Fraction \longrightarrow Mixed fraction

$$\frac{13}{5}$$

=

$$2\frac{3}{5}$$

remainder

$$5 \times 2 = 10$$

$$13 - 10 = 3$$

$$\text{remainder} = 3$$

bottom

Bottom goes into top

M8 - 6.4 - Adding Subtracting Fractions Notes

Steps: Get the same bottom (LCD), do to top, do to bottom, add or subtract tops.

Lowest common denominator (LCD): the lowest common multiple of the denominators

$$\frac{1}{2} + \frac{1}{2} =$$

If the denominators are the same, we already have the LCD.

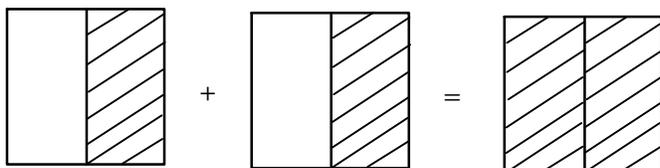
$$LCD = 2$$

$$\frac{1+1}{2} =$$

Add numerators: $1 + 1 = 2$

$$\frac{2}{2} = \textcircled{1}$$

Simplify.



$$\frac{1}{2} + \frac{1}{3} =$$

Multiply the top and bottom of each fraction by the denominator of the other fraction.

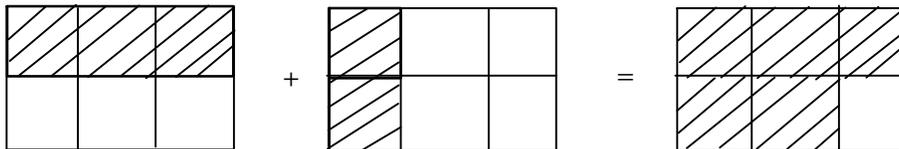
$$LCD = 6$$

$$\frac{3 \times 1}{3 \times 2} + \frac{1 \times 2}{3 \times 2} =$$

This will always give you a common denominator (not necessarily the LCD).

$$\frac{3}{6} + \frac{2}{6} = \textcircled{\frac{5}{6}}$$

Add the numerators.



$$\frac{1}{2} + \frac{1}{3} =$$

$$LCD = 6$$

$$\frac{3}{6} + \frac{2}{6} =$$

$$\frac{3 \times 1}{3 \times 2} + \frac{1 \times 2}{3 \times 2} =$$

$$\frac{3}{6} + \frac{2}{6} = \frac{5}{6}$$

$$\frac{3}{4} - \frac{1}{6} =$$

Multiples of 4: 4, 8, $\textcircled{12}$, 16, 20

$$LCD = 12$$

Multiples of 6: 6, $\textcircled{12}$, 18, 24

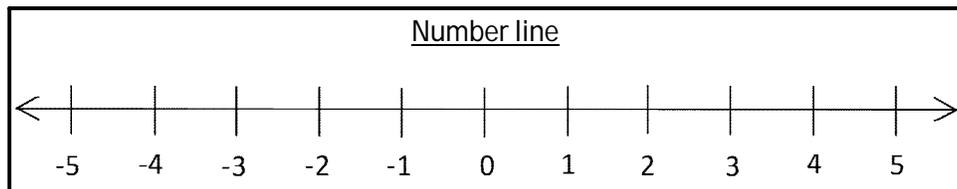
$$\frac{3 \times 3}{3 \times 4} - \frac{1 \times 2}{6 \times 2} =$$

Multiply top and bottom of first fraction by 3 to get 12 in the denominator.
Multiply top and bottom of second fraction by 2 to get 12 in the denominator.

$$\frac{9}{12} - \frac{2}{12} = \textcircled{\frac{7}{12}}$$

Subtract the numerators.

M8 - 8.1 - Adding/Subtracting Number Line Notes

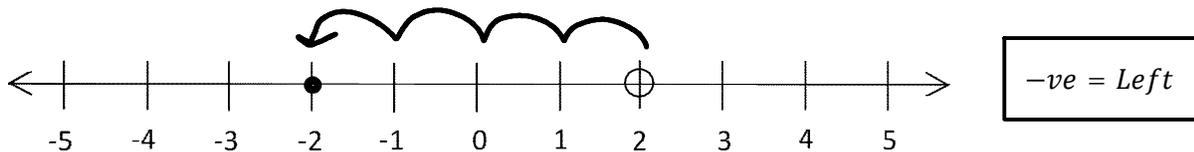


$2 - 4 =$

2 → starting point, place pen on 2 ○

- → "left," move left

4 → move left 4



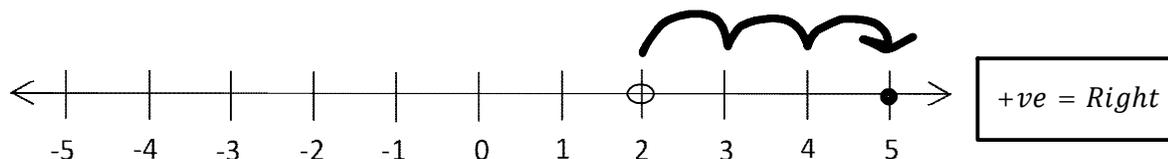
$2 - 4 =$ -2 Check on Calculator!

$2 + 3 =$

2 → starting point, place pen on 2 ○

+ → "right," move right

3 → move right 3



$2 + 3 =$ 5 Check on Calculator!

Same Plus

$5 + (+2)$ $5 \text{ } \textcircled{+} \text{ } \textcircled{+} 2$ \downarrow $5 + 2$ <p>Same Plus</p> 7 Check on Calculator!	$7 - (-4)$ $7 \text{ } \textcircled{-} \text{ } \textcircled{-} 4$ \downarrow $7 + 4$ <p>Same Plus</p> 11 Check on Calculator!
--	---

Different Minus

$10 + (-3)$ $10 \text{ } \textcircled{+} \text{ } \textcircled{-} 3$ \downarrow $10 - 3$ <p>Different Minus</p> 7 Check on Calculator!	$-2 + (-3)$ $-2 \text{ } \textcircled{+} \text{ } \textcircled{-} 3$ \downarrow $-2 - 3$ <p>Different Minus</p> -5 Check on Calculator!
---	--

If you have two of the same sign side-by-side it becomes a positive sign.

If you have two different signs side-by-side it becomes negative

After that use the number line!

*-2 - 2 ≠ 4 = -4

M8 - 8.2 - $\times \div$ Same Plus, Different Minus Notes

If you multiply or divide numbers with two of the same sign we follow the rule "Same Plus."

$$\begin{array}{l} 3 \times 3 = \\ +3 \times +3 = \end{array} \textcircled{+9}$$

"Same plus"

$$\begin{array}{l} -9 \\ -3 \\ -9 \\ -3 \end{array} = \textcircled{+3}$$

"Same plus" Same Plus

$+$	\times	$+$	$=$	$+$
$-$	\times	$-$	$=$	$+$
$+$	\div	$+$	$=$	$+$
$-$	\div	$-$	$=$	$+$

"Same plus"

If you multiply or divide numbers with two different signs we follow the rule "Different Minus."

$$\begin{array}{l} 3 \times -3 = \\ +3 \times -3 = \end{array} \textcircled{-9}$$

"Different minus"

$$\begin{array}{l} -9 \\ 3 \\ -9 \\ +3 \end{array} = \textcircled{-3}$$

"Different minus" Different Minus

$+$	\times	$-$	$=$	$-$
$-$	\times	$+$	$=$	$-$
$+$	\div	$-$	$=$	$-$
$-$	\div	$+$	$=$	$-$

"Different minus"

Don't forget about signs side-by-side!

M8 - 8.3 - BEDMAS: Order of Operations Notes

Check on
Calculator!

- | | |
|---------------------------|------------------|
| B - brackets | Brackets first |
| E - exponents | Exponents second |
| D - division | Division |
| M - multiplication | Multiplication |

A - addition	Addition	}	In order from left to right
S - subtraction	Subtraction		

$$3 \times 4 + 2 =$$

$$12 + 2 = 14$$

Multiply first
Add second

$$10 - 4 \div 2 =$$

$$10 - 2 = 8$$

Divide first
Subtract second

$$2^3 + 4 =$$

$$\textcircled{2^3} + 4 =$$

$$8 + 4 = 12$$

Multiply the number on the base the number of times of the exponent

$2^3 = 2 \times 2 \times 2 = 8$
 ↙ Exponent
 ↘ Base

Check on Calculator!

$7^2 = 7 \times 7 = 49$

Exponents first
Addition second

$$2(3 + 4)^2 =$$

$$2(7)^2 =$$

$$2\textcircled{(7)^2} =$$

$$2(49) = 98$$

Brackets first
Exponents second
Multiply third

$$2 - 3 + 4 =$$

$$-1 + 4 = 3$$

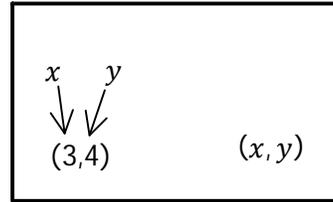
Subtraction First
Addition Second



Do side work Off to the Right

M8 - 9.1 - Plotting Points Graph Notes

(x, y) A point on a graph is given by an "ordered pair"



Plot the following table of values:

x	y
2	-3
-4	-1
-3	5
0	0
4	0
0	-3

Ordered Pairs

$(2, -3)$

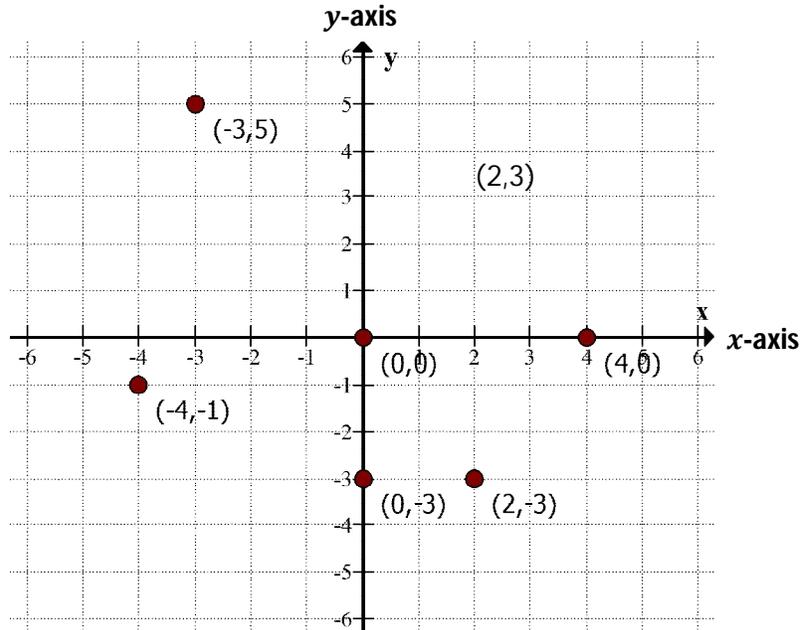
$(-4, -1)$

$(-3, 5)$

$(0, 0)$

$(4, 0)$

$(0, -3)$



Steps to plot a point:

1. Find the x location on the x -axis. (The number in the left of the brackets.)
2. Go straight up or down to the y value. (The number on the right of the brackets).
3. Draw and label the point.

M9 - 9.2 - Graphing TOV: $y = x, y = x + 1$ Notes

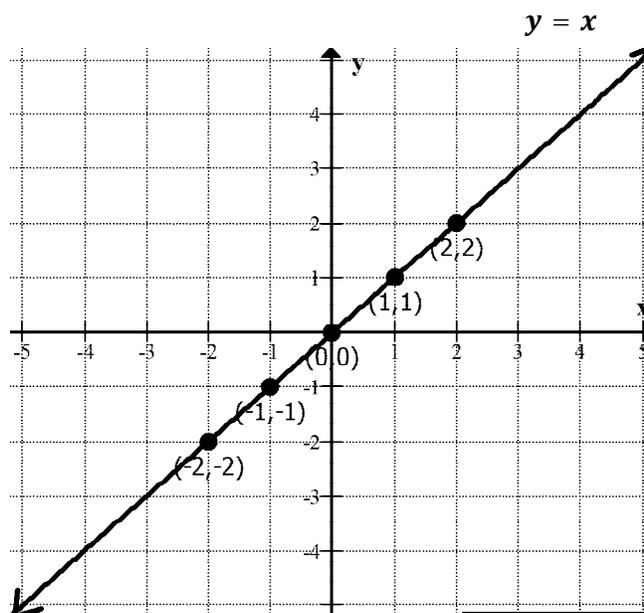
Graph: $y = x$

Start with an empty Table of Values

		$y = x$	
x	y	x	y
-2		-2	-2
-1		-1	-1
0		0	0
1		1	1
2		2	2

Ordered Pairs

- $(-2, -2)$
- $(-1, -1)$
- $(0, 0)$
- $(1, 1)$
- $(2, 2)$



Choose Logical x Values

$$y = x$$

$$y = (-2)$$

$(-2, -2)$

$$y = x$$

$$y = (-1)$$

$(-1, -1)$

$$y = x$$

$$y = (0)$$

$(0, 0)$

$$y = x$$

$$y = (1)$$

$(1, 1)$

$$y = x$$

$$y = (2)$$

$(2, 2)$

Write the Formula

(Substitute with Brackets)

Substitute (x) values in the Formula

Put the y value into the Table

Write the Point (x, y)

Graph and Label the Points (x, y)

Draw and Label the Line

(with Arrow Tips)

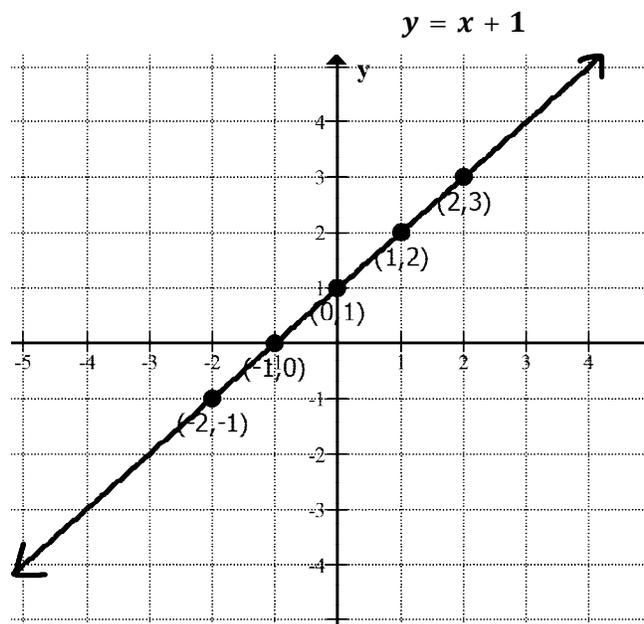
Graph: $y = x + 1$

$$y = x + 1$$

x	y
-2	-1
-1	0
0	1
1	2
2	3

Ordered Pairs

- $(-2, -1)$
- $(-1, 0)$
- $(0, 1)$
- $(1, 2)$
- $(2, 3)$



OR Do it in your head!

$$y = x + 1$$

$$y = (-2) + 1$$

$$y = -1$$

$(-2, -1)$

$$y = x + 1$$

$$y = (-1) + 1$$

$$y = 0$$

$(-1, 0)$

$$y = x + 1$$

$$y = (0) + 1$$

$$y = 1$$

$(0, 1)$

$$y = x + 1$$

$$y = (1) + 1$$

$$y = 2$$

$(1, 2)$

Notice: the graph of $y = x + 1$ is the graph of $y = x$, moved up 1. (Or Left One*)

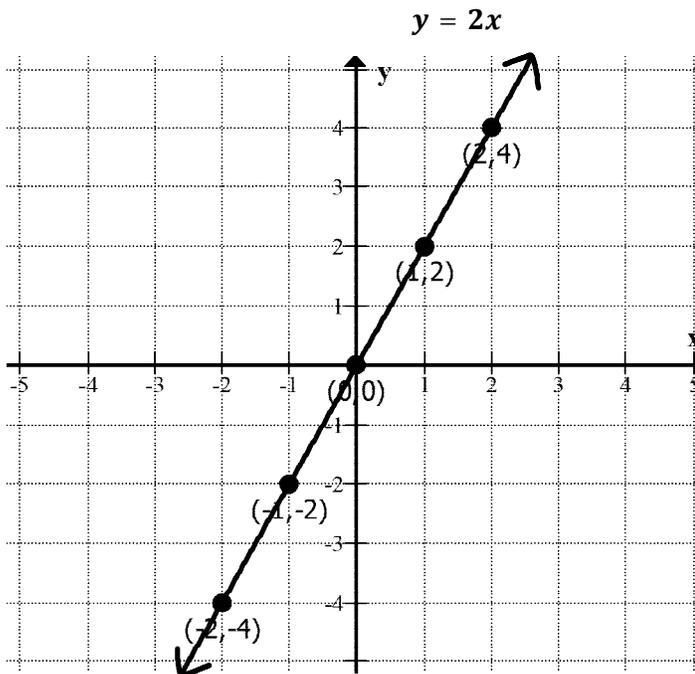
M9 - 9.2 - Graphing TOV: $y=2x$, $y = 2x + 1$ Notes

Graph: $y = 2x$

$y = 2x$	
x	y
-2	-4
-1	-2
0	0
1	2
2	4

Ordered Pairs

- $(-2, -4)$
- $(-1, -2)$
- $(0, 0)$
- $(1, 2)$
- $(2, 4)$



$$y = 2x$$

$$y = 2(-2)$$

$$y = -4$$

$(-2, -4)$

$$y = 2x$$

$$y = 2(-1)$$

$$y = -2$$

$(-1, -2)$

$$y = 2x$$

$$y = 2(0)$$

$$y = 0$$

$(0, 0)$

$$y = 2x$$

$$y = 2(1)$$

$$y = 2$$

$(1, 2)$

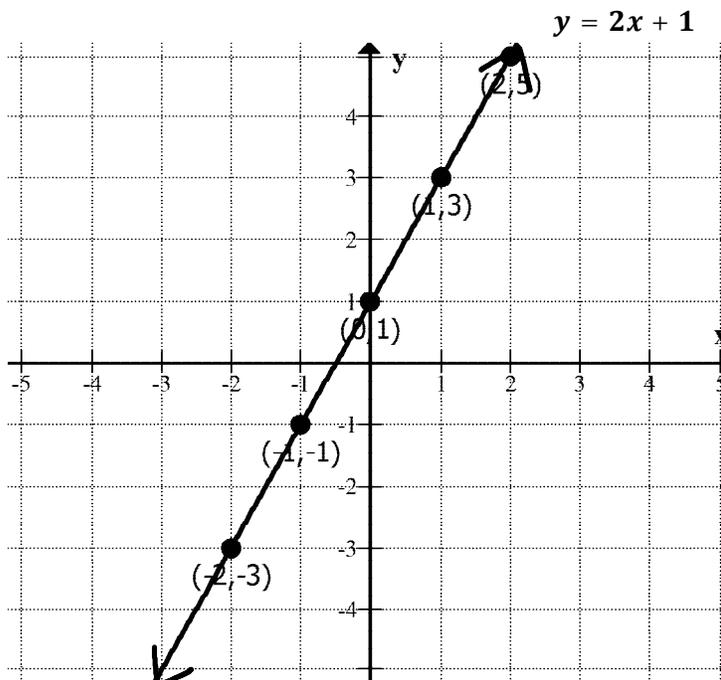
Notice: the graph of $y = 2x$ is twice as steep as the graph of $y = x$.

Graph: $y = 2x + 1$

$y = 2x + 1$	
x	y
-2	-3
-1	-1
0	1
1	3
2	5

Ordered Pairs

- $(-2, -3)$
- $(-1, -1)$
- $(0, 1)$
- $(1, 3)$
- $(2, 5)$



$$y = 2x + 1$$

$$y = 2(-2) + 1$$

$$y = -4 + 1$$

$$y = -3$$

$(-2, -3)$

$$y = 2x + 1$$

$$y = 2(-1) + 1$$

$$y = -2 + 1$$

$$y = -1$$

$(-1, -1)$

$$y = 2x + 1$$

$$y = 2(0) + 1$$

$$y = 0 + 1$$

$$y = 1$$

$(0, 1)$

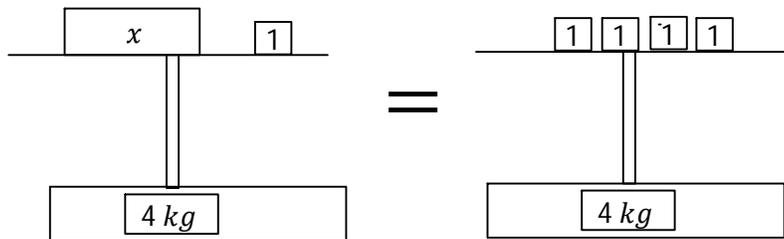
Notice: the graph of $y = 2x + 1$ is the graph of $y = 2x$ up 1.

M8 - 10.0 - Golden Rule Scale Picture Notes

$\boxed{1} = 1\text{kg}$

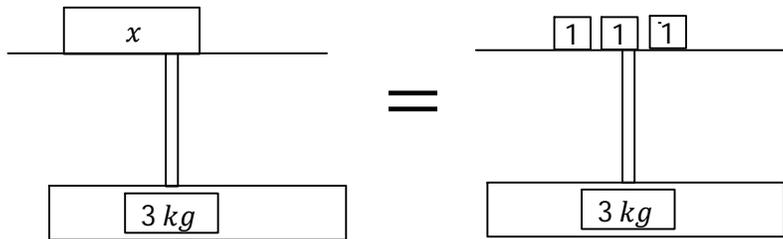
The Golden Rule: Whatever you do to the right side of the equal sign, do to the left side.

What plus 1 = 4?



$$x + 1 = 4$$

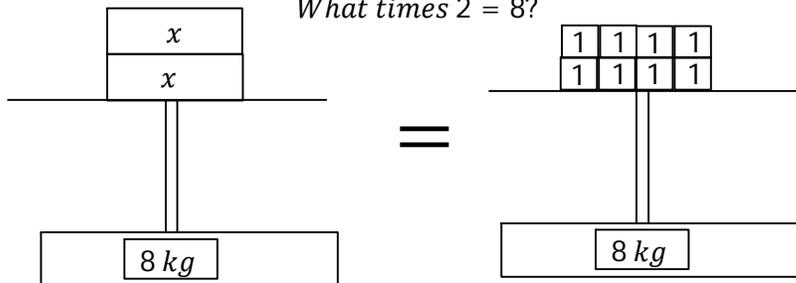
$$\begin{array}{r} x + 1 = 4 \\ -1 \quad -1 \\ \hline x = 3 \end{array}$$



Take off one from both sides

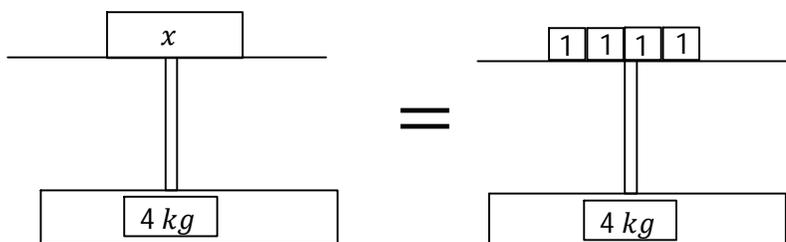
$$x = 3$$

What times 2 = 8?



$$2x = 8$$

$$\begin{array}{r} 2x = 8 \\ \hline \frac{2x}{2} = \frac{8}{2} \\ x = 4 \end{array}$$



Divide both sides by two

$$x = 4$$

M8 - 10.2 - " $ax = b$ " " $\frac{x}{a} = b$ " " $\frac{ax}{b} = c$ " Notes

Answer should say $x = \underline{\quad}$

Solve for x , by dividing to both sides.

$$2x = 4$$

$$\frac{2x}{2} = \frac{4}{2}$$

Divide both sides by 2

Divide both sides by the coefficient on x .

~~$$\frac{2x}{2} = \frac{4}{2}$$~~

Cross it off

$$\frac{2}{2} = 1$$

$$x = \frac{4}{2}$$

$$x = 2$$

Check Answer

$$\begin{aligned} 2x &= 4 \\ 2(2) &= 4 \\ 4 &= 4 \quad \checkmark \end{aligned}$$

Question
Substitute
Left Must Equal Right

Short Forms

~~$$2x = 4$$~~

$$x = 2$$

~~$$2x = \frac{4}{2}$$~~

$$x = 2$$

Solve for x , by multiplying to both sides.

$$\frac{x}{3} = 6$$

$$3 \times \frac{x}{3} = 6 \times 3$$

Multiply both sides by 3

Multiply both sides by number below the letter

~~$$3 \times \frac{x}{3} = 6 \times 3$$~~

Cross it off

$$\frac{3}{3} = 1$$

$$x = 6 \times 3$$

$$x = 18$$

Check Answer

$$\begin{aligned} \frac{x}{3} &= 6 \\ \frac{18}{3} &= 6 \\ 6 &= 6 \quad \checkmark \end{aligned}$$

Short Forms

~~$$3 \times \frac{x}{3} = 6 \times 3$$~~

$$x = 18$$

~~$$\frac{x}{3} = 6 \times 3$$~~

$$x = 18$$

Solve for x

$$\frac{5}{4}x = 10$$

$$4 \times \frac{5}{4}x = 10 \times 4$$

Multiply both sides by 4

~~$$4 \times \frac{5}{4}x = 10 \times 4$$~~

$$5x = 40$$

$$\frac{5x}{5} = \frac{40}{5}$$

Divide both sides by 5

~~$$\frac{5x}{5} = \frac{40}{5}$$~~

$$x = \frac{40}{5}$$

$$x = 8$$

Check Answer

$$\begin{aligned} \frac{5}{4}x &= 10 \\ \frac{5}{4}(8) &= 10 \\ 10 &= 10 \quad \checkmark \end{aligned}$$

Short Form

$$\begin{aligned} 4 \times \frac{5}{4}x &= 10 \times 4 \\ \frac{5x}{5} &= \frac{40}{5} \\ x &= 8 \end{aligned}$$

~~$$\frac{5}{4}x = 10 \times \frac{4}{5}$$~~

$$x = 8$$

M8 - 10.3 - " $\frac{a}{x} = b$ " " $\frac{a}{bx} = c$ " Notes

Solve for x

$$\frac{8}{x} = 4$$

$$x \times \frac{8}{x} = 4 \times x$$

Multiply x to both sides

Multiply both sides by the denominator

Short Form

$$\begin{array}{l} \frac{8}{x} = 4 \\ \frac{x}{x} \\ \frac{8}{4} = x \\ \boxed{x = 2} \end{array}$$

~~$$x \times \frac{8}{x} = 4 \times x$$~~

Cross it off

$$8 = 4x$$

$$\frac{8}{4} = \frac{4x}{4}$$

Divide both sides by 4

$$\boxed{2 = x}$$

Check Answer

$$\begin{array}{l} \frac{8}{x} = 4 \\ \frac{x}{x} \\ \frac{8}{2} = 4 \\ 4 = 4 \quad \checkmark \end{array}$$

Solve for x

$$\frac{24}{2x} = 3$$

~~$$2x \times \frac{24}{2x} = 3 \times 2x$$~~

Multiply $2x$ to both sides

Short Form

$$\begin{array}{l} \frac{24}{2x} = 3 \\ \frac{2x}{2x} \\ \frac{24}{2(3)} = x \\ \boxed{x = 4} \end{array}$$

$$24 = 6x$$

$$\frac{24}{6} = \frac{6x}{6}$$

Divide both sides by 6

$$\boxed{4 = x}$$

Check Answer

$$\begin{array}{l} \frac{24}{2x} = 3 \\ \frac{2x}{2x} \\ \frac{24}{2(4)} = 3 \\ \frac{24}{8} = 3 \\ 3 = 3 \quad \checkmark \end{array}$$

M8 - 10.4 - " $\frac{ax}{bx} = \frac{c}{d}$ " Cross Multiply Notes

Solve for x , by multiplying both sides by the opposite denominator.

$$\frac{x}{6} = \frac{4}{3}$$

$$\frac{x}{\cancel{6}} = \frac{4}{\cancel{3}}$$

$$3 \times x = 4 \times 6$$

$$3x = 24$$

$$\frac{3x}{3} = \frac{24}{3}$$

$$x = 8$$

Denominators Multiply to Opposite Side Numerator

Divide both sides by 3

Check Answer

$$\frac{x}{6} = \frac{4}{3}$$

$$\frac{8}{6} = \frac{4}{3}$$

$$\frac{6}{6} = \frac{4}{3}$$

$$\frac{4}{4} = \frac{4}{3}$$

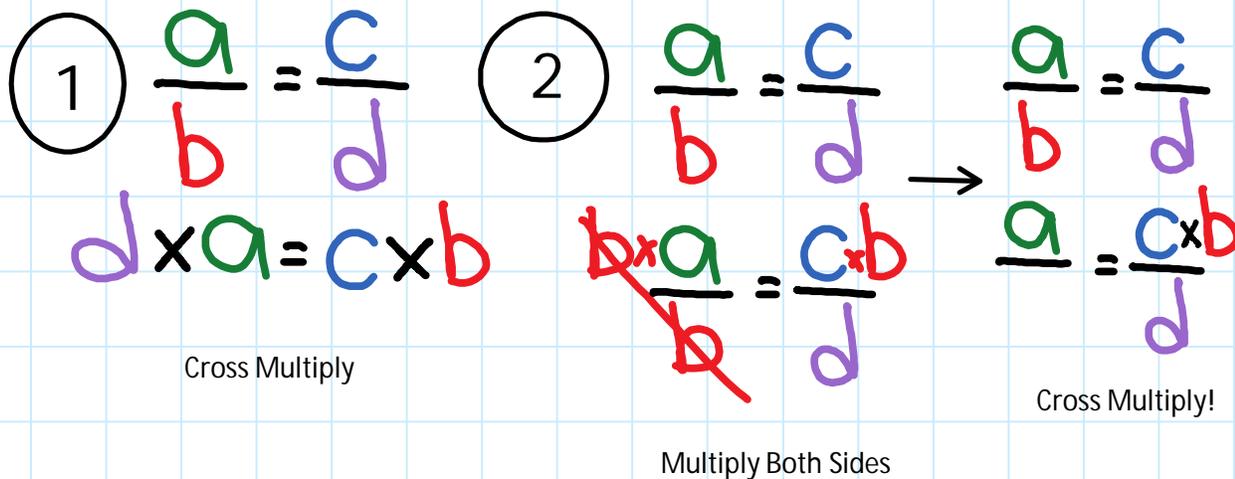
$$\frac{1}{3} = \frac{1}{3} \quad \checkmark$$

Short Form

$$\frac{x}{6} = \frac{4}{3}$$

$$3 \times x = 4 \times 6$$

...



Equivalent Fractions	Algebra	Cross Multiplication
$\frac{x}{2} = 4$ $\frac{x}{2} = \frac{4}{1}$ $x = 4 \times 2$ $\frac{x}{2} = \frac{1 \times 2}{1}$ $\frac{x}{2} = \frac{8}{1}$ $\frac{x}{\cancel{2}} = \frac{8}{\cancel{1}}$ $x = 8$	$\frac{x}{2} = 4$ $\frac{x}{2} = \frac{4}{1}$ $2 \times \frac{x}{2} = \frac{4}{1} \times 2$ $\frac{x}{\cancel{2}} = \frac{4}{\cancel{1}} \times 2$ $x = 4 \times 2$ $x = 8$	$\frac{x}{2} = 4$ $\frac{x}{2} = \frac{4}{1}$ $1 \times x = 4 \times 2$ $1x = 8$ $x = 8$
$\frac{x}{2} = 4$ $\frac{x}{2} = \frac{8}{1}$ $\frac{x}{\cancel{2}} = \frac{8}{\cancel{1}}$ $x = 8$	$\frac{x}{2} = 4 \times 2$ $\frac{x}{2} = 8$ $x = 8$	$\frac{x}{2} = 4$ $\frac{x}{2} = \frac{4}{1}$ $1x = 4 \times 2$ $x = 8$

M8 - 10.5 - " $\pm ax + b = c, \frac{x}{a} + b = c$ " Notes

Solve for x

$$6x + 8 = 50$$

$$\begin{array}{r} 6x + 8 = 50 \\ -8 \quad -8 \end{array}$$

Subtract 8 from both sides

$$6x = 42$$

$$\frac{6x}{6} = \frac{42}{6}$$

Divide both sides by 6

$$\frac{\cancel{6}x}{\cancel{6}} = \frac{42}{6}$$

Cross it off

$$x = \frac{42}{6}$$

$$x = 7$$

Short Form

$$\begin{array}{l} 6x + 8 = 50 \\ 6x = 50 - 8 \\ 6x = 42 \\ \boxed{x = 7} \end{array}$$

Check Answer

$$\begin{array}{l} 6x + 8 = 50 \\ 6(7) + 8 = 50 \\ 42 + 8 = 50 \\ 50 = 50 \quad \checkmark \end{array}$$

Solve for x

$$\frac{x}{3} - 8 = -3$$

$$\begin{array}{r} \frac{x}{3} - 8 = -3 \\ +8 \quad +8 \end{array}$$

Add 8 to both sides

$$\frac{x}{3} = 5$$

$$\frac{\cancel{x}}{\cancel{3}} \times 3 = 5 \times 3$$

Multiply both sides by 3

$$x = 5 \times 3$$

$$x = 15$$

Short Form

$$\begin{array}{l} \frac{x}{3} - 8 = -3 \\ \frac{x}{3} = -3 + 8 \\ \frac{x}{3} = 5 \\ x = 15 \end{array}$$

Check Answer

$$\begin{array}{l} \frac{x}{3} - 8 = -3 \\ \frac{15}{3} - 8 = -3 \\ 5 - 8 = -3 \\ -3 = -3 \quad \checkmark \end{array}$$

M8 - 10.6 - " $a(x + b) = c, \frac{a}{x+b} = c$ " Distribution Notes

Solve for x , by **Distributing a into $x + b$.**

$$-4(x - 3) = -8$$

$$\begin{array}{l} \curvearrowright \\ -4(x - 3) = -8 \end{array}$$

$$-4x + 12 = -8$$

$$\begin{array}{l} -4x + 12 = -8 \\ \cancel{-12} \quad \cancel{-12} \end{array}$$

$$-4x = -20$$

$$\begin{array}{l} \cancel{-4x} \quad \cancel{-20} \\ \cancel{-4} \quad \cancel{-4} \end{array}$$

$$x = \frac{-20}{-4}$$

$$x = 5$$

Distribute

$$\begin{array}{c} \text{Distribution} \\ \curvearrowright \\ -4(x - 3) = -4x + 12 \end{array}$$

Multiply the number in front of the brackets into both numbers inside the brackets.

Check Answer

$$-4(x - 3) = -8$$

$$-4(5 - 3) = -8$$

$$-4(2) = -8$$

$$-8 = -8 \quad \checkmark$$

OR

Divide 1st

$$-4(x - 3) = -8$$

$$\begin{array}{l} \cancel{-4} \cancel{(x - 3)} = \frac{-8}{\cancel{-4}} \\ \phantom{\cancel{-4} \cancel{(x - 3)}} = -4 \end{array}$$

$$x - 3 = 2$$

$$\begin{array}{l} x - 3 = 2 \\ \cancel{-3} \quad \cancel{+3} \end{array}$$

$$x = 5$$

Short Forms

$$-4(x - 3) = -8$$

$$x - 3 = 2$$

$$4x = 20$$

$$x = 5$$

$$-4(x - 3) = -8$$

$$-4x + 12 = -8$$

$$-4x = -20$$

$$x = 5$$

Solve for x , by **Distributing a into $x + b$.**

$$\begin{array}{l} \curvearrowright \\ \frac{1}{2}(x + 4) = 6 \end{array}$$

$$\begin{array}{l} x \quad 4 \\ \frac{1}{2} + \frac{1}{2} = 6 \end{array}$$

$$\begin{array}{l} \cancel{x} \quad \cancel{4} \\ \frac{1}{2} + 2 = 6 \end{array}$$

$$\begin{array}{l} \cancel{-2} \quad \cancel{-2} \\ \frac{x}{2} = 4 \end{array}$$

$$\begin{array}{l} x \\ \frac{1}{2} = 4 \end{array}$$

$$\begin{array}{l} \cancel{2} \times \cancel{\frac{1}{2}} = 4 \times 2 \end{array}$$

$$x = 8$$

Distribute

Check Answer

$$\frac{1}{2}(x + 4) = 6$$

$$\frac{1}{2}(8 + 4) = 6$$

$$\frac{1}{2}(12) = 6$$

$$6 = 6 \quad \checkmark$$

OR

Multiply 1st

$$\frac{1}{2}(x + 4) = 6$$

$$\begin{array}{l} \cancel{2} \times \cancel{\frac{1}{2}} (x + 4) = 6 \times 2 \end{array}$$

$$\begin{array}{l} x + 4 = 12 \\ \cancel{-4} \quad \cancel{-4} \end{array}$$

$$x = 8$$

Short Forms

$$\frac{1}{2}(x + 4) = 6$$

$$x + 4 = 12$$

$$x = 8$$

$$\frac{1}{2}(x + 4) = 6$$

$$\frac{x}{2} + 2 = 6$$

$$\frac{x}{2} = 4$$

$$x = 8$$

Solve for x , by **multiplying to both sides by $x + b$.**

$$\frac{14}{x - 3} = 2$$

$$(x - 3) \times \frac{14}{x - 3} = 2 \times (x - 3)$$

$$\begin{array}{l} \cancel{(x - 3)} \times \frac{14}{\cancel{x - 3}} = 2 \times (x - 3) \end{array}$$

$$14 = 2x - 6$$

$$\begin{array}{l} +6 \quad +6 \\ 20 = 2x \end{array}$$

$$20 = 2x$$

$$\frac{20}{2} = \frac{2x}{2}$$

$$\frac{10}{1} = \frac{x}{1}$$

$$10 = x$$

$$x = 10$$

Multiply $x - 3$ to both sides

Cross it off

Distribute

Check Answer

$$\frac{14}{x - 3} = 2$$

$$\frac{14}{10 - 3} = 2$$

$$\frac{14}{7} = 2$$

$$2 = 2 \quad \checkmark$$

Short Form

$$\frac{14}{x - 3} = 2$$

$$14 = 2(x - 3)$$

$$14 = 2x - 6$$

$$20 = 2x$$

$$x = 10$$

M8 - 10.7 - LCD " $\frac{x}{a} + \frac{b}{c} = \frac{d}{e}$ " Notes

Solve for x by multiplying each term by the LCD

$$\begin{aligned}
 x - 1 &= \frac{1}{2} \\
 2 \times (x - 1) &= \frac{1}{2} \times 2 \\
 2x - 2 &= 1 \\
 +2 &+2 \\
 2x &= 3 \\
 \frac{2x}{2} &= \frac{3}{2} \\
 x &= \frac{3}{2}
 \end{aligned}$$

LCD = 2

Multiply both sides by 2
Distribute
Add 2 to both sides
Divide both sides by 2

Check Answer

$$\begin{aligned}
 x - 1 &= \frac{1}{2} \\
 \frac{3}{2} - 1 &= \frac{1}{2} \\
 \frac{3}{2} - \frac{2}{2} &= \frac{1}{2} \\
 \frac{1}{2} &= \frac{1}{2} \quad \checkmark
 \end{aligned}$$

Short Form

$$\begin{aligned}
 x - 1 &= \frac{1}{2} \\
 2(x - 1) &= 1 \\
 2x - 2 &= 1 \\
 2x &= 3 \\
 x &= \frac{3}{2}
 \end{aligned}$$

OR

Algebra	Add Fractions
$ \begin{aligned} x - 1 &= \frac{1}{2} \\ +1 &+1 \\ x &= \frac{3}{2} \end{aligned} $	$ \begin{aligned} \frac{1}{2} + 1 &= \frac{1}{2} + \frac{2}{2} \\ &= \frac{3}{2} \end{aligned} $
	<p>Expand $1 = \frac{1}{1} = \frac{1 \times 2}{1 \times 2} = \frac{2}{2}$</p> <p>LCD = 2</p>

Solve for x by multiplying each term by the LCD

$$\begin{aligned}
 x - \frac{1}{4} &= \frac{1}{2} \\
 4 \times (x - \frac{1}{4}) &= \frac{1}{2} \times 4 \\
 4x - \frac{4}{4} &= \frac{4}{2} \\
 4x - 1 &= 2 \\
 +1 &+1 \\
 4x &= 3 \\
 \frac{4x}{4} &= \frac{3}{4} \\
 x &= \frac{3}{4}
 \end{aligned}$$

LCD = 4

Multiply both sides by 4
Distribute
Add 1 to both sides
Divide both sides by 4

Check Answer

$$\begin{aligned}
 x - \frac{1}{4} &= \frac{1}{2} \\
 \frac{3}{4} - \frac{1}{4} &= \frac{1}{2} \\
 \frac{2}{4} &= \frac{1}{2} \\
 \frac{1}{2} &= \frac{1}{2} \quad \checkmark
 \end{aligned}$$

Short Form

$$\begin{aligned}
 x - \frac{1}{4} &= \frac{1}{2} \\
 (x - \frac{1}{4} = \frac{1}{2}) \times 4 & \\
 4x - 1 &= 2 \\
 4x &= 3 \\
 x &= \frac{3}{4}
 \end{aligned}$$

Instead of actually multiplying by the LCD we are going to multiply and simplify at the same time.

Solve for x by multiplying each term by the LCD

$$\begin{aligned}
 \frac{x}{2} + \frac{1}{4} &= \frac{1}{3} \\
 (\frac{x}{2} + \frac{1}{4} = \frac{1}{3}) \times 12 & \\
 \frac{12x}{2} + \frac{12}{4} &= \frac{12}{3} \\
 6x + 3 &= 4 \\
 -3 &-3 \\
 6x &= 1 \\
 x &= \frac{1}{6}
 \end{aligned}$$

LCD = 12

Multiply both sides by 12
Distribute
Simplify
Algebra

Check Answer

$$\begin{aligned}
 \frac{x}{2} + \frac{1}{4} &= \frac{1}{3} \\
 (\frac{1}{6}) + \frac{1}{4} &= \frac{1}{3} \\
 \frac{1}{12} + \frac{3}{12} &= \frac{4}{12} \\
 \frac{4}{12} &= \frac{1}{3} \quad \checkmark
 \end{aligned}$$

Fractions $\div +$

$$\begin{aligned}
 (\frac{1}{6}) & \quad \frac{1}{12} + \frac{1}{4} \\
 \frac{1}{6} \div 2 & \quad \frac{1}{12} + \frac{3}{12} \\
 \frac{1}{6} \times \frac{1}{2} & \quad \frac{4}{12} \\
 \frac{1}{12} & \quad \frac{1}{3}
 \end{aligned}$$

Short Form

$$\begin{aligned}
 (\frac{x}{2} + \frac{1}{4} = \frac{1}{3}) \times 12 & \\
 6x + 3 &= 4 \\
 6x &= 1 \\
 x &= \frac{1}{6}
 \end{aligned}$$

M8 - 10.8 - Combining Like Terms Notes

Combine the like terms: Add/Subtract like Terms

$$x + x = (2x) \quad x + 2x = (3x) \quad 2x + 4x = (6x) \quad 6x - 4x = (2x) \quad 2x - 5x = (-3x) \quad x - x = (0)$$

Solve for x

$$\begin{aligned} x &= 1 + 2 \\ x &= 3 \end{aligned}$$

Combine Like Terms

$$\begin{aligned} x + x &= 4 \\ 2x &= 4 \\ \cancel{2x} & \quad \cancel{4} \\ x &= 2 \end{aligned}$$

Check Answer

$$\begin{aligned} x + x &= 4 \\ 2 + 2 &= 4 \\ 4 &= 4 \quad \checkmark \end{aligned}$$

$$3x + 3x = 4 + 8$$

$$\begin{aligned} 6x &= 12 \\ \cancel{6x} & \quad \cancel{12} \\ x &= 2 \end{aligned}$$

Check Answer

$$\begin{aligned} 3x + 3x &= 4 + 8 \\ 3(2) + 3(2) &= 4 + 8 \\ 6 + 6 &= 12 \\ 12 &= 12 \quad \checkmark \end{aligned}$$

Solve for x, by combining like terms by adding and subtracting to both sides

$$2x = 4 + x$$

$$\begin{aligned} 2x &= 4 + x \\ -x & \quad -x \\ x &= 4 \end{aligned}$$

Work on the complicated side!

Subtract x from both sides

Check Answer

$$\begin{aligned} 2x &= 4 + x \\ 2(4) &= 4 + (4) \\ 8 &= 8 \quad \checkmark \end{aligned}$$

Short Form

$$\begin{aligned} 2x &= 4 + x \\ x &= 4 \end{aligned}$$

$$\begin{aligned} 2x &= 4 + x \\ -4 & \quad -4 \\ 2x - 4 &= x \\ -2x & \quad -2x \\ -4 &= -x \\ -4 & \quad -x \\ -1 &= -1 \end{aligned}$$

$$x = 4$$

Not Optimal!!!

Solve for x, by combining like terms

$$3x + 2 = 2x + 6$$

$$\begin{aligned} 3x + 2 &= 2x + 6 \\ -2 & \quad -2 \\ 3x &= 2x + 4 \\ -2x & \quad -2x \end{aligned}$$

$$x = 4$$

Subtract 2 from both sides

Subtract 2x from both sides

Check Answer

$$\begin{aligned} 3x + 2 &= 2x + 6 \\ 3(4) + 2 &= 2(4) + 6 \\ 12 + 2 &= 8 + 6 \\ 14 &= 14 \quad \checkmark \end{aligned}$$

Short Form

$$\begin{aligned} 3x + 2 &= 2x + 6 \\ x &= 4 \end{aligned}$$

Solve for x, by combining like terms

$$3x - 1 + 4x = x + 11$$

$$\begin{aligned} 3x + 4x - 1 &= x + 11 \\ 7x - 1 &= x + 11 \\ +1 & \quad +1 \\ 7x &= x + 12 \\ -x & \quad -x \\ 6x &= 12 \\ \frac{6x}{6} &= \frac{12}{6} \\ x &= 2 \end{aligned}$$

Rearrange Order of Terms (Signs!!!)
Combine Like Terms

Algebra

Check Answer

$$\begin{aligned} 3x - 1 + 4x &= x + 11 \\ 3(2) - 1 + 4(2) &= (2) + 11 \\ 6 - 1 + 8 &= 2 + 11 \\ 13 &= 13 \quad \checkmark \end{aligned}$$

Short Form

$$\begin{aligned} 3x - 1 + 4x &= x + 11 \\ 6x &= 12 \\ x &= 2 \end{aligned}$$

M8 - 10.9 - Creating/Solving Equations Notes

Pick a Number.

Let $x = \text{the number}$

Let Statements

Word	Meaning
Sum, More, Add, Increased	+
Difference, Less, Subtract, Decreased, Take away	-
Product, Times, Multiplied	×
Quotient, Divide, Split	÷

Words Problems

Let Statements
Equation
Isolate
Solve (Algebra)
Answer!
Check Answer!

Expressions

Three more than a number

$$x + 3$$

Eight less than a number

$$x - 8$$

A number less than four

$$4 - x$$

Five times a number

$$5x$$

A third of a number

$$\frac{1}{3}x$$

Eight divided by a number

$$\frac{8}{x}$$

Twice the sum of a number and three

$$2(x + 3)$$

A number plus four "ALL" divided by two

$$\frac{x + 4}{2}$$

Create and Solve the following:

Five more than a number is 8. What is the number?

Let $x = \text{the \#}$

Let Statements

$$x + 5 = 8$$

Create Equation

$$\begin{array}{r} x + 5 = 8 \\ -5 \quad -5 \\ \hline x = 3 \end{array}$$

Solve

Check Answer

$$\begin{array}{l} x + 5 = 8 \\ (3) + 5 = 8 \\ 8 = 8 \quad \checkmark \end{array}$$

The number is 3

Answer the question

Twice the "SUM" of a number and three is 12. What is the number?

Let $x = \text{the number}$

$$2(x + 3) = 12$$

$$\begin{array}{r} 2(x + 3) = 12 \\ 2x + 6 = 12 \\ -6 \quad -6 \\ \hline 2x = 6 \\ \frac{2x}{2} = \frac{6}{2} \end{array}$$

$$x = 3$$

Check Answer

$$\begin{array}{l} 2(x + 3) = 12 \\ 2((3) + 3) = 12 \\ 2(6) = 12 \quad \checkmark \end{array}$$

The number is 3

Three less than twice a number is 7. What is the number?

Let $x = \#$

$$2x - 3 = 7$$

$$\begin{array}{r} 2x - 3 = 7 \\ +3 \quad +3 \\ \hline 2x = 10 \\ \frac{2x}{2} = \frac{10}{2} \end{array}$$

$$x = 5$$

The number is 5

Check Answer

$$\begin{array}{l} 2x - 3 = 7 \\ 2(5) - 3 = 7 \\ 10 - 3 = 7 \\ 7 = 7 \quad \checkmark \end{array}$$

Five times a number plus three "ALL" divided by two equals triple the number. What is the number?

Let $x = \#$

$$\frac{(5x + 3)}{2} = 3x$$

$$\begin{array}{r} \cancel{2} \times \frac{5x + 3}{\cancel{2}} = 3x \times 2 \\ 5x + 3 = 6x \\ -5x \quad -5x \end{array}$$

$$x = 3$$

The number is 3

Check Answer

$$\begin{array}{l} \frac{5x + 3}{2} = 3x \\ \frac{5(3) + 3}{2} = 3(3) \\ \frac{18}{2} = 9 \\ 9 = 9 \quad \checkmark \end{array}$$

M8 - 10.9 - One vs Two Variable Equations Notes

Create and Solve the following:

One number is two more than another and their sum is 12. What are the numbers?

Let $x = 1st \#$
Let $x - 2 = 2nd \#$

One Variable!

$$\begin{aligned} x + (x - 2) &= 12 \\ x + x - 2 &= 12 \\ 2x - 2 &= 12 \\ +2 \quad +2 & \\ 2x &= 14 \\ 2x &= 14 \\ \frac{2x}{2} &= \frac{14}{2} \end{aligned}$$

$$x = 7$$

$$\begin{aligned} 1st \# &= 7 \\ 2nd \# &= 5 \end{aligned}$$

$$\begin{aligned} 2nd\# &= x - 2 \\ &= (7) - 2 \\ 2nd\# &= 5 \end{aligned}$$

Let $x = 1st \#$
Let $x + 2 = 2nd \#$

$$\begin{aligned} x + (x + 2) &= 12 \\ x + x + 2 &= 12 \\ 2x + 2 &= 12 \\ -2 \quad -2 & \\ 2x &= 10 \\ 2x &= 10 \\ \frac{2x}{2} &= \frac{10}{2} \end{aligned}$$

$$x = 5$$

$$\begin{aligned} 1st \# &= 5 \\ 2nd \# &= 7 \end{aligned}$$

OR

$$\begin{aligned} 2nd\# &= x + 2 \\ &= (5) + 2 \\ 2nd\# &= 7 \end{aligned}$$

Words Problems

Let Statements
Equation
Solve (Algebra)
Answer!
Check Answer!

One number is two more than another and their sum is 12. What are the numbers?

Let $x = 1st \#$
Let $y = 2nd \#$

Two Variable!

OR

$$x + y = 12$$

Equation #1

$$x - y = 2$$

Equation #2

Words Problems

$$\begin{aligned} x + y &= 12 \\ -x & \quad -x \end{aligned}$$

$$y = (12 - x) \text{ Isolate a Variable}$$

(Substitute into other Equation)

$$\begin{aligned} x - (12 - x) &= 2 \\ x - 12 + x &= 2 \\ 2x - 12 &= 2 \\ +12 \quad +12 & \\ 2x &= 14 \\ 2x &= 14 \\ \frac{2x}{2} &= \frac{14}{2} \end{aligned}$$

$$\begin{aligned} x - y &= 2 \\ y - x &= 2 \\ y - 2 &= x \\ x - 2 &= y \\ x + 2 &= y \\ y + 2 &= x \end{aligned}$$

It Doesn't Matter!

Solve

Let Statements
Equation/s
Isolate
Substitute
Solve (Algebra)
Substitute
Solve
Answer!
Check Answer!

$$\begin{aligned} y &= 12 - x \\ y &= 12 - (7) \end{aligned}$$

(Substitute)

$$x = 7$$

$$y = 5 \text{ Solve}$$

Answer

$$\begin{aligned} 1st \# &= 7 \\ 2nd \# &= 5 \end{aligned}$$

Check Answer

$$\begin{aligned} 5 + 2 &= 7 \quad \checkmark \\ 5 + 7 &= 12 \quad \checkmark \end{aligned}$$

M8 - 10.9 - 2/3 Number/Consecutive Equations Notes

Create and Solve the following:

The sum of three numbers is 67. The 2nd number one less than is twice the 1st. The 3rd number is four more than the 1st.

$$\begin{aligned} \text{Let } x &= \text{1st \#} \\ \text{Let } 2x - 1 &= \text{2nd \#} \\ \text{Let } x + 4 &= \text{3rd \#} \end{aligned}$$

$$x + 2x - 1 + x + 4 = 67$$

$$x + 2x - 1 + x + 4 = 67$$

$$4x + 3 = 67$$

$$\begin{array}{r} -3 \quad -3 \\ 4x = 64 \end{array}$$

$$4x = 64$$

$$\frac{4x}{4} = \frac{64}{4}$$

$$\frac{4x}{4} = \frac{64}{4}$$

$$\text{1st \#} = 16$$

$$x = 16$$

$$\begin{aligned} \text{2nd\#} &= 2x - 1 \\ &= 2(16) - 1 \\ &= 32 - 1 \end{aligned}$$

$$\text{2nd\#} = 31$$

$$\begin{aligned} \text{3rd\#} &= x + 4 \\ &= (16) + 4 \end{aligned}$$

$$\text{3rd\#} = 20$$

$$\text{1st \#} = 16$$

$$\text{2nd \#} = 31$$

$$\text{3rd \#} = 20$$

Check Answer

$$16 + 31 + 20 = 67 \quad \checkmark$$

The sum of three consecutive integers is 24.

$$\begin{aligned} \text{Let } x &= \text{1st \#} \\ \text{Let } x + 1 &= \text{2nd \#} \\ \text{Let } x + 2 &= \text{3rd \#} \end{aligned}$$

$$x + x + 1 + x + 2 = 24$$

$$x + x + 1 + x + 2 = 24$$

$$3x + 3 = 24$$

$$\begin{array}{r} -3 \quad -3 \\ 3x = 21 \end{array}$$

$$3x = 21$$

$$\frac{3x}{3} = \frac{21}{3}$$

$$\frac{3x}{3} = \frac{21}{3}$$

$$\text{1st \#} = 7$$

$$x = 7$$

$$\begin{aligned} \text{2nd\#} &= x + 1 \\ &= (7) + 1 \end{aligned}$$

$$\text{2nd\#} = 8$$

$$\begin{aligned} \text{3rd\#} &= x + 2 \\ &= (7) + 2 \end{aligned}$$

$$\text{3rd\#} = 9$$

$$\text{1st \#} = 7$$

$$\text{2nd \#} = 8$$

$$\text{3rd \#} = 9$$

Check Answer

$$7 + 8 + 9 = 24 \quad \checkmark$$

Find three consecutive odd integers where five less than triple the 2nd is quadruple the 1st.

$$\begin{aligned} \text{Let } x &= \text{1st \#} \\ \text{Let } x + 2 &= \text{2nd \#} \\ \text{Let } x + 4 &= \text{3rd \#} \end{aligned}$$

$$3(x + 2) - 5 = 4x$$

$$3(x + 2) - 5 = 4x$$

$$3x + 6 - 5 = 4x$$

$$3x + 1 = 4x$$

$$\begin{array}{r} -3x \quad -3x \\ 1 = x \end{array}$$

$$1 = x$$

$$\text{1st \#} = 1$$

$$x = 1$$

$$\begin{aligned} \text{2nd\#} &= x + 2 \\ &= (1) + 2 \end{aligned}$$

$$\text{2nd\#} = 3$$

$$\begin{aligned} \text{3rd\#} &= x + 4 \\ &= (1) + 4 \end{aligned}$$

$$\text{3rd\#} = 5$$

$$\text{1st \#} = 1$$

$$\text{2nd \#} = 3$$

$$\text{3rd \#} = 5$$

Check Answer

$$3(3) - 5 = 4(1)$$

$$9 - 5 = 4$$

$$4 = 4 \quad \checkmark$$

M8 - 10.9 - Age/Now-Then Equations Notes

Create and Solve the following:

Four years less than triple Mark's age equals fourteen years more than double his age. How old is Mark?

Let $m = \text{Mark's age}$

$$3m - 4 = 2m + 14$$

$$\begin{array}{r} 3m - 4 = 2m + 14 \\ -2m \quad -2m \\ \hline m - 4 = 14 \\ +4 \quad +4 \end{array}$$

$$m = 18$$

Answer

Mark is 18 years old

Check Answer

$$3(18) - 4 = 2(18) + 14 \checkmark$$

If Nicole were triple her age she was three years ago she would be twice her current age. How old is Nicole now?

Let $n = \text{Nicole's age}$

Let $n - 3 = \text{Nicole's age 3 years ago}$

Let $2n = \text{Twice Nicole's age}$

$$3(n - 3) = 2n$$

$$\begin{array}{r} 3(n - 3) = 2n \\ 3n - 9 = 2n \\ -3n \quad -3n \\ \hline -9 = -n \\ -9 \quad -n \\ \hline -1 = -1 \\ 9 = n \end{array}$$

$$n = 9$$

Check Answer

$$\begin{array}{l} 3(9 - 3) = 2(9) \\ 3(6) = 2(9) \\ 18 = 18 \checkmark \end{array}$$

Answer

Nicole is 9 years old now

M8 - 11.1 - Probability Notation/Rules Notes

Probability Notation

Event	Sample Space	Notation
For a coin toss	Heads, Tails	$S = \{H, T\}$
Six-sided die?	1, 2, 3, 4, 5, 6	$S = \{1,2,3,4,5,6\}$

Sample Space:

The set of all possible outcomes.

$P(E)$ is the probability of event E taking place.

$$P(H) = \frac{1}{2} = 0.5 = 50\%$$

$$P(1) = \frac{1}{6} = 0.1\bar{6} = 16.67\%$$

Probabilities can be expressed: as decimals or fractions between 0 and 1
: as percentages between 0 and 100%.

$$0 \leq P(E) \leq 1$$

$$0\% \leq P(E) \leq 100\%$$

If an event can't happen it has a probability of 0.

The probability of rolling a 7 on a standard six-sided die has a probability of 0.

$$P(7) = 0$$

$$P(E) \neq > 1 \text{ or } 100\%$$

The probability can never be less than 0% or greater than 100%.

If an event will happen with certainty, it has a probability of 1.

The probability of getting a head or a tail when flipping a coin is 1.

$$P(H \cup T) = 1$$

\cup : OR

If the probability of an event occurring is $P(E)$, then the probability that it DOESN'T occur is:

The probability of NOT rolling a 6 is:

$$P(\bar{6}) = 1 - P(6)$$

\bar{E} : Not E

$$P(\bar{E}) = 1 - P(E)$$

Compliment

$$= 1 - \left(\frac{1}{6}\right)$$

$$P(\bar{6}) = \frac{5}{6}$$

The sum of probabilities of all outcomes in the sample space must sum to 1.

When rolling a dice the sample space is $S = \{1,2,3,4,5,6\}$ and the sum of probabilities of all possible outcomes is:

$$P(1,2,3,4,5 \text{ or } 6) = P(1) + P(2) + P(3) + P(4) + P(5) + P(6)$$

$$= \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6}$$

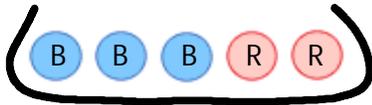
$$= \frac{6}{6}$$

$$= 1$$

$$P(1,2,3,4,5 \text{ or } 6) = 1$$

M8 - 11.1 - Marbles Probability Notes

You have 3 blue marbles and 2 red marbles in a bag, a total of 5 marbles.



$$\text{Probability} = \frac{\text{number of desired outcomes}}{\text{total outcomes}}$$

If you randomly take a marble out of the bag what is the probability that it will be:

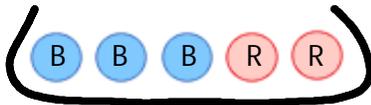
A blue marble? $P(B) = ?$

A red marble? $P(R) = ?$

	$P(B) = \frac{3 \text{ blue}}{5 \text{ total}}$		$P(R) = \frac{2 \text{ red}}{5 \text{ total}}$
	$P(B) = 0.6$		$P(R) = 0.4$

$$P(B) + P(R) = 1$$

You replace the marble. You now take a another marble out of the bag. Find the probability of:



With Replacement w/ rep

A blue marble (given blue)? $\text{given: } |$

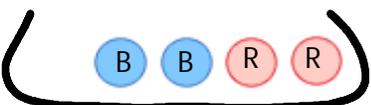
A red marble (given blue)?

$$P(B|B) = 0.6$$

$$P(R|B) = 0.4$$

$P(B|B) = P(B)$ **Independent** $P(R|B) = p(R)$
 Probability does not depend

You now take a blue marble out of the bag.



Without Replacement w/o rep

What is now the probability that your next drawn marble will be:

A blue marble (given blue)?

A red marble (given blue)?

	$p(b b) = \frac{2 \text{ blue}}{4 \text{ total}}$		$p(r b) = \frac{2 \text{ blue}}{4 \text{ total}}$
	$p(b b) = 0.5$		$p(r b) = 0.5$

$p(b|b) \neq p(b^*)$ **Dependent** $p(r|b) \neq p(r^*)$
 Probability depends

M8 - 11.1 - Coin Flip Probability Notes

What is the probability of flipping a Head?

$$P(H) = \frac{1}{2}$$

← 1 Head
← 2 possible outcomes (Heads or Tails)

What is the probability of flipping a Tail?

$$P(T) = \frac{1}{2}$$

← 1 Tail
← 2 possible outcomes (Heads or Tails)



If you flip a coin 2 times in a row:

Sample Space: HH, HT, TH, TT

Sample Space: Possible Outcomes

What is the probability of flipping?

Independent

$\cdot = \times$

$TH \neq HT$

Two Heads in a row?

Two Tails in a row?

A Tail THEN a Head?

A Head THEN a Tail?

$$P(HH) = P(H) \cdot P(H)$$

$$P(TT) = P(T) \cdot P(T)$$

$$P(TH) = P(T) \cdot P(H)$$

$$P(HT) = P(H) \cdot P(T)$$

$$= \frac{1}{2} \times \frac{1}{2}$$

$$= \frac{1}{2} \times \frac{1}{2}$$

$$= \frac{1}{2} \times \frac{1}{2}$$

$$= \frac{1}{2} \times \frac{1}{2}$$

$$P(HH) = \frac{1}{4}$$

$$P(TT) = \frac{1}{4}$$

$$P(TH) = \frac{1}{4}$$

$$P(HT) = \frac{1}{4}$$

$P(HH)$ or $P(2H)$

Method 1: Multiply

Method 2: Table

	H	T
H	H,H	H,T
T	T,H	T,T

$$P(TT) = \frac{1}{4}$$

Method 3: Tree

A Head AND a Tail?

\cap : And

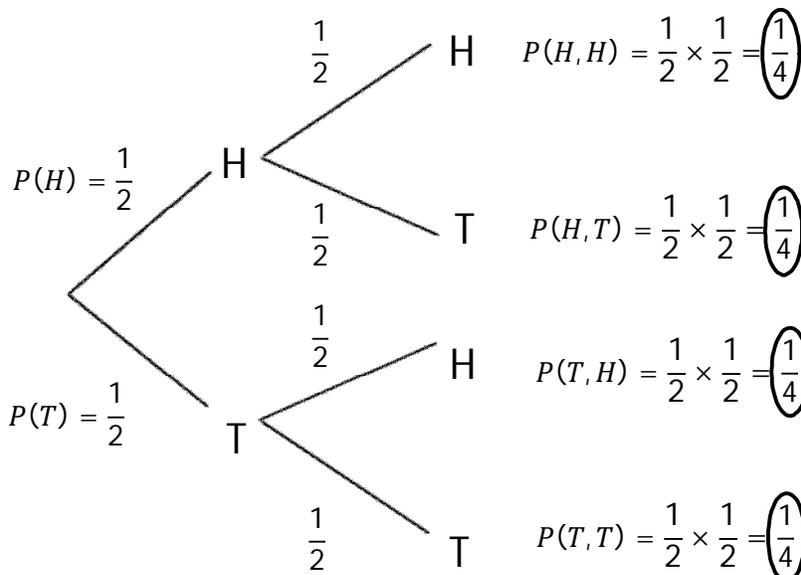
$$\begin{aligned} P(H \cap T) &= P(HT) + P(TH) \\ &= P(H) \times P(T) + P(T) \times P(H) \\ &= \frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{2} \\ &= \frac{1}{4} + \frac{1}{4} \end{aligned}$$

$$P(H \cap T) = \frac{1}{2}$$

$$P(H \cap T) = P(T \cap H)$$

$$P(HT \cup TH) = P(H \cap T)$$

\cup : OR



Multiply Branches

Add Leaves

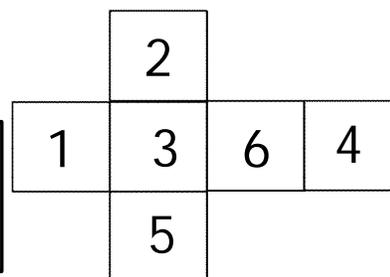
$$P(H \cap T) = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$$

M8 - 11.1 - Rolling a Dice Probability Notes

What is the probability of rolling a 6 with a die?

$$P(6) = \frac{1}{6}$$

← 1 Six
← 6 total numbers



What is the probability of rolling two 6's?

In a row = two separate dice

Method 1: Multiply

Independent

$$P(66) = P(6) \times P(6)$$

$$= \frac{1}{6} \times \frac{1}{6}$$

$$P(66) = \frac{1}{36}$$

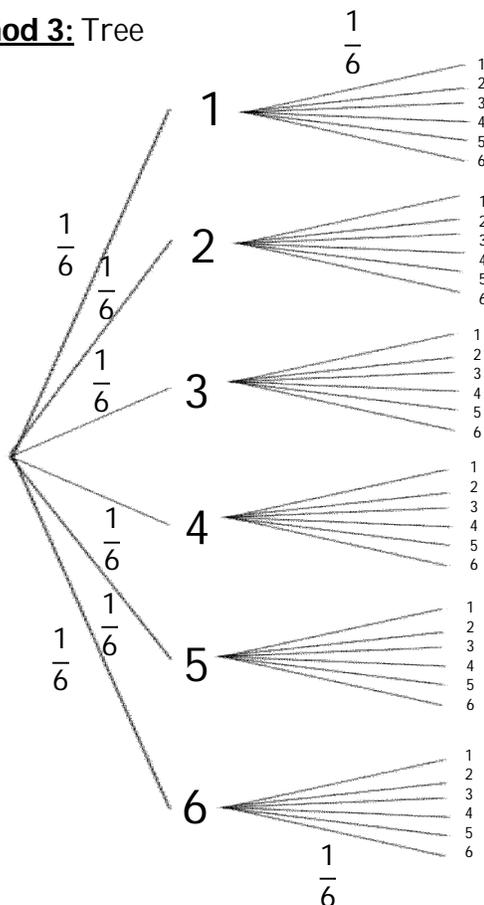


Method 2: Table

Sample Space

	1	2	3	4	5	6
1	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
2	(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
3	(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
4	(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
5	(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
6	(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

Method 3: Tree



$$P(6,6) = \frac{1}{36}$$

$$P(6,6) = \frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$$

M8 - 11.1 - Rolling a Dice and Coin Flip Probability Notes

What is the probability of flipping a Tail with a coin and rolling a 4 with a die?

Method 1: Multiply $P(T4) = P(T) \times P(4)$

$$= \frac{1}{2} \times \frac{1}{6}$$

$$P(T4) = \frac{1}{12}$$



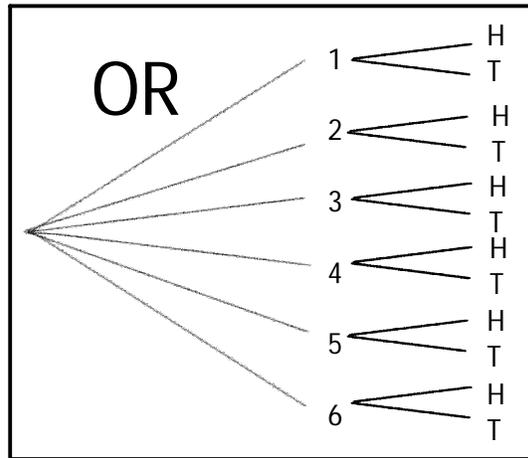
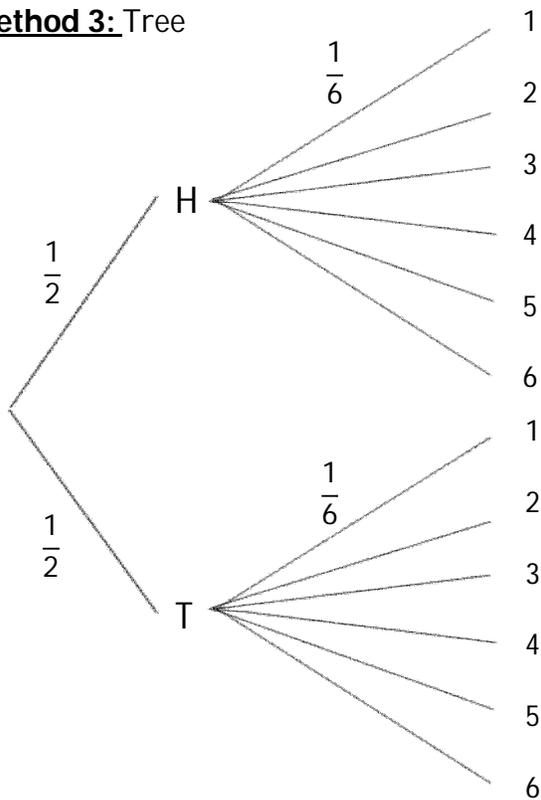
Method 2: Table

	1	2	3	4	5	6
H	H,1	H,2	H,3	H,4	H,5	H,6
T	T,1	T,2	T,3	T,4	T,5	T,6

Sample Space

$$P(T, 4) = \frac{1}{12}$$

Method 3: Tree



$$P(T, 4) = \frac{1}{2} \times \frac{1}{6} = \frac{1}{12}$$

M8 - 11.1 - Mean, Median, Mode, Range Notes

Mode: Most occurring number.

Mode = 3

0, 1, 1, 2, 2, 3, 3, 3, 4, 4, 10

Range: $10 - 0 = 10$

Range: Top# - Bottom#

Median: Middle number

Median = 3

Mean = $\frac{\text{All Numbers Added}}{\text{Number of Numbers}}$ (Average)

$$\text{Mean} = \frac{0 + 1 + 1 + 2 + 2 + 3 + 3 + 3 + 4 + 4 + 10}{11}$$

$$\text{Mean} = \frac{33}{11}$$

Mean = 3

No Mode

1, 3, 5, 7

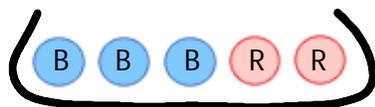
$$\text{Median} = \frac{3 + 5}{2}$$

$$= \frac{8}{2}$$

Median = 4

M8 - 11.1 - Odds Probability Notes

You have 3 blue marbles and 2 red marbles in a bag, a total of 5 marbles.



Odds: Odds in Favor : Odds Against

Choose a Marble. What are the odds?

Odds in favour Blue = Odds against Red

Odds in favour Red = Odds against Blue

3Blue: 2Red

3Red: 2Blue

$3 : 2 \leftarrow 5 - 3 = 2$

$2 : 3 \leftarrow 5 - 2 = 3$

Odds Against = Total - Odds in favour

Pick a Card.

SAMPLE SPACE

Hearts ♥	Diamonds ♦	Spades ♠	Clubs ♣
Ace ♥	Ace ♦	Ace ♠	Ace ♣
2 ♥	2 ♦	2 ♠	2 ♣
3 ♥	3 ♦	3 ♠	3 ♣
4 ♥	4 ♦	4 ♠	4 ♣
5 ♥	5 ♦	5 ♠	5 ♣
6 ♥	6 ♦	6 ♠	6 ♣
7 ♥	7 ♦	7 ♠	7 ♣
8 ♥	8 ♦	8 ♠	8 ♣
9 ♥	9 ♦	9 ♠	9 ♣
10 ♥	10 ♦	10 ♠	10 ♣
Jack ♥	Jack ♦	Jack ♠	Jack ♣
Queen ♥	Queen ♦	Queen ♠	Queen ♣
King ♥	King ♦	King ♠	King ♣

What are the odds of choosing an Ace?

4 Aces : 48 Other Cards

$4 : 48 \leftarrow 52 - 4 = 48$

Odds Against = Total - Odds in favour

What are the odds of choosing a Heart?

13 Hearts: 39 Other Cards

$13 : 39 \leftarrow 52 - 13 = 39$

Odds Against = Total - Odds in favour

(4 Suits/13 Cards per Suit/52 Cards)

The End

