# Math 10 Notes

Knack Academics

Nicholas Cragg
Knack Publishing
www.knackacademics.com
nick@knackacademics.com
604.505.2867

# M10 - 1.1 - SI/Imperial Conversion Factors vs Equal Fractions Notes

**How many centimeters around a 400m track?**

1

$$\text{Given} \rightarrow \frac{?}{400m} = \boxed{\frac{100cm}{1m}} \leftarrow \text{Conversion Factor}$$

$\times 400$

$\times 400$

$100cm \times 400 = 40000cm$

There are 40000 cm around a 400 m track.

OR

**How many centimeters around a 400m track?**

2

$$400\cancel{m} \times \boxed{\frac{100cm}{1\cancel{m}}} = 40000cm$$

Given

Conversion Factor

$\boxed{\begin{array}{c}\frac{m}{m} = 1 \\ \text{Cross it off.}\end{array}}$

Notice: choose a conversion factor that allows you to cross off the units you're given to get the units you want.

**How many inches in 1m?**

$$1\cancel{m} \times \boxed{\frac{100cm}{1\cancel{m}}} = 100cm$$

$$100\cancel{cm} \times \boxed{\frac{1in}{2.54\cancel{cm}}} = 39.37in$$

OR

$$1\cancel{m} \times \boxed{\frac{100\cancel{cm}}{1\cancel{m}}} \times \boxed{\frac{1in}{2.54\cancel{cm}}} = \frac{100in}{2.54} = 39.37in$$

Notice: sometimes we need to use two conversion factors to get from what we are given to get the units we want or all in one step.

**How many meters squared ($m^2$) in 2 kilometers squared ($km^2$)?**

$$2\cancel{km^2} \times \boxed{\frac{1000m}{1\cancel{km}}} \times \boxed{\frac{1000m}{1\cancel{km}}} = 2000000m^2$$

OR

$$\boxed{2km^2 \times \left(\frac{1000m}{1km}\right)^2 = 2000000m^2}$$

$$km^2 = \cancel{km} \times \cancel{km} \times \frac{m}{\cancel{km}} \times \frac{m}{\cancel{km}} = m^2$$

*Notice: in order to cross off $km^2$ we must multiply by the conversion factor 2 times.*

**How many centimeters cubed ($cm^3$) in 1 meter cubed ($m^3$)**

$$1\cancel{m^3} \times \boxed{\frac{100cm}{1\cancel{m}}} \times \boxed{\frac{100cm}{1\cancel{m}}} \times \boxed{\frac{100cm}{1\cancel{m}}} = 10000cm^3$$

OR

$$\boxed{1m^3 \times \left(\frac{100cm}{1m}\right)^3 = 10000cm^3}$$

*Notice: in order to cross off $m^3$ we must multiply by the conversion factor 3 times.*

# M10 - 1.2 - Conversion 1st vs 2nd Notes

***Find the Area in*** $\boldsymbol{cm^2}$

$3m$

$2m$

$$3m \times \frac{100cm}{1m} = 300cm$$

$$2m \times \frac{100cm}{1m} = 200cm$$

$300cm$

$200cm$

*OR*

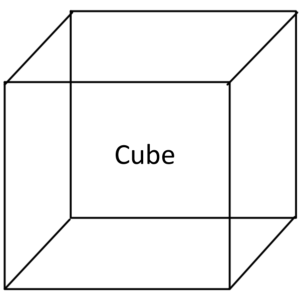
$A = l \times w$
$A = 3 \times 2$
$A = 6m^2$

$$6m^2 \times \frac{100cm}{1m} \times \frac{100cm}{1m} = 60000cm^2$$

$A = l \times w$
$A = 300 \times 200$

$A = 60000cm^2$

**How many litres of water can fit in this cube?**




$10m$

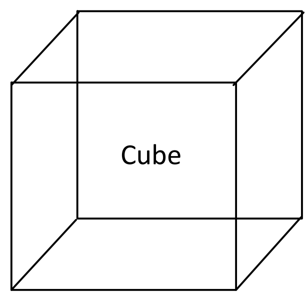
$V = l \times w \times h$
$V = 10m \times 10m \times 10m$
$V = 1000m^3$

$$1000m^3 \times \frac{100cm}{1m} \times \frac{100cm}{1m} \times \frac{100cm}{1m} = 1000000000cm^3$$

$$1000000000cm^3 \times \frac{1mL}{cm^3} = 1000000000mL$$

$$1000000000mL \times \frac{1L}{1000mL} = 1000000L$$

*OR*




$1000cm$

$$10m \times \frac{100cm}{m} = 1000cm$$

$V = l \times w \times h$
$V = 1000cm \times 1000cm \times 1000cm$
$V = 1000000000cm^3$

# M10 - 1.3 - Scientific Notation Conversion Factors Notes

**Conversion Factors**

**How many Litres are in 50 Millilitres?**

$$50\ mL \times \frac{1\ L}{1000\ mL} = 0.05\ L = 5 \times 10^{-2}\ L$$

OR

**Prefixes**

$$50\ mL \times \frac{10^{-3}\ L}{1\ mL} = 0.05\ L = 5 \times 10^{-2}\ L$$

**Attach Prefix Exponent to the Base Unit!**

**How many Micrometers in 4 Meters?**

$$4\ m \times \frac{1000000\ \mu m}{1\ m} = 4000000\ \mu m$$

$$4000000\ \mu m = 4 \times 10^{6}\ \mu m$$

OR

$$4\ m \times \frac{1\ \mu m}{10^{-6}\ m} = 4000000\ \mu m$$

$$4000000\ \mu m = 4 \times 10^{6}\ \mu m$$

**How many millimeters in 24 kilometers?**

$$24km \times \frac{1000m}{1km} \times 24000m$$

$$24000m \times \frac{100cm}{1m} = 2400000cm$$

$$2400000cm \times \frac{10mm}{1cm} = 24000000mm$$

OR

$$24km \times \frac{1000m}{1km} \times \frac{100cm}{1m} \times \frac{10mm}{1m} = 24000000mm$$

$$24000000mm = 2.4 \times 10^{7}mm$$

OR

**Base Unit 1st**

$$24km \times \frac{10^{3}m}{1km} = 24000m$$

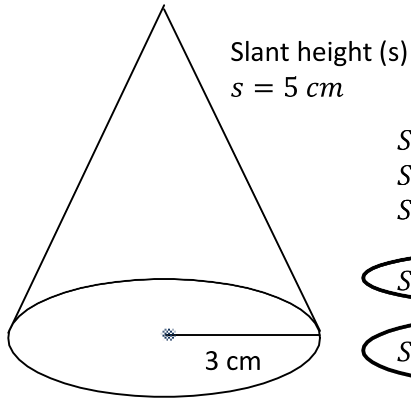
$$24000m \times \frac{1mm}{10^{-3}m} = 24000000mm$$

OR

$$24km \times \frac{10^{3}m}{1km} \times \frac{1mm}{10^{-3}m} = 24000000mm$$

# M10 - 2.1 - Cone Surface Area/Volume Notes

**Cone Surface Area**

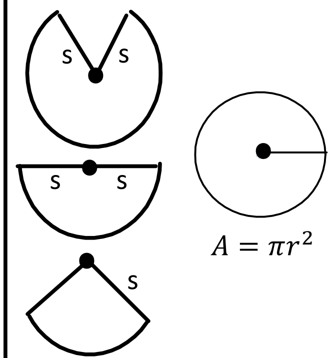



$SA = \pi r^2 + \pi rs$
$SA = (3.14)(3)^2 + (3.14)(3)(5)$
$SA = 28.27 + 47.12$

$SA = 75.40\ cm^2$

$SA = 24\pi\ cm^2$ Terms of Pie

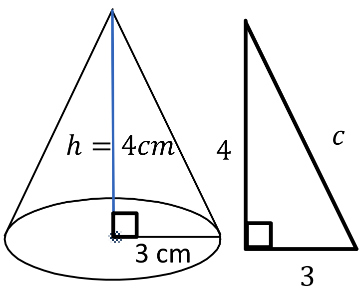
**Net Area**




$A = \pi r^2$

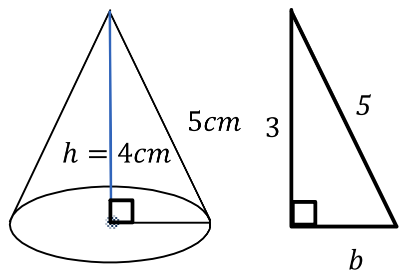
$A = \pi rs$

**Pythagoras (Same as Above)**




$a^2 + b^2 = c^2$
$3^2 + 4^2 = c^2$
$9 + 16 = c^2$
$c^2 = 25$
$c = \sqrt{25}$

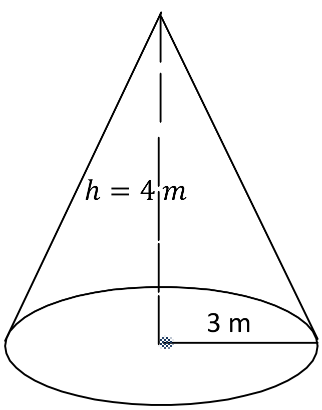
$c = 5$




$a^2 + b^2 = c^2$
$3^2 + b^2 = 5^2$
$9 + b^2 = 25$
$-9 \qquad -9$
$b^2 = 16$
$\sqrt{b^2} = \sqrt{16}$

$b = 4$

**Cone Volume**




$V = \frac{1}{3} \times (area\ of\ base) \times h$

$V = \frac{1}{3} \times (\pi r^2) \times h$

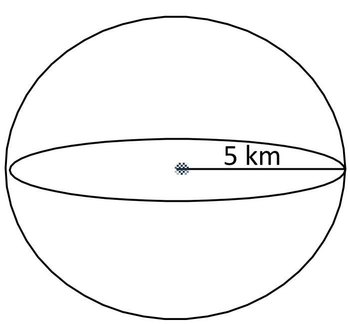
$V = \frac{1}{3} \times \left((3.14)(3)^2\right) \times 4$

$\boxed{V = \pi r^2 h}$

$V = 37.7\ m^3$

$V = 12\pi\ m^3$ Terms of Pie

**Sphere Surface Area and Volume**




$SA = 4\pi r^2$
$SA = 4(3.14)(5)^2$

$SA = 314\ km^2$

$SA = 100\pi\ km^2$ Terms of Pie

$V = \frac{4}{3}\pi r^3$

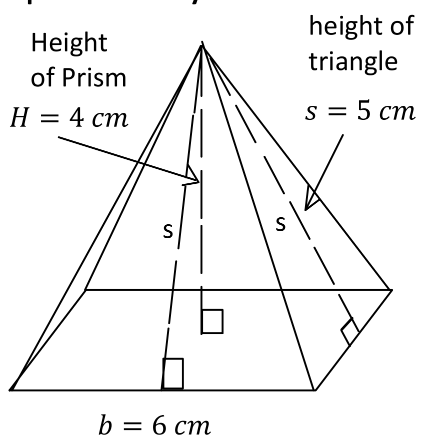
$V = \frac{4}{3}(3.14)(5)^3$

$V = 523.6\ km^3$

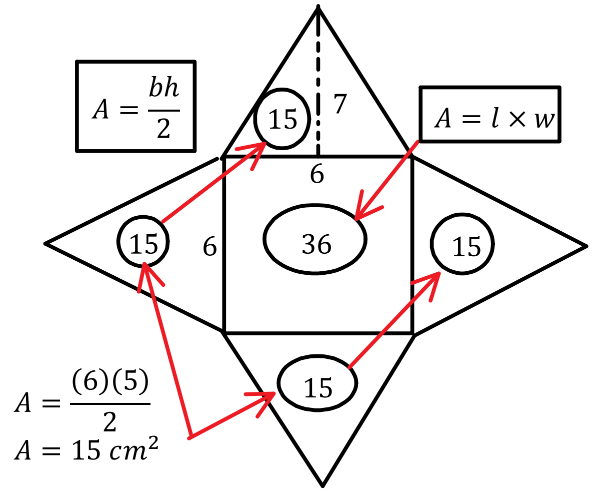
$V = \frac{100}{3}\pi\ km^3$

# M10 - 2.2 - Square Pyramid Notes

**Square Based Pyramid Surface Area and Volume**




OR




$SA = 2bs + b^2$
$SA = 2(6)(5) + (6)^2$
$SA = 60 + 36$

$SA = 96\ cm^2$

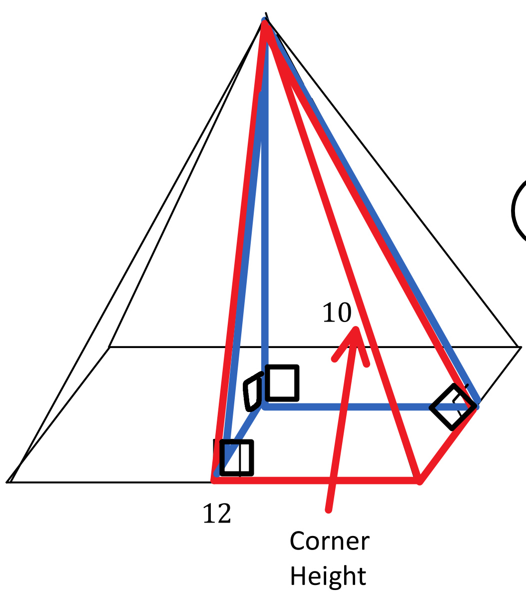
$SA = 15 + 15 + 15 + 15 + 36$

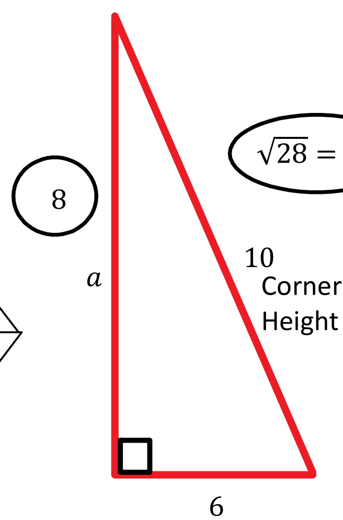
$SA = 96\ cm^2$

$V = \frac{1}{3} \times (area\ of\ base) \times h$
$V = \frac{1}{3} \times (l \times w) \times h$
$V = \frac{1}{3} \times (6 \times 6) \times 4$

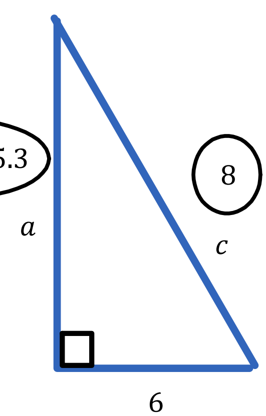
$V = 48\ cm^3$

**Pythagoras (Different than Above)**




$\sqrt{28} = 5.3$




$a^2 + b^2 = c^2$
$a^2 + 6^2 = 10^2$
$a^2 + 36 = 100$
$-36 \quad -36$
$a^2 = 64$
$a = \sqrt{64}$

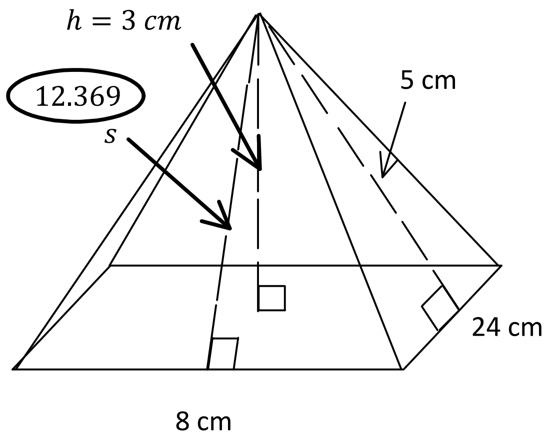
$a = 8$

$a^2 + b^2 = c^2$
$a^2 + 6^2 = 8^2$
$a^2 + 36 = 64$
$-36 \quad -36$
$a = 28$
$a = \sqrt{28}$

$a = \sqrt{28} = 5.3$

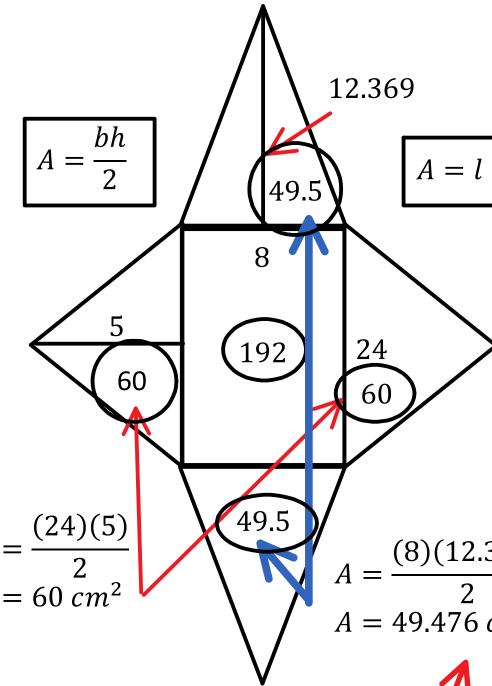
# M10 - 2.3 - Rectangular Pyramid Notes

**Rectangular Based Pyramid Surface Area and Volume**




$A = \dfrac{bh}{2}$

$A = l \times w$

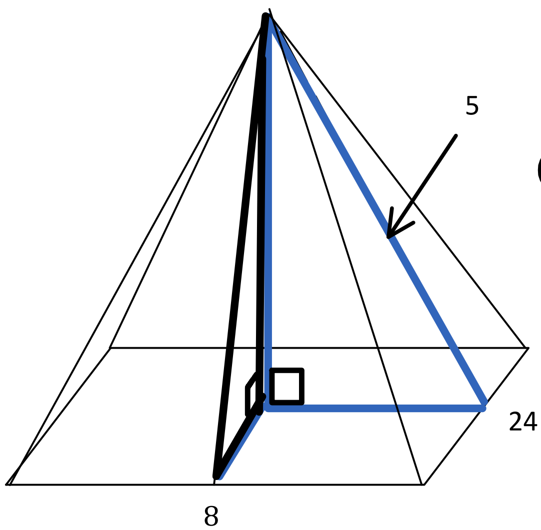


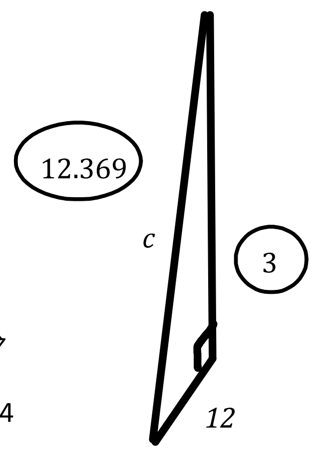

$A = \dfrac{(24)(5)}{2}$
$A = 60\ cm^2$

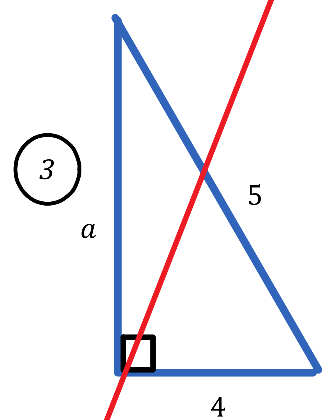
$A = \dfrac{(8)(12.369)}{2}$
$A = 49.476\ cm^2$

$SA = 60+60+49.5+49.5+192$
$SA = 412\ cm^2$

**Pythagoras (Same as Above)**




$a^2 + b^2 = c^2$
$3^2 + 12^2 = c^2$
$9 + 144 = c^2$
$153 = c^2$
$\sqrt{153} = c$
$c = \sqrt{153} = 12.369$

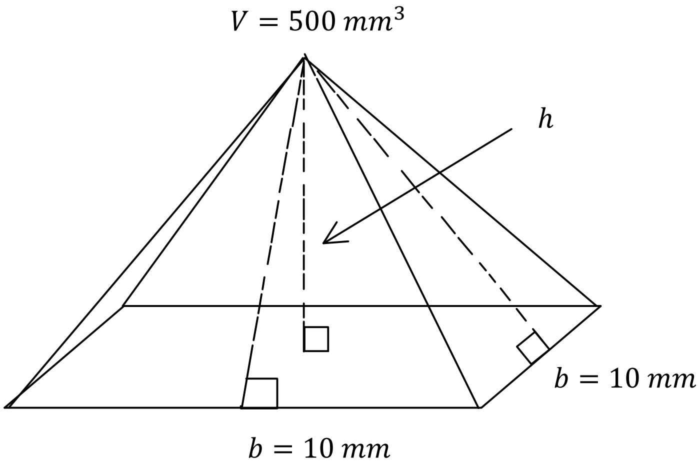
$a^2 + b^2 = c^2$
$a^2 + 4^2 = 5^2$
$a^2 + 16 = 25$
$-16 \quad -16$
$a^2 = 9$
$a = \sqrt{9}$
$a = 3$

If Corner Height
See page before

$V = \dfrac{1}{3} \times (area\ of\ base) \times h$
$V = \dfrac{1}{3} \times (l \times w) \times h$
$V = \dfrac{1}{3} \times 8 \times 24 \times 3$
$V = 192\ cm^3$

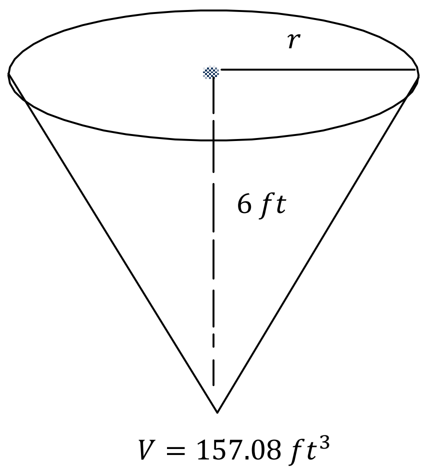
# M10 - 2.4 - Volume/Surface Area Missing Length Notes

**Find the missing length for the shapes below.**




$$\begin{aligned}
V &= \frac{1}{3} \times (area\ of\ base) \times h \\
V &= \frac{1}{3} \times (l \times w) \times h \\
500 &= \frac{1}{3} \times 10 \times 10 \times h \\
500 &= \frac{100h}{3} \\
3 \times 500 &= \frac{100h}{3} \times 3 \\
1500 &= 100h \\
\frac{1500}{100} &= \frac{100h}{100}
\end{aligned}$$

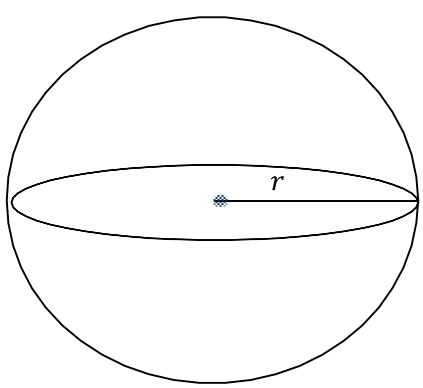
$$h = 15\ mm$$




$$\begin{aligned}
V &= \frac{1}{3} \times (area\ of\ base) \times h \\
V &= \frac{1}{3} \times (\pi r^2) \times h \\
157.08 &= \frac{1}{3} \times \left((3.14)r^2\right) \times 6 \\
157.08 &= 6.28r^2 \\
\frac{157.08}{6.28} &= \frac{6.28r^2}{6.28} \\
25 &= r^2 \\
\sqrt{25} &= r
\end{aligned}$$

$$r = 5\ ft$$

$SA = 196\pi\ in^2$ Terms of pie

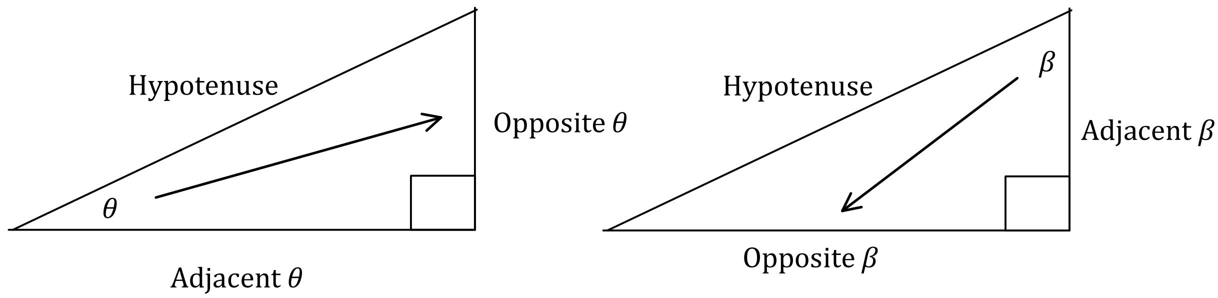



$$\begin{aligned}
SA &= 4\pi r^2 \\
196\pi &= 4\pi r^2 \\
\frac{196\pi}{\pi} &= \frac{4\pi r^2}{\pi} \\
\frac{196}{4} &= \frac{4r^2}{4} \\
49 &= r^2 \\
\sqrt{49} &= r
\end{aligned}$$

$$r = 7\ in$$

# Sides

$(\theta \;\&\; \beta \text{ are Angles})$




**Hypotenuse**: The Longest Side, Opposite of the 90° Angle.

**Adjacent**: The side touching angle $\theta$.

**Opposite**: The side opposite of angle $\theta$.

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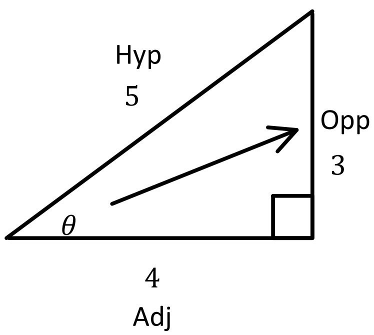
**Sine Ratio**




Label Hyp/Opp/Adj

1 $\boxed{sin\theta = \frac{Opposite}{Hypotenuse}}$

$sin\theta = \frac{opp}{hyp}$

$\boxed{sin\theta = \frac{O}{H}}$

$sin\theta = \frac{O}{H}$

$sin\theta = \frac{3}{5}$

$sin\theta = 0.6$

**Find an Angle**

Calculator Degree Mode! (Not Radians)

Mode | Degree

$sin\theta = \frac{3}{5}$

$\theta = \sin^{-1}(0.6)$

$\theta = 36.9^o$

---

# SOH CAH TOA

| S | O | H | C | A | H | T | O | A |
|---|---|---|---|---|---|---|---|---|
| INE | OPPOSITE | HYPOTENUSE | OS | ADJACENT | HYPOTENUSE | AN | OPPOSITE | ADJACENT |

Choose the part of **SOH CAH TOA** that has 2 pieces of info that we have, and one we want.

**Cosine Ratio**

$\boxed{cos\theta = \frac{Adjacent}{Hypotenuse}}$

$\boxed{cos\theta = \frac{A}{H}}$ 2

$cos\theta = \frac{4}{5}$

$cos\theta = 0.8$

$\theta = \cos^{-1}(0.8)$

$\theta = 36.9^o$

**Tangent Ratio**

$\boxed{tan\theta = \frac{Opposite}{Adjacent}}$

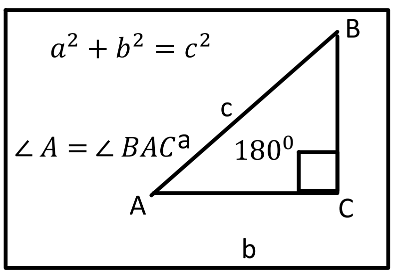
$\boxed{tan\theta = \frac{O}{A}}$ 3

$tan\theta = \frac{3}{4}$

$tan\theta = 0.75$

$\theta = \tan^{-1}(0.75)$

$\theta = 36.9^o$




Measure the angle with a protractor!

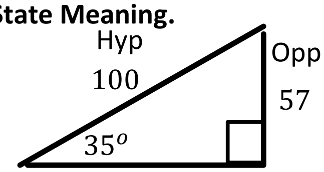
# M10 - 3.2 - SOH CAH TOA Trigonometry Algebra Notes

**Plug into your Calculator, Draw a Triangle, State Meaning.**

$sin35 = 0.57$ | sin | 35 | Calculator Buttons

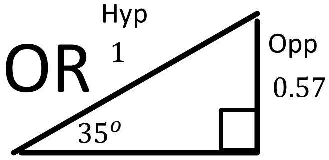
$$\boxed{\frac{0.57}{1} = \frac{57}{100}}$$

A Right Triangle with an angle of 35° has a ratio of:




Opposite = 57
Hypotenuse = 100

OR




Opposite = 0.57
Hypotenuse = 1

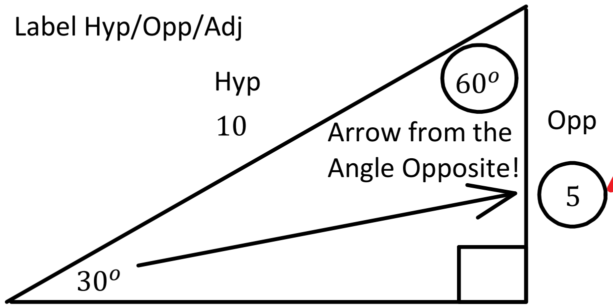
---

**Solve the Triangle Using Trigonometry**

**Solve the Opposite**

Circled info is found!

Label Hyp/Opp/Adj




Adj 8.660 3 Decimal Places

$$\boxed{sin\theta = \frac{Opp}{Hyp}}$$

$$sin30 = \frac{Opp}{10}$$

$$10 \times sin30 = \frac{Opp}{\cancel{10}} \times \cancel{10}$$

$$5 = Opp$$

Formula
Substitute
Angle/Hypotenuse
× 10 Both Sides
Cross it Off
Calculator Buttons

| 10 | sin | 30 |
|---|---|---|

$$sin\theta = \frac{Opp}{Hyp}$$

$$sin30 = \frac{5}{10}$$

$$0.5 = \frac{1}{2} ✓$$

Check Answer
Formula
Substitute
Left = Right
Check Mark

---

**Algebra Review**

$$2 = \frac{6}{A}$$

$$A \times 2 = \frac{6}{\cancel{A}} \times \cancel{A}$$

$$2A = 6$$

$$\frac{\cancel{2}A}{\cancel{2}} = \frac{6}{2}$$

$$A = \frac{6}{2}$$

$$A = 3$$

Multiply $A$
Divide 2
Both Sides

$$2 = \frac{6}{A}$$

$$A = \frac{6}{2}$$

$$A = 3$$

Cross
Multiply
Switch $A$ and 2

**Pythagoras**

$$a^2 + b^2 = c^2$$

$$5^2 + b^2 = 10^2$$

$$25 + b^2 = 100$$

$$-25 \qquad -25$$

$$b^2 = 75$$

$$\sqrt{b^2} = \sqrt{75}$$

$$b = 8.66$$

**$180^o$ in a Triangle**

$$180^o = \angle a + \angle b + \angle c$$

$$\angle a = 180^o - 90^o - 30^o$$

$$\angle a = 90^o - 30^o$$

$$\angle a = 60^o$$

$$\boxed{90^o - \theta}$$

Measure the Angle with a Protractor!
Measure Hypotenuse Double Opposite!

**Solve the Adjacent**

**Using Tan**

$$tan\theta = \frac{Opp}{Adj}$$

$$tan30 = \frac{5}{A}$$

$$A \times tan30 = \frac{5}{\cancel{A}} \times \cancel{A}$$

$$Atan30 = 5$$

$$\frac{A\cancel{tan}30}{\cancel{tan}30} = \frac{5}{tan30}$$

$$A = \frac{5}{tan30}$$

$$A = 8.660$$

Multiply 5
Divide $tan30$
Both Sides!

OR

Cross
Multiply

$$tan30 = \frac{5}{A}$$

$$A = \frac{5}{tan30}$$

$$A = 8.660$$

**Using Cos**

$$cos\theta = \frac{Adj}{Hyp}$$

$$10 \times cos30 = \frac{Adj}{\cancel{10}} \times \cancel{10}$$

$$Adj = 8.660$$

Calculator Buttons

| 5 | ÷ | tan | 30 |
|---|---|---|---|

**Find Other Angle β**

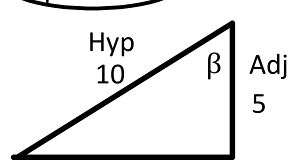
Opp and Adj Switch

$$cos\beta = \frac{Adj}{Hyp}$$

$$cos\beta = \frac{5}{10}$$

$$\beta = \cos^{-1}\left(\frac{5}{10}\right)$$

$$\beta = 60^o$$



# M10 - 4.1 - Entire to Mixed Radicals Notes

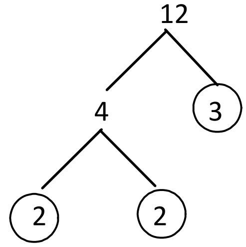
**Simplify** **Prime Factorization**

$\sqrt[2]{12} = \sqrt[2]{2 \times 2 \times 3}$

$= 2\sqrt[2]{3}$

Check Answer

$3.46 = 3.46$




$\sqrt[2]{2 \times 2} = \sqrt[2]{4} = 2$

Two Identical Numbers Under a Square Root: One on Outside

**Perfect Squares**

$\sqrt[2]{12} = \sqrt[2]{4 \times 3}$

$= \sqrt[2]{4} \times \sqrt[2]{3}$

$\frac{12}{4} = 3$

$= 2\sqrt[2]{3}$

Find Two Numbers that Multiply to the Number Underneath the Square Toot such that you know the Square Root of One of them.

**Perfect Squares**

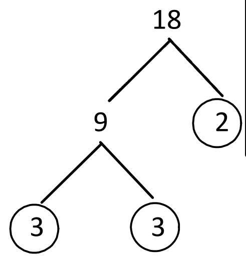
1,4,9,16,25,36,49 ...

$\sqrt[2]{18} = \sqrt[2]{3 \times 3 \times 2}$

$= 3\sqrt[2]{2}$

Check Answer

$4.25 = 4.24$




Check on Calculator

2nd $\sqrt[2]{\square}$ $\sqrt[3]{\square}$

Ti84 Math $\sqrt[x]{\square}$

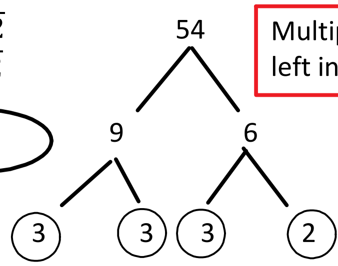
$\sqrt[2]{54} = \sqrt[2]{3 \times 3 \times 3 \times 2}$

$= 3\sqrt[2]{3 \times 2}$

$= 3\sqrt[2]{6}$

Check Answer

$7.35 = 7.35$




Multiply what's left inside.

**Cant Even Root a Negative**

$\sqrt[2]{-9} =$ No Solution

**Can Odd Root a Negative**

$\sqrt[3]{-27} = \sqrt[3]{-3 \times -3 \times -3}$

$= -3$

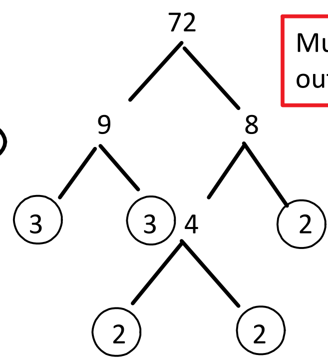
$\sqrt[2]{72} = \sqrt[2]{3 \times 3 \times 2 \times 2 \times 2}$

$= 3 \times 2\sqrt[2]{2}$

$= 6\sqrt[2]{2}$

Check Answer

$8.49 = 8.49$




Multiply numbers outside of root.

$5\sqrt[2]{12} = 5\sqrt[2]{2 \times 2 \times 3}$

$= 5 \times 2\sqrt[2]{3}$

$= 10\sqrt[2]{3}$

Check Answer

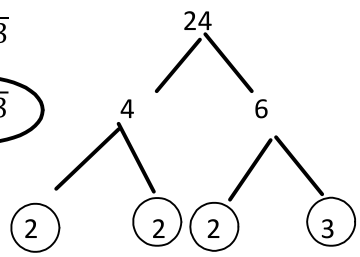
$17.32 = 17.32$

$\sqrt[3]{24} = \sqrt[3]{2 \times 2 \times 2 \times 3}$

$= 2\sqrt[3]{3}$

Check Answer

$2.88 = 2.88$




$\sqrt[3]{2 \times 2 \times 2} = \sqrt[3]{8} = 2$

Three Identical Numbers Under a Cube Root: One on Outside.

$\sqrt[3]{54} = \sqrt[3]{3 \times 3 \times 3 \times 2}$

$= 3\sqrt[3]{2}$

**See Above**

Check Answer

$3.78 = 3.78$

**Perfect Cubes**

$\sqrt[3]{24} = \sqrt[3]{8 \times 3}$

$= \sqrt[3]{8} \times \sqrt[3]{3}$

$\frac{24}{8} = 3$

$= 2\sqrt[3]{3}$

What are Two Numbers that Multiply to the Number Underneath the Cube Root that you know the Cube Root of One of them.

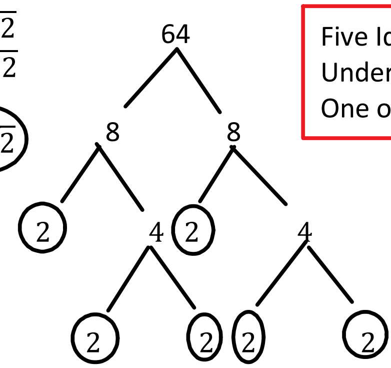
**Perfect Cubes**

1,8,27,64,125,216 ...

# M10 - 4.2 - Mixed to Entire/Variables Radicals Notes

**Simplify**

$$\sqrt[5]{64} = \sqrt[5]{2 \times 2 \times 2 \times 2 \times 2 \times 2}$$
$$= \sqrt[5]{2 \times 2 \times 2 \times 2 \times 2 \times 2}$$
$$= 2\sqrt[5]{2}$$




Five Identical Numbers Under a Fifth Root: One on Outside.

$$5 = \sqrt{5^2}$$
$$5x = \sqrt{5^2x^2}$$
$$5 = \sqrt[3]{5^3}$$

Check Answer

$2.30 = 2.30$

Check on Calculator OR

2nd | 5 | $\sqrt[x]{\square}$ | $x^{y}$ with $\frac{1}{}$

Ti84 | Math | 5 | $\sqrt[x]{\square}$

$$\sqrt[2]{x^2} = \sqrt[2]{x \times x}$$
$$= x$$

$x^2$

$x$ $x$

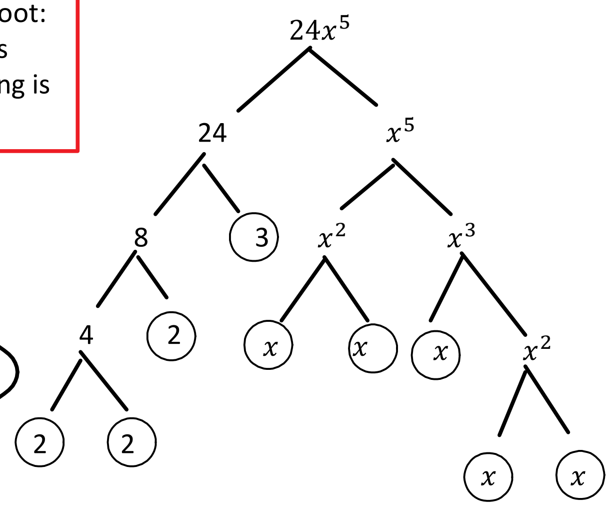
Two Identical Variables Under a Square Root: One Comes Out. Nothing is left.

Check Answer

$\sqrt[2]{5^2} = 5$

Arbitrary Number

$$\sqrt[3]{24x^5} = \sqrt[3]{2 \times 2 \times 2 \times 3 \times x \times x \times x \times x \times x}$$
$$= 2x\sqrt[3]{3x^2}$$




**Expand**

$$5\sqrt[2]{2} = \sqrt[2]{5 \times 5 \times 2}$$
$$= \sqrt[2]{25 \times 2}$$
$$= \sqrt[2]{50}$$

One number Outside of a Square Root: Two Inside.

Check Answer

$7.08 = 7.07$

$$5\sqrt[3]{2} = \sqrt[3]{5 \times 5 \times 5 \times 2}$$
$$= \sqrt[3]{125 \times 2}$$
$$= \sqrt[3]{250}$$

One Number Outside of a Cube root: Three Inside.

Check Answer

$8.55 = 8.55$

$$-7\sqrt[2]{3} = -\sqrt[2]{7 \times 7 \times 3}$$
$$= -\sqrt[2]{49 \times 3}$$
$$= -\sqrt[2]{147}$$

A Negative may Not go Inside an Even Root

Check Answer

$-12.12 = -12.12$

$$-4\sqrt[5]{5} = \sqrt[5]{-4 \times 4 \times 4 \times 4 \times 4 \times 5}$$
$$= \sqrt[5]{-4^5 \times 5}$$
$$= \sqrt[5]{-5120}$$

One Number Outside of a Fifth Root: Five Inside.

Check Answer

$-5.52 = -5.52$

A Negative may go Inside an Odd Root

# M10 - 4.3 - Add/Sub/Multiply Exponents Laws Notes

Coefficient $\longrightarrow 4x^2 \longrightarrow$ Exponent

$\uparrow$ Base

**Remember:**
**-Never multiply the base by the exponent**
**-Must have same base to use laws.**

**Multiplying with the Same Base, Add Exponents.**

$$x^3 \times x^2 = (x \times x \times x) \times (x \times x) = x^5$$

$$x^3 \times x^2 = x^{3+2} = x^5$$

Add Exponents

Check Answer

$$2^3 \times 2^2 = 32 = 2^5$$

Arbitrary Numbers

**Dividing with the Same Base, Subtract Exponents.**

$$\frac{x^5}{x^2} = \frac{\cancel{x \times x} \times x \times x \times x}{\cancel{x \times x}} = x^3$$

$$\frac{x^5}{x^2} = x^{5-2} = x^3$$

Subtract Exponents

Check Answer

$$\frac{3^5}{3^2} = 27 = 3^3$$

Arbitrary Numbers

**Exponents to Exponents, Multiply Exponents**

$$(x^2)^3 = (x \times x)^3 = (x \times x) \times (x \times x) \times (x \times x) = x^6$$

$$(x^2)^3 = x^{2 \times 3} = 2^6$$

Multiply Exponents

Check Answer

$$(5^2)^3 = 15625 = 5^6$$

Arbitrary Numbers

Ultimately you will either use:

Exponent Laws

OR

Repeated Multiplication/Division Theory

**Product/Quotients to Exponents, Multiply Exponents**

$$(x^1 \times y^1)^2 = x^2y^2$$

Multiply Exponents

$$(2x)^3 = (2x) \times (2x) \times (2x) = 8x^3$$

$$(2x)^3 = 2^3x^3 = 8x^3$$

$$\left(\frac{2^1x^1}{y^3}\right)^2 = \frac{2^2x^2}{y^{2\times 3}} = \frac{4x^2}{y^6}$$

Cannot distribute into a sum!

$(3+4)^2 \neq 3^2 + 4^2 = 25$

$(3+4)^2 = (3+4)(3+4) = 7 \times 7 = 7^2 = 49$

$$\left(\frac{6mn^3}{4m^2n}\right)^3$$

$$\left(\frac{3n^2}{2m}\right)^3$$

Simplify 1st

$$\frac{3^3n^6}{2^3m^3}$$

$$\frac{27n^6}{8m^3}$$

OR

$$\left(\frac{6mn^3}{4m^2n}\right)^3$$

$$\frac{6^3m^3n^9}{4^3m^6n^3}$$

Multiply Exponents 1st

$$\frac{216n^6}{64m^3}$$

$$\frac{27n^6}{8m^3}$$

# M10 - 4.4 - Negative Exponents Laws Notes

Negative Exponents

$x^{-2} = \frac{1}{x^2}$ *Bring to the bottom, make exponent positive* $\boxed{x^{-a} = \frac{1}{x^a}}$

$\frac{1}{x^{-2}} = \frac{x^2}{1}$ *Bring to the top, make exponent positive* $\boxed{\frac{1}{x^{-a}} = x^a}$

$3a^{-2} = \frac{3}{a^2}$ *Bring to the bottom, make exponent positive*

Notice the 3 doesn't come down

$3^{-3}a^{-2} = \frac{1}{3^3a^2}$ *Bring to the bottom, make exponent positive*

$(2x)^{-3} = \frac{1}{(2x)^3} = \frac{1}{8x^3}$ *Bring to the bottom, make exponent positive*

$$\boxed{\frac{x^{-2}+5}{3} \neq \frac{5}{3x^2}}$$

## Step 1

$\overline{\qquad\qquad}$ ← Over

$\frac{2x^5y^{-2}}{z^{-3}} = \_\_\_\_ = \frac{2x^5z^3}{y^2}$

When working with negative exponents:

Start with a fraction "Over" sign.
Put anything not moved!
Move whatever needs to be moved.
If nothing is left on the top, put a 1.

**When you can flip it!**

$\left(\frac{x}{y}\right)^{-2} = \frac{x^{-2}}{y^{-2}} = \frac{y^2}{x^2}$

Distribute Exponents
*Bring to the bottom, make exponent positive*
*Bring to the top, make exponent positive*

OR

$\left(\frac{x}{y}\right)^{-2} = \left(\frac{y}{x}\right)^2 = \frac{y^2}{x^2}$ Flip it and make the exponent positive

**Alternate Subtraction Methods**

$\frac{x^2}{x^5} = x^{2-5} = x^{-3} = \frac{1}{x^3}$

Subtract from the top

Theory

$$\boxed{\frac{x^2}{x^5} = \frac{\cancel{x \times x}\ 1}{\cancel{x \times x} \times x \times x \times x} = \frac{1}{x^3}}$$

$\frac{x^2}{x^5} = \frac{1}{x^{5-2}} = \frac{1}{x^3}$

Subtract from Bottom

$\frac{x^2}{x^{-3}} = x^2x^3 = x^5$

$\frac{x^2}{x^{-3}} = x^{2-(-3)} = x^5$

Bring Up, Add

OR

Subtract

$\frac{x^{-2}}{x^3} = \frac{1}{x^3x^2} = \frac{1}{x^5}$

$\frac{x^{-2}}{x^3} = \frac{1}{x^{3-(-2)}} = \frac{1}{x^5}$

Bring Down, Add

OR

Subtract From Bottom

# M10 - 4.5 - Fraction Exponents/Radical/Root Form Notes

**Change from exponential form to radical/root form. Simplify if necessary.**

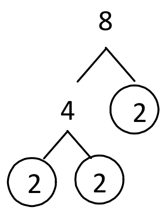
$5^{\frac{3}{4}} = \sqrt[4]{5^3}$

Check on Calculator

$5^{\frac{3}{4}} = 3.34 = \sqrt[4]{5^3}$ ✓

$x^{\frac{2}{3}} = \sqrt[3]{x^2}$

$8^{\left(\frac{1}{3}\right)} = \sqrt[3]{8^1} = 2$



$\sqrt[3]{8}$
$\sqrt[3]{2 \times 2 \times 2}$
$2$

$\sqrt[3]{8} = 2$

$$x^{\frac{m}{n}} = \sqrt[n]{x^m}$$

Check on Calculator

$8^{\frac{1}{3}} = 2 = \sqrt[3]{8^1}$ ✓

$8^{\frac{2}{3}} =$
$\sqrt[3]{8^2}$
$2^2$
$4$

Change to Radical/Root Form
Cube Root 1st
Square 2nd

$\sqrt[3]{8} = 2$

Check on Calculator $8^{\frac{2}{3}} = 4$ ✓

OR

$8^{\frac{2}{3}} =$
$\sqrt[3]{8^2}$
$\sqrt[3]{64}$
$4$

Change to Radical/Root Form
Square 1st
Cube Root 2nd

$8^2 = 64$
$\sqrt[3]{64} = 4$

Easier to Root 1st

$(-27)^{\frac{4}{3}}$
$\sqrt[3]{(-27)^4}$
$(-3)^4$
$81$

Change to Radical/Root Form
Cube Root 1st
Square 2nd

$\sqrt[3]{-27} = -3$

Check on Calculator

$(-27)^{\frac{4}{3}} = 81$ ✓

**Simplify by exponents laws. Answer in root form.**

$\left(2^{\frac{1}{2}}\right)\left(2^{\frac{1}{4}}\right) = 2^{\frac{3}{4}} = \sqrt[4]{2^3} = \sqrt[4]{8}$

Add Exponents

$$\frac{1}{2} + \frac{1}{4} = \frac{3}{4}$$

$(3)^{\frac{3}{2}} \div (3)^{\frac{3}{5}} = (3)^{\frac{9}{10}} = \sqrt[10]{3^9}$

Subtract Exponents

$$\frac{3}{2} - \frac{3}{5} = \frac{9}{10}$$

$\left(\sqrt[2]{2^3}\right)^{\frac{1}{4}} =$

$\left(2^{\frac{3}{2}}\right)^{\frac{1}{4}}$

$2^{\frac{3}{8}}$

$\sqrt[8]{2^3} = \sqrt[8]{8}$

$$\frac{3}{2} \times \frac{1}{4} = \frac{3}{8}$$

Multiply Exponents

Check Answer

$\left(\sqrt[2]{2^3}\right)^{\frac{1}{4}} = 1.30 = \sqrt[8]{8}$

Check on Calculator

$\left(2^{\frac{1}{2}}\right)\left(2^{\frac{1}{4}}\right) = 1.68 = \sqrt[4]{8}$ ✓

$(3)^{\frac{3}{2}} \div (3)^{\frac{3}{5}} = 2.69 = \sqrt[10]{3^9}$ ✓

$\left(\sqrt[2]{2^3}\right)^{\frac{1}{4}} = 1.30 = \sqrt[8]{2^3}$ ✓

$(-27x^9y^{-3})^{\frac{4}{3}}$
$\sqrt[3]{(-27)^4}x^{12}y^{-4}$

$$\frac{81x^{12}}{y^4}$$

$9 \times \frac{4}{3} = 12$
$-3 \times \frac{4}{3} = -4$
$\sqrt[3]{(-27)^4} = (-3)^4 = 81$

$\left(\frac{-27a^3}{8}\right)^{\frac{1}{3}}$

$$\frac{(-27)^{\frac{1}{3}}a^{3\times\frac{1}{3}}}{8^{\frac{1}{3}}}$$

$$\frac{-3a}{2}$$

$(5x^3)^{\frac{1}{2}}$
$\sqrt[2]{5^1}\sqrt[2]{x^3}$
$\sqrt{5}x\sqrt{x}$

$x\sqrt{5x}$

# M10 - 5.1 - Factoring GCF Notes

***Remove Greatest Common Factor "GCF."***

$12x + 8 =$

$4(3x + 2)$ $GCF = 4$

4 times what is 12x

4 times what is 8

Divide both terms by GCF

$\frac{12x}{4} + \frac{8}{4} = 3x + 2$

Answer goes in brackets

Check your answer by Distribution

$4(3x + 2)$
$12x + 8$

The answer should be the same as the original question.

$2x^2 + 3x =$

$x(2x + 3)$ $GCF = x$

$12x^2 + 8x =$

$4x(3x + 2)$ $GCF = 4x$

$x^2 + x^3 =$

$x^2(1 + x)$ $GCF = x^2$

$8x^2y + 4xy =$

$4xy(2x + 1)$ $GCF = 4xy$

$-2x + 8$

$-2(x - 4)$ $GCF = -2$

$\boxed{8 \div -2 = -4}$

$(-x - 2) =$
$-1(x + 2) =$ $GCF = -1$

$-(x + 2)$

$(2 - x) =$ $GCF = -1$
$-1(-2 + x) =$

$-(x - 2)$

Rearrange Order of the Terms

$ab + cb$ $GCF = b$

$b(a + c)$

They both have a $b$
Take out a $b$

$x(x + 2) + 4(x + 2) =$

$(x + 2)(x + 4)$

$GCF = (x + 2)$
They both have a $(x + 2)$
Take out a $(x + 2)$

Poetry

$2x^2 + 3x + 4x + 6$
$(2x^2 + 3x) \mid (+4x + 6)$
$x(2x + 3) + 2(2x + 3)$

$(2x + 3)(x + 2)$

Group
GCF, GCF

GCF

$2x^2 - 6 + 3x + 4x$
$2x^2 + 3x + 4x + 6$

$\cdots$

Rearrange Order of Terms

$2x - \frac{1}{2}$ $GCF = 2$

$\frac{1}{2} \div \frac{2}{1} = \frac{1}{2} \times \frac{1}{2} = 1/4$

$2(x - \frac{1}{4})$

$\left(\frac{1}{2}x + 4\right)$ $GCF = \frac{1}{2}$

$4 \div \frac{1}{2} = 4 \times \frac{2}{1} = 8$

$\frac{1}{2}(x + 8)$

# M10 - 5.2 - Factoring (a=1) Trinomials Notes

"a" is the number to the left of the $x^2$ term.
"b" is the number to the left of the $x$ term.
"c" is the number by itself.

**Factor by Decomposition**

$\mathbf{a = 1}$

$1x^2 + 2x - 3$ $\quad a = 1,\ b = 2,\ c = -3$

$\mathbf{a \neq 1}$

$2x^2 - 3x + 4$ $\quad a = 2,\ b = -3,\ c = 3$

**Identifying "a", "b", and "c" in:**
$\boldsymbol{ax^2 + bx + c}$

$y = kx^2 + mx + h$ $\quad a = k,\ b = m,\ c = h$

**Binomials**

$\mathbf{b = 0}$

$2x^2 + 4$

$\mathbf{c = 0}$

$x^2 + 4x$

$$\overset{a}{1x^2} + \overset{b}{5x} - \overset{c}{6}$$ Label a,b & c

**REARRANGE**

$6 - 5x + x^2$
$x^2 - 5x + 6$

$\overset{a}{1x^2} + \overset{b}{5x} + \overset{c}{6}$ $\quad \boxed{a = 1}$ ✓

Setup

$x^2 + 2x + 3x + 6$ *Decompose*
$(x^2 + 2x)|(+3x + 6)$ *Group*
$x(x + 2) + 3(x + 2)$ *GCF*
*Switch*

$(x + 2)(x + 3)$

| | |
|---|---|
| ____ X ____ = c | __2__ X __3__ = ~~c~~ 6 |
| ____ + ____ = b | __2__ + __3__ = ~~b~~ 5 |

Products
1,6
2,3

What are two numbers that:
Multiply to "c", the last number
Add together to get "b", the middle number.

Quick Method

| | |
|---|---|
| $x^2 + 5x + 6$<br>$(x + 2)(x + 3)$ | The numbers go in the brackets. |

*Step 1 Decompose*: What are two numbers that: multiply to get "$a \times c$" and add to get "$b$."
"b" gets split up into the two numbers above on the right.
*Step 2 Group*: Place brackets around the first two terms, and the second two terms.
*Step 3 GCF*: Remove a GCF from Groups.
*Step 3 GCF*: Remove a GCF from each.

They both have a $(x + 2)$
Take out a $(x + 2)$ Poetry

Check by Multiplying out

In your Head

# FOIL

$(x + 2)(x + 3)$

$x^2 + 3x + 2x + 6$

$x^2 + 5x + 6$ ✓

The answer should be the same as the original question.

$x^2 + 6x + 8$ $\quad a = 1$ ✓

__2__ X __4__ = ~~c~~ 8
__2__ + __4__ = ~~b~~ 6

$(x + 2)(x + 4)$

$x^2 - 3x - 10$ $\quad a = 1$ ✓

__−5__ X __2__ = ~~c~~ −10
__−5__ + __2__ = ~~b~~ −3

$(x - 5)(x + 2)$

Remember the sign of the numbers you choose goes in the bracket along with the number.

$x^2 + 4x + 15$ Cannot factor

# M10 - 5.3 - Factor by Decomposition $ax^2 + bx + c$ $(a \neq 1)$ Notes

**Factor by Decomposition**

$\times$

Put an Arrow from $a$ *to* $\times\, c$

$2x^2 + 7x + 6$

$\boxed{a \neq 1}$

Setup $a \times c$

$2x^2 + 3x + 4x + 6$ — *1) Decompose*

$(2x^2 + 3x) \mid (+4x + 6)$ — *2) Group*

$x(2x+3) + 2(2x+3)$ — *3) GCF*

$(2x+3)(x+2)$ — *4) GCF*

| | | | | |
|---|---|---|---|---|
| ______ | X | ______ | = *ac* | |
| ______ | + | ______ | = *b* | |

| | | | | | Products |
|---|---|---|---|---|---|
| 3 | X | 4 | = ~~*ac*~~ | 12 | 1,12 |
| 3 | + | 4 | = ~~*b*~~ | 7 | 2,6 |
| | | | | | 3,4 |

*Step 1 Decompose*: What are two numbers that: multiply to get "$a \times c$" and add to get "$b$."
"b" gets split up into the two numbers above on the right.
*Step 2 Group*: Place brackets around the first two terms, and the second two terms.
*Step 3 GCF*: Remove a GCF from Groups.
*Step 3 GCF*: Remove a GCF from each.

$(2x+3)(x+2)$

In your Head

**FOIL**

$2x^2 + 4x + 3x + 6$

$2x^2 + 7x + 6$

Quick Method

$2x^2 + 7x + 6$
$(2x+3)(x+2)$

Think by FOIL

$2x^2 + 7x + 6$

$(\qquad)(\qquad)$ Set Up Brackets

$(2x \qquad)(x \qquad)$ $2x \times x = 2x^2$

Then Figure out what works!

---

$2x^2 + 5x + 2$

*$a \neq 1$*

$2x^2 + 4x + 1x + 2$

$(2x^2 + 4x)(+x + 2)$

$2x(x+2) + 1(x+2)$

$(2x+1)(x+2)$

| | | | | |
|---|---|---|---|---|
| 4 | X | 1 | = ~~*ac*~~ | 4 |
| 4 | + | 1 | = ~~*b*~~ | 5 |

*Decompose*
*Group*

*GCF* $\boxed{GCF = 1}$
*Factor GCF out each set of brackets*

$2x^2 + 3x - 2$

*$a \neq 1$*

$2x^2 + 4x - x - 2$

$(2x^2 + 4x)(-x - 2)$

$2x(x+2) - 1(x+2)$

$(2x-1)(x+2)$

| | | | | |
|---|---|---|---|---|
| 4 | X | $-1$ | = ~~*ac*~~ | $-4$ |
| 4 | + | $-1$ | = ~~*b*~~ | 3 |

*Decompose*
*Group*

Don't Cut off a negative!

*GCF*
*Factor GCF out each set of brackets*

# M10 - 5.4 - Differences of Squares Notes

**Differences of Squares: A Subtraction Sign in Between two Squared Things**

$x^2 - 9$

$(\ + \ )(\ - \ )$ Step 1 Set Up Two Sets of Brackets with a +(Plus) and a − (Minus) Sign.

$(x + \ )(x - \ )$ Step 2 What squared is $x^2$? $x$. That answer goes first in each set of brackets.

$(x + 3)(x - 3)$ Step 3 What squared is 9? 3. That number goes second in each set of brackets.

$(x + 3)(x - 3)$

$x^2 - 3x + 3x - 9$

$x^2 - 9$

In your Head

**FOIL**

$x^2 + 4$ Cannot Factor a Sum of Squares

Cannot Factor

$4x^2 - 36$

$4(x^2 - 9)$ $GCF$

$4(x + 3)(x - 3)$ Factor

$4(x + 3)(x - 3)$ FOIL

$4(x^2 - 3x + 3x - 9)$

$4(x^2 - 9)$

$4x^2 - 36$

$x^4 - 1$

$(x^2 - 1)(x^2 + 1)$

$(x + 1)(x - 1)(x^2 + 1)$

$\boxed{\begin{array}{l} x^4 = x^2 \times x^2 \\ \text{Factor Twice} \end{array}}$

$x^4 - 81$

$(x^2 - 9)(x^2 + 9)$

$(x + 3)(x - 3)(x^2 + 9)$

$\boxed{\begin{array}{l} a^4 - b^4 \\ (a^2 + b^2)(a^2 - b^2) \\ (a^2 + b^2)(a + b)(a - b) \end{array}}$

$4x^2 - 49$

$(2x)^2 - 7^2$

$(2x + 7)(2x - 7)$

Figure Out what is being Squared

Change of base $\boxed{4x^2 = (2x)^2}$

Do this in your Head

Factor

$(2x + 7)(2x - 7)$ FOIL

$4x^2 - 14x + 14x - 49$

$4x^2 - 49$

$9x^2 - y^2$

$(3x)^2 - y^2$ $\boxed{9x^2 = (3x)^2}$

$(3x + y)(3x - y)$ Factor

$(3x + y)(3x - y)$ FOIL

$9x^2 - 3xy + 3xy - y^2$

$9x^2 - y^2$

$-x^2 + 49$

$49 - x^2$ Rearrange

$(7 + x)(7 - x)$ Factor

$(7 + x)(7 - x)$ FOIL

$49 - 7x + 7x - x^2$

$49 - x^2$

$49 - x^2$

$-(-49 + x^2)$ GCF= −1

$-(x^2 - 49)$ Rearrange

$-(x - 7)(x + 7)$ Factor

$-(x^2 + 7x - 7x - 49)$ FOIL

$-(x^2 - 49)$

$-x^2 + 49$

$49 - x^2$

$\boxed{\begin{array}{l} (1 - x^{10}) \\ (1 - x^5)(1 + x^5) \end{array}}$

# M10 - 5.5 - Factoring Combo Trinomials Notes

**Factoring Combinations**

$2x^2+10x+12$
$2(x^2+5x+6)$

$GCF=2$
$a=1$
Factor

$2(x+2)(x+3)$

FOIL

$2(x+2)(x+3)$
$2(x^2+3x+2x+6)$
$2(x^2+5x+6)$

$2x^2+10x+12$

*OR*

Decomposition

$2\ x^2+10x+12$
$2(x^2+5x+6)$
$x^2+2x+3x+6$
$(x^2+2x)(+3x+6)$
$x(x+2)+3(x+2)$
$2(x+2)(x+3)$

$GCF=2$
Forget about the 2
Put the 2 down
Below I the
Answee

$-x^2-5x-6$
$-(x^2+5x+6)$

$a=-1$
$GCF=-1$

$-(x+2)(x+3)$

Factor

$-(x^2+3x+2x+6)$
$-(x^2+5x+6)$

FOIL

$-x^2-5x-6$ ✓

$x^3+5x^2+6x$
$x(x^2+5x+6)$

$GCF=x$
Factor

$x(x+2)(x+3)$

$x(x+2)(x+3)$
$x(x^2+3x+2x+6)$
$x(x^2+5x+6)$

$x^3+5x^2+6x$ ✓

$x^4+5x^2+6$

$(x^2+3)(x^2+2)$

Factor

$(x^2+3)(x^2+2)$
$x^4+2x^2+3x^2+6$

FOIL

$x^4+5x^2+6$ ✓

$x^4-5x^2-36$
$(x^2-9)(x^2+4)$

Factor Trinomials

$(x-3)(x+3)(x^2+4)$

Factor Differences of Squares

$x^2-3xy-10y^2$

$(x-5y)(x+2y)$

$a=1$
*Factor*

$(x+2y)(x-5y)$
$x^2-5xy+2xy-10y^2$

FOIL

$x^2-3xy-10y^2$ ✓

$\underline{-5}$ X $\underline{2}$ = $\cancel{c}$ $-10$

$\underline{-5}$ + $\underline{2}$ = $\cancel{b}$ $-3$

Decomposition

$x^2-3xy-10y^2$
$x^2-5xy+2xy-10y^2$
$(x^2-5xy)+(+2xy-10y^2)$
$x(x-5y)+2y(x-5y)$

$(x+2y)(x-5y)$

# M10 - 5.6 - Factoring Substitution Let x = m+1 Notes

**Substitution Factoring**

$(m+1)^2+5(m+1)+6$

$\boxed{Let\ x = m+1}$

$x^2+5x+6$
$(x+2)(x+3)$
$\big((m+1)+2\big)\big((m+1)+3\big)$

*Put "x" in for "m + 1"*
Factor
*Put "m + 1" back in for "x"*
Substitute with Brackets

$(m+3)(m+4)$

OR

FOIL then Factor

$(m+1)^2+5(m+1)+6$
$(m+1)(m+1)\ldots$
$m^2+2m+1+5m+5+6$
$m^2+7m+12$
$(m+3)(m+4)$

$4x^2-(x+2)^2$
$(2x)^2-(x+2)^2$

$a^2-b^2$

*let $a = 2x$*
*let $b = (x+2)$*

*Put "a" in for "2x"*
*Put "b" in for "x + 2"*

Figure Out what is being Squared
Change of base
Do this in your Head

$\boxed{4x^2 = (2x)^2}$

$(a+b)(a-b)$

Factor

*Put "2x" back in for "a"*
*Put "x + 2" back in for "b"*
Substitute with Brackets

$(2x+(x+2))(2x-(x+2))$

Distribute
Combine Like Terms

$(3x+2)(x-2)$

OR

FOIL then Factor

$4x^2-(x+2)^2$
$4x^2-(x+2)(x+2)$
$4x^2-(x^2+4x+4)$
$4x^2-x^2-4x-4$
$3x^2-4x-4$
...
$(3x+2)(x-2)$

$9(x+2)^2-16(x-1)^2$

Let $a = x+2$
Let $b = x-1$

$9a^2-16b^2$
$(3a+4b)(3a-4b)$

$\big(3(x+2)+4(x-1)\big)\big(3(x+2)-4(x-1)\big)$
$(3x+6+4x-4)(3x+6-4x+4)$
$(7x+2)(-x+10)$

$-(7x+2)(x-10)$

$x^2-6x+9-y^2$
$(x^2-6x+9)-y^2$
$(x-3)^2-y^2$
...
$(x-3+y)(x-3-y)$

Group First/Last 3 Terms
Factor
Differences of Squares
...

$9x^4-9x^2+6xy-y^2$
$9x^4-(9x^2-6xy+y^2)$
$9x^4-(3x-1)^2$
$(3x^2)^2-(3x-1)^2$
$\big(3x^2+(3x-1)\big)\big(3x^2-(3x-1)\big)$

$(3x^2+3x-1)(3x^2-3x+1)$

# M10 - 6.1 - Linear/Continuous Notes

**Table of Values (Linear/Non-Linear)**

| | | $x$ | $y$ | | |
|---|---|---|---|---|---|
| | | $-4$ | $0$ | | |
| $\Delta x$ | $+2$ | $-2$ | $3$ | $+3$ | $\Delta y$ |
| $\Delta x$ | $+2$ | $0$ | $6$ | $+3$ | $\Delta y$ |
| $\Delta x$ | $+4$ | $4$ | $12$ | $+6$ | $\Delta y$ |
| $\Delta x$ | $+4$ | $8$ | $18$ | $+6$ | $\Delta y$ |

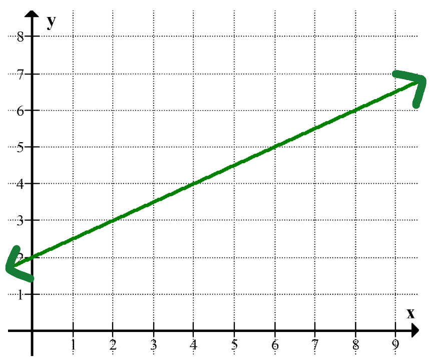
$If\ the\ fraction\ \frac{\Delta y}{\Delta x} = \frac{\Delta y}{\Delta x}, it\ is\ \boxed{Linear.}$

$\frac{3}{2} = \frac{3}{2}\ \boxed{Linear}$ ✓

$\frac{3}{2} = \frac{6}{4}$

$\frac{3}{2} = \frac{3}{2}\ \boxed{Linear}$ ✓

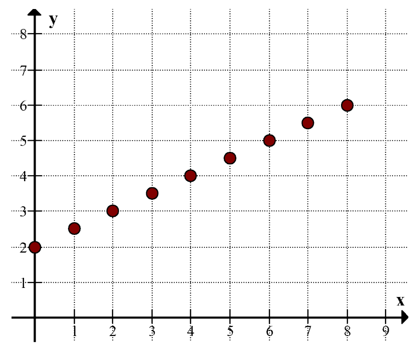
**Graph (Linear/Non-Linear)(Continuous/Discrete)**



**Linear**

If the line is straight, the relation is linear

**Continuous**

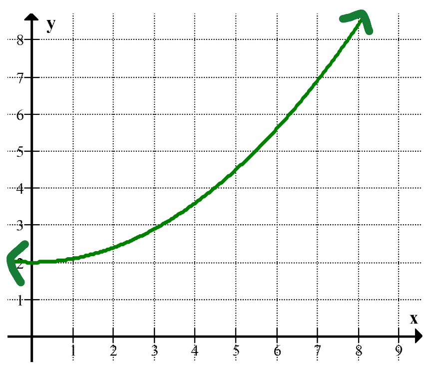


**Linear**

If the points are in a straight line, the relation is linear

**Discrete**

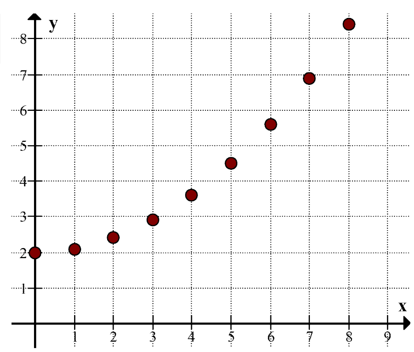
**Continuous: Points are connected**



**Non-Linear**

**Continuous**

If the points are/can be connected



**Non-Linear**

**Discrete**

If the points are not/cannot be connected

**Information: (Continuous/Discrete)**

**Continuous**

Walking to school
Filling a cup with water

The points can be connected because you are at each point throughout time.

**Discrete**

Counting the weight of apples
Counting number of Humans

The point not connected because you cannot have half an apple* or half a human.

**Linear/Non-Linear** Make a table of values or graph information and see.

**Equations (Linear/Non-Linear)**

**Linear**

If the equation is degree/exponents 0 or 1

$y = 3x + 1$ $\qquad$ $2y + 3x - 4 = 0$

**Non-Linear**

$y = x^2$ $\qquad$ $y^2 + x^2 = 1$ $\qquad$ $y = x^3 - 2x + 4$

$No\ y - int$

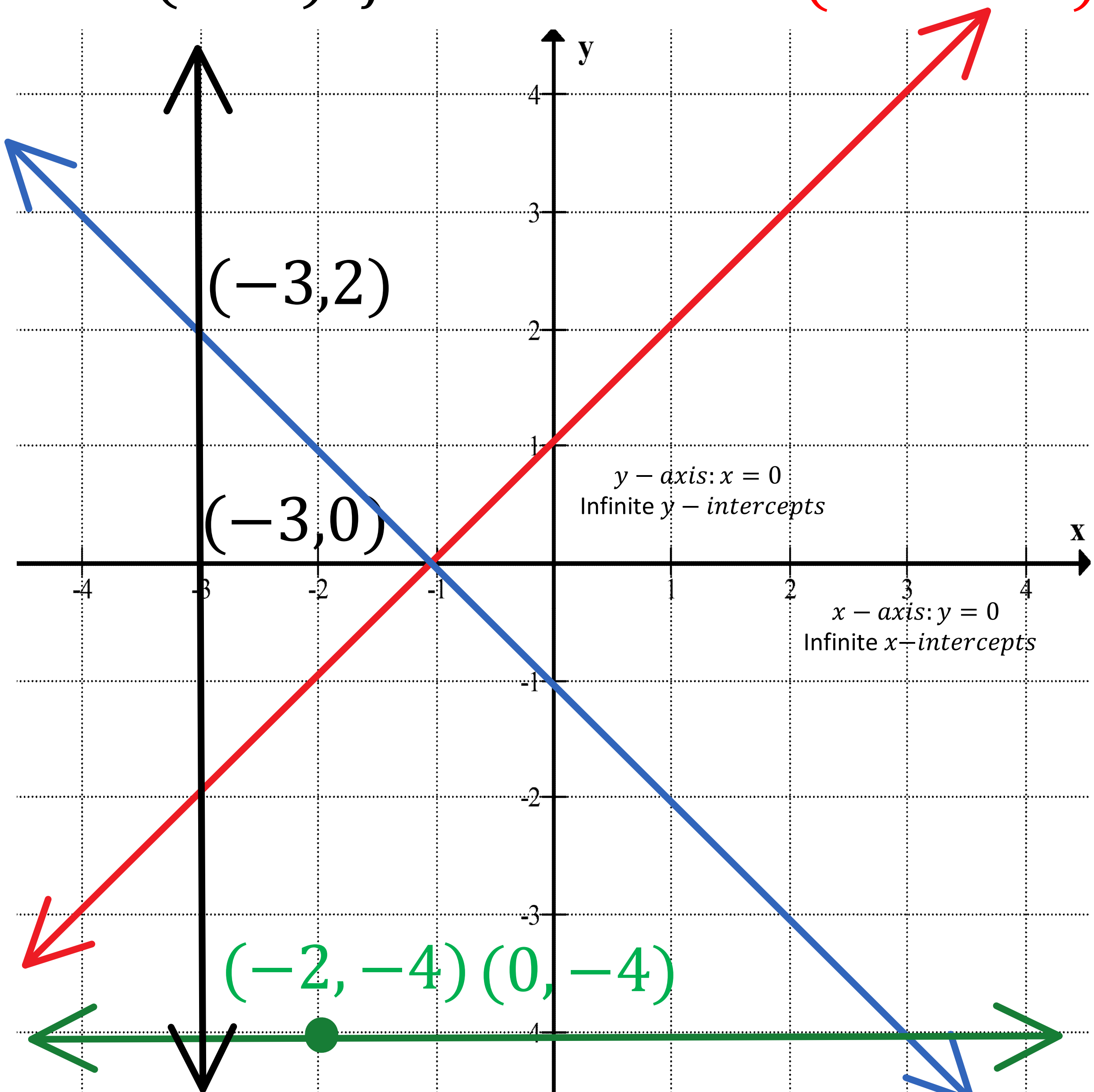
$x = -3$

Vertical

$m = (Und)efined$

Up to Right

$m = +(Positive)$




$m = 0\ (Zero)$

Flat - Horizontal

$y = -4$

$No\ x - int$

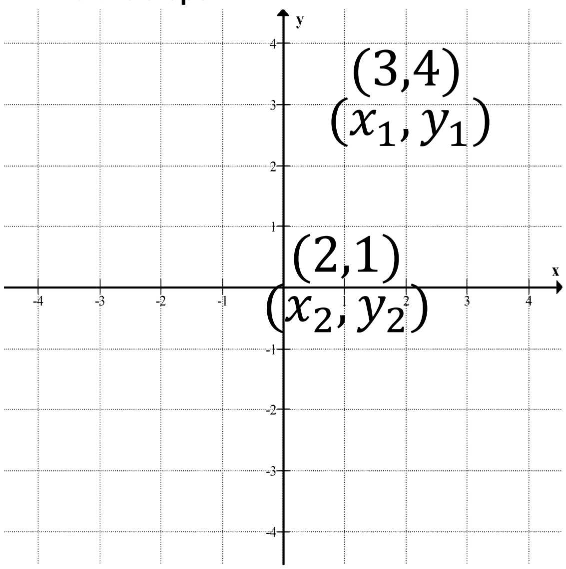
$m = +(Negative)$

Down to Right

(Up to Left)

# M10 - 6.3 - Slope Formula Notes

**Find the Slope**




**Slope Formula**

$$\boxed{Slope\ = \frac{rise}{run} = \frac{y_2 - y_1}{x_2 - x_1}}$$

← Vertical distance

← Horizontal distance

$(3,4)$ $(2,1)$

$(x_1, y_1)$ $(x_2, y_2)$

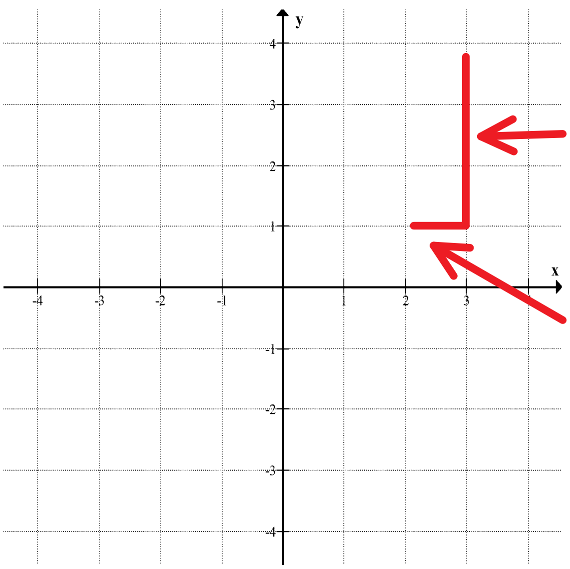
$$Slope\ = \frac{rise}{run} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{(5) - (2)}{(2) - (1)}$$

$$= \frac{3}{1}$$

Substitute with brackets

Slope = 3



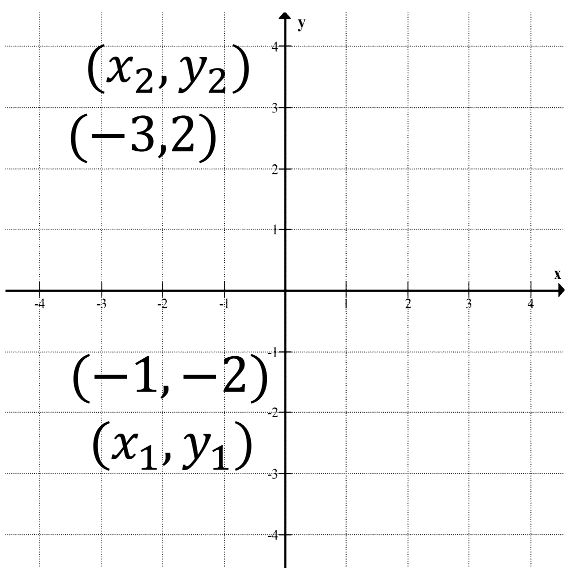
Vertical distance

Horizontal distance

Slope is how much you go up by over how much you go over by.

1) Start at the point on the Left
2) Count straight up to the next point
3) count straight over to the next point

$$\boxed{Slope = \frac{Up\ or\ Down}{Left\ or\ Right}}$$




$(-1,-2)$ $(-3,2)$

$(x_1, y_1)$ $(x_2, y_2)$

$$Slope\ = \frac{rise}{run} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{(2) - (-2)}{(-3) - (-1)}$$

$$= \frac{2 + 2}{-3 + 1}$$

$$= \frac{4}{-2}$$

$Slope = -2$

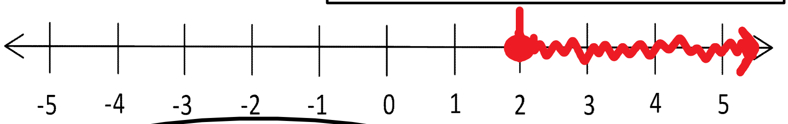
# M10 - 6.4 - Domain Range Notes

**Domain**: All possible $x$ values. $x$

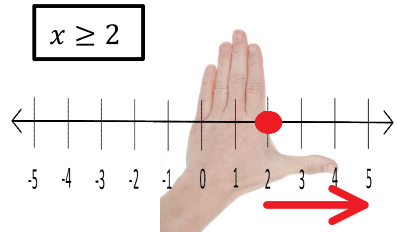
**Range**: All possible $y$ values. $y$

$x \geq 2$

$\leq, \geq$ • [ ] ———
Equal to (closed, square, solid)




$x \geq 2$




Left Hand
Thumb Points Greater Than

Words: $x$ is Greater than Equal to 2

Set Notation: Domain: $\{\, x \mid x \geq 2, x \in \mathbb{R}\}$

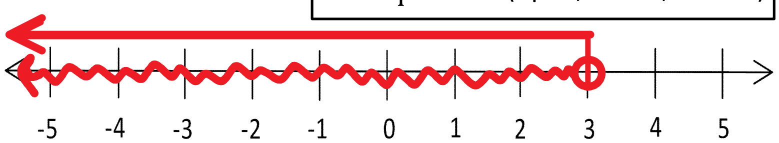
Interval Notation $[2, \infty)$

$x \in \mathbb{R}$ :
$x$ can be all
Real Numbers

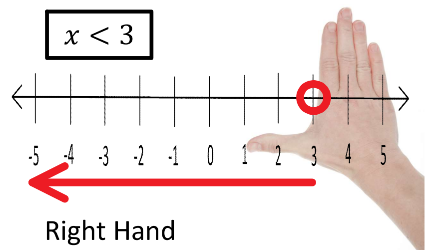
$(\infty, \infty)$ Infinity Not Included

$x < 3$

$<, >$ ◉ ( ) $(-\infty, \infty)$ – – – –
Not Equal to (open, round, dotted)




$x < 3$




Right Hand
Thumb Points Less Than

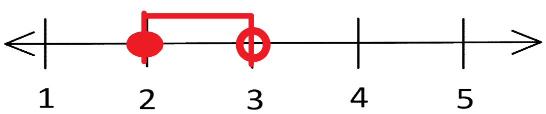
Words: $x$ is Less than 3

Set Notation: Domain: $\{\, x \mid x < 3, x \in \mathbb{R}\}$

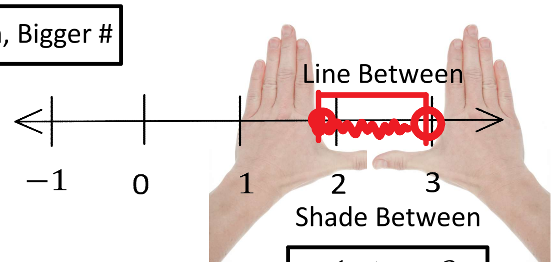
Interval Notation $(\infty, 3)$

$2 \leq x < 3$

Smaller #, Less Than*, Variable, Less Than, Bigger #




$x \geq 2$ $x < 3$

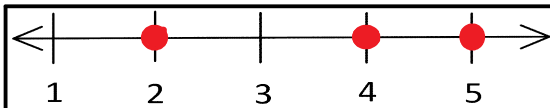



$-1 \leq x < 3$

Words: $x$ is Greater than or Equal to 2 and Less Less than 3

Set Notation: Domain: $\{\, x \mid 2 \leq x < 3, x \in \mathbb{R}\}$

Interval Notation [2,3)

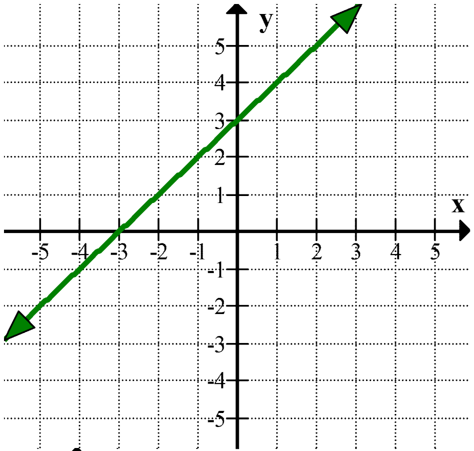



Words: $x = 2{,}4{,}5$ **A List**

Domain: $\{\, x \mid x = 2{,}4{,}5, \mathbb{Z} \in \mathbb{R}\}$

$\mathbb{Z} \in \mathbb{R}$
$x$ can be all
Real Integers

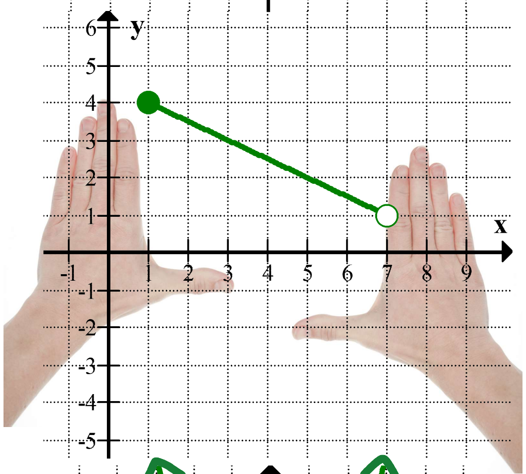
# M10 - 6.5 - Graph: Domain and Range Notes



**Domain**:
Number Line:
Set Notation: $\{x|\ x \in \mathbb{R}\}$
Interval Notation: $(-\infty, \infty)$

$\boxed{\{x|\ -\infty < x < \infty\}}$

**Range:**
Number Line:
Set Notation: $\{y|\ y \in \mathbb{R}\}$
Interval Notation: $(-\infty, \infty)$

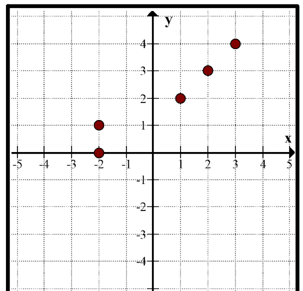


Use your Hands

**Domain**:
Number Line:
Set Notation: $\{x|1 \leq x < 7, x \in \mathbb{R}\}$
Interval Notation: $[1,7)$

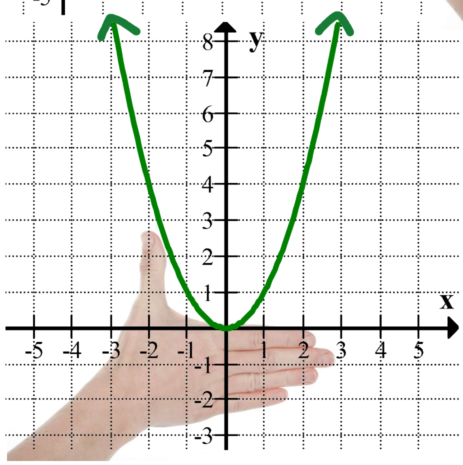
Rotate the Page!

**Range**:
Number Line:
Set Notation: $\{y|1 < y \leq 4, y \in \mathbb{R}\}$
Interval Notation: $(1,4]$



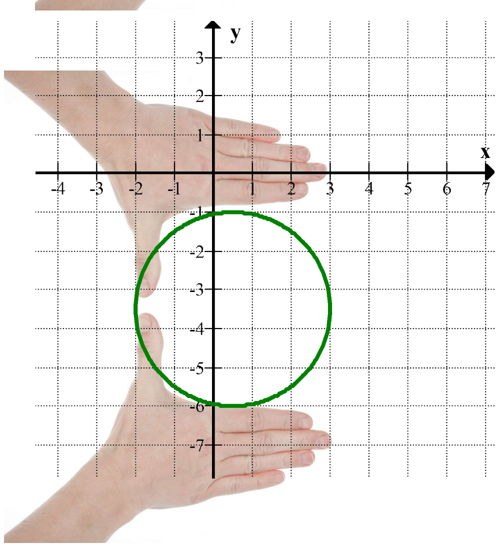
Domain: $\{-2,1,2,3\}$
Range: $\{0,1,2,3,4\}$

| x | 7 |
|---|---|
| $-2$ | 0 |
| $-2$ | 1 |
| 1 | 2 |
| 2 | 3 |
| 3 | 4 |



**Domain**:
Number Line:
Set Notation: $\{x|\ x \in \mathbb{R}\}$
Interval Notation: $(-\infty, \infty)$

**Range:**
Number Line:
Set Notation: $\{y|\ y \geq 0, y \in \mathbb{R}\}$
Interval Notation: $[0, \infty)$



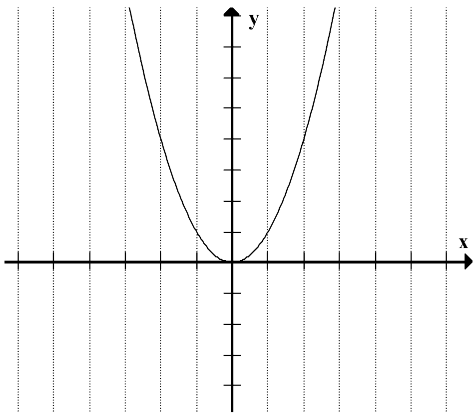
**Domain**:
Number Line:
Set Notation: $\{x| -2 \leq x \leq 3, x \in \mathbb{R}\}$
Interval Notation: $[-2,3]$

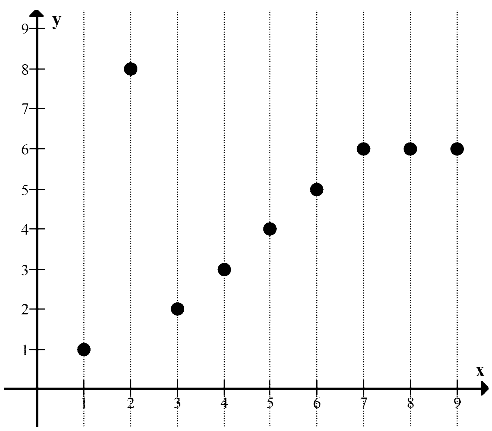
**Range**:
Number Line:
Set Notation: $\{y|\ -6 \leq y \leq -1, y \in \mathbb{R}\}$
Interval Notation: $[-6, -1]$

# M10 - 6.6 - Function Vertical Line Test Notes

A **Relation** is a **Function** if you only have one $y$ value for every $x$ value.

**Is a function**





$(0,1), (1,2), (2,3), (3,3), (4,5)$

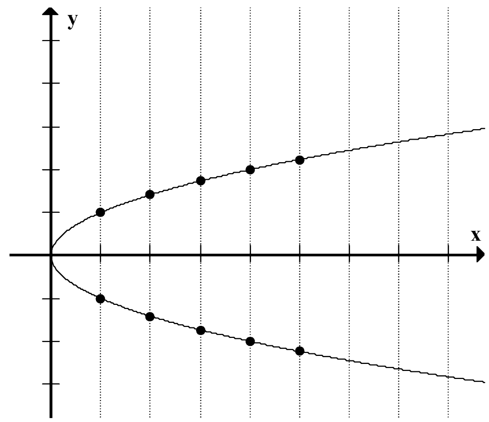
| $x$ | $y$ |
|---|---|
| 1 | 1 |
| 2 | 2 |
| 4 | 3 |
| 5 | 6 |

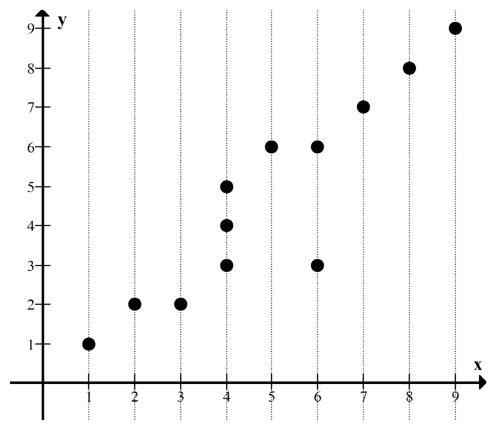
Each $x$ value only has one $y$ value

A **Relation** is a **Function** if you run your pencil vertically along the page and only cross the line once.

A **Relation** is **NOT** a **Function with more than** one $y$ value for any $x$ value.

**Not a function**





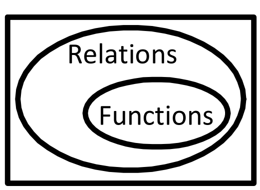
$(0,1), (1,2), (1,3), (2,4), (3,5)$

| $x$ | $y$ |
|---|---|
| 1 | 1 |
| 2 | 3 |
| 2 | 5 |
| 3 | 9 |

An x value with more than one y value

A **Relation** is a **Function** if you run your pencil vertically along the page and ever hits the line more than once.

Venn Diagram




All Functions are Relations
Not all Functions are Relations

# M10 - 7.1 - Standard/General Form Notes

**Graph the Line in Standard Form:**

$3x + 2y = 6$

**OR**

**x and y intercept method**

$3x + 2y - 6 = 0$
Subtract 6 on Both Sides

$Ax + By = C$
$Ax + By - C = 0$

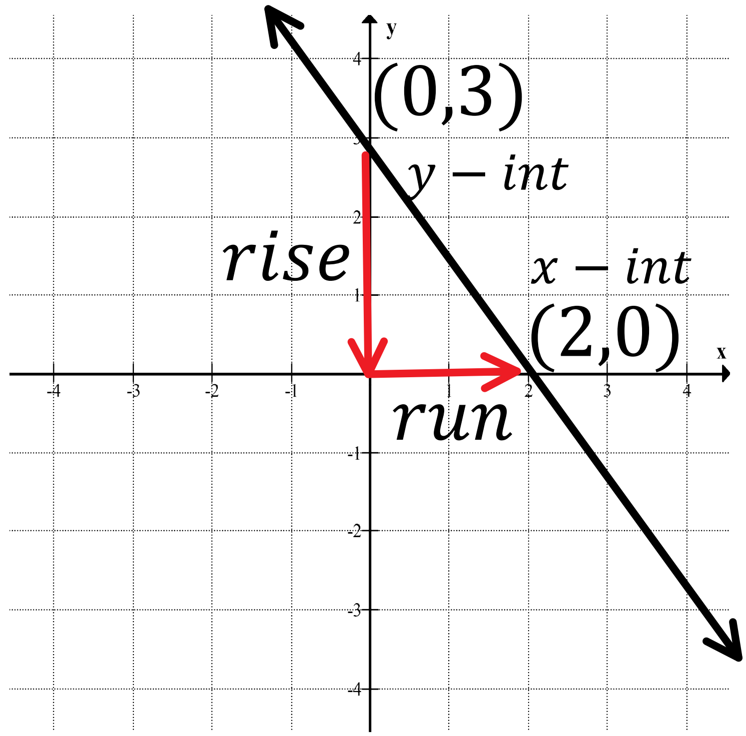
| $x$ | $y$ |
|---|---|
| 0 | |
| | 0 |

**Y Intercept:**

| | |
|---|---|
| $3x + 2y = 6$ | Equation |
| $2(0) + 2y = 6$ | Put Zero in for $x$ |
| $2y = 6$ | Solve |
| $\frac{2y}{2} = \frac{6}{2}$ | |
| | $(x, y)$ |
| $y = 3$ | $(0,3)$ |

**X Intercept:**

| | |
|---|---|
| $3x + 2y = 6$ | Equation |
| $3x + 2(0) = 6$ | Put Zero in for $y$ |
| $3x = 6$ | Solve |
| $\frac{3x}{3} = \frac{6}{3}$ | |
| | $(x, y)$ |
| $x = 2$ | $(2,0)$ |




**Converting Forms**

**Standard to Slope Intercept**

$Ax + By + C = 0 \longrightarrow y = mx + b$

| | |
|---|---|
| $3x + 2y = 6$ | Equation |
| $-3x \quad -3x$ | Subtract $3x$ to Both Sides |
| $2y = -3x + 6$ | |
| $\frac{2y}{2} = -\frac{3x}{2} + \frac{6}{2}$ | Divide Both Sides by 2 |
| $y = -\frac{3}{2}x + 3$ | Slope Intercept Equation |

$Slope = -\frac{3}{2}$ $\quad y - int: (0,3)$

$y = mx + b \longleftarrow y - intercept: (0, b)$

$Slope = \frac{rise}{run}$

**Slope Intercept to Standard**

$y = mx + b \longrightarrow Ax + By + C = 0$

| | |
|---|---|
| $y = -\frac{3}{2}x + 3$ | Equation |
| $\left(y = -\frac{3}{2}x + 3\right) \times 2$ | Multiply Both Sides by 2 $(LCD^*)$ |
| $2y = -3x + 6$ | |
| $+3x \quad +3x$ | Add $3x$ to Both Sides |
| $3x + 2y = 6$ | Standard From Equation |
| $-6 \quad -6$ | Subtract 6 from Both Sides |
| $3x + 2y - 6 = 0$ | Standard Form Equation |

$Ax + By = C$
$Ax + By - C = 0$
$+x$ coefficient
$x, y, \#/= 0$ Order
No Fractions

# M10 - 7.2 - Slope Intercept Form ($y = mx + b$) Notes

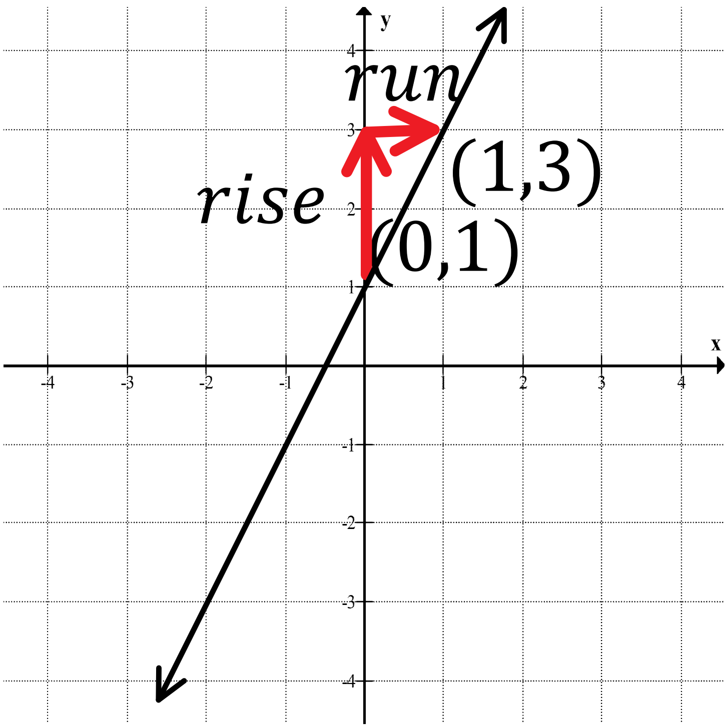
**Graphing Slope Intercept Form.** **Slope Intercept Method**

$y = 2x + 1 \leftarrow$ $y - intercept$: $(0,1)$

$\uparrow$

$Slope = \frac{2}{1}$

$\boxed{y = mx + b \leftarrow y - intercept: (0,b) \quad \uparrow \quad Slope = \frac{rise}{run}}$




Steps:

Plot $y - intercept$: $(0,1)$

Use slope: $\frac{2}{1}$ $\leftarrow$ Rise $\leftarrow$ Run

Plot new Point: (1,3)

Put Point in Other Direction

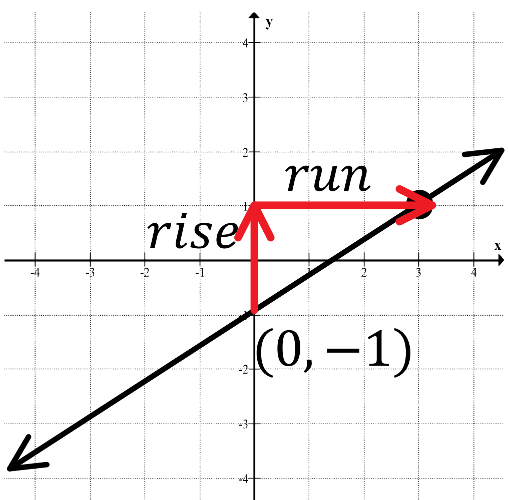
Draw New Points

Draw line

Arrow Tips

| $x$ | $y$ |
|---|---|
| $-1$ | $-1$ |
| $0$ | $1$ |
| $1$ | $3$ |
| $-2$ | $-3$ |

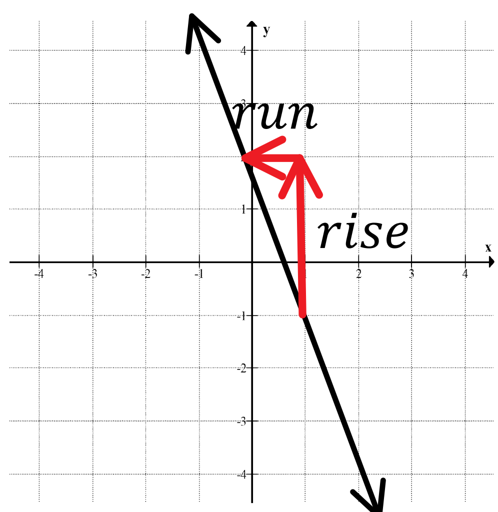
**Find Equation in Slope Intercept Form**




$y - int$: $(0,-1)$ $\quad$ $slope = m = \frac{2}{3}$

$y = mx + b$ $\quad$ Equation

$y = \frac{2}{3}x - 1$ $\quad$ Substitute b,m



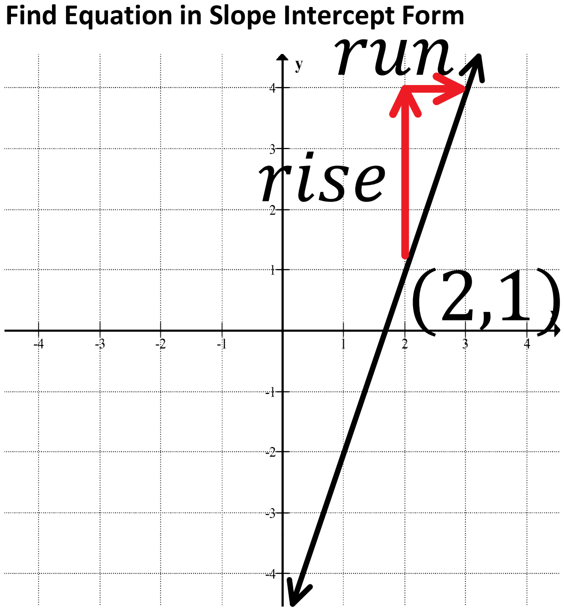

$y - int$: $(0,2)$ $\quad$ $slope = m = -\frac{3}{1}$

$y = mx + b$ $\quad$ $\boxed{\frac{-3}{1} = \frac{3}{-1} = -\frac{3}{1}}$

$y = -\frac{3}{1}x + 2$

# M10 - 7.3 - Slope Point Form $y - y_1 = m(x - x_1)$ *Notes*

**Find Equation in Slope Intercept Form**




$Point\ (x_1, y_1)$

$y - y_1 = m(x - x_1)$

$Slope = \dfrac{rise}{run}$

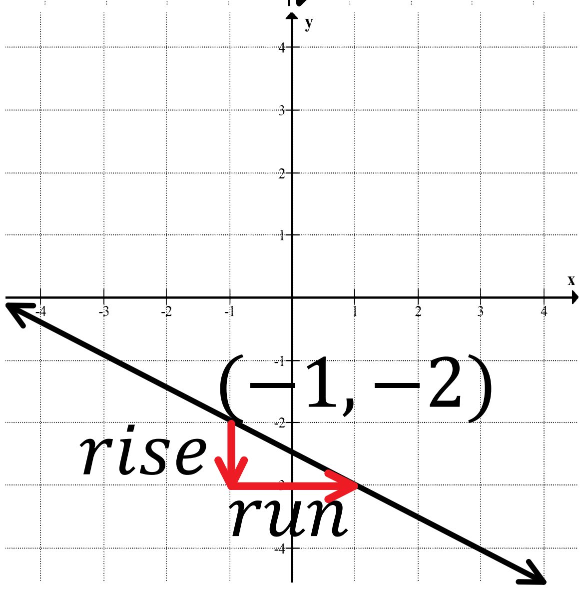
Steps:

Find Point $\quad Point\ (2,1)\ (x_1, y_1)$

Find Slope $\quad slope = m = \dfrac{3}{1}$

Equation $\quad y - y_1 = m(x - x_1)$

Substitute m
Point $\quad y - 1 = \dfrac{3}{1}(x - 2)$




Steps:

Find Point $\quad Point\ (-1, -2)\ (x_1, y_1)$

Find Slope $\quad slope = m = -\dfrac{1}{2}$

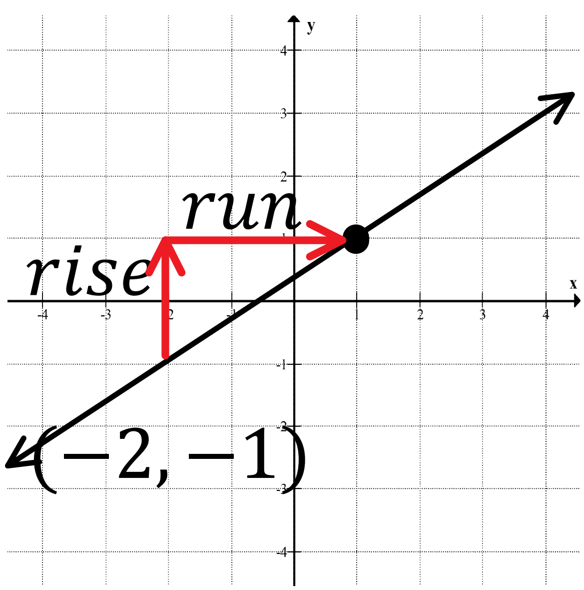
Equation $\quad y - y_1 = m(x - x_1)$

Substitute with Brackets

Substitute m
Point $\quad y - (-2) = -\dfrac{1}{2}(x - (-1))$

Simplify $\quad y + 2 = -\dfrac{1}{2}(x + 1)$

**Graph Slope Intercept Form**




Steps:

Equation $\quad y + 1 = \dfrac{2}{3}(x + 2)$

Write Form $\quad y - y_1 = m(x - x_1)$

Find Point
Graph Point $\quad Point\ (-2, -1)\ (x_1, y_1)$

Notice it's the Opposite of what's Inside the Brackets

Find Slope
Graph Slope $\quad slope = m = \dfrac{2}{3}$

# M10 - 7.4 - Find Equation Slope Int/Slope Pt Form Algebra Notes

**Given a point and the slope:** $(1,3)$ $m = 2$
$(x, y)$

$$\boxed{y - y_1 = m(x - x_1) \longrightarrow y = mx + b}$$

<u>Slope Intercept Form:</u>

| | |
|---|---|
| $y = mx + b$ | Slope Intercept Form |
| $y = (2)x + b$ | Substitute m |
| $(3) = (2)(1) + b$ | Substitute x and y |
| $3 = 2 + b$ | |
| $-2 \quad -2$ | |
| $1 = b$ | Solve for b |
| $y = mx + b$ | Slope Intercept Form |
| $y = (2)x + (1)$ | Substitute m and b |
| $y = 2x + 1$ | |

<u>Slope Point Form:</u>

| | |
|---|---|
| $y - y_1 = m(x - x_1)$ | Slope Point Form |
| $y - y_1 = 2(x - x_1)$ | Substitute m |
| $y - (3) = 2\big(x - (1)\big)$ | Substitute x and y |
| $y - 3 = 2(x - 1)$ | **Slope Point to Slope Intercept Form** |
| $y - 3 = 2(x - 1)$ | |
| $y - 3 = 2x - 2$ | Distribute |
| $+3 \quad +3$ | Add 3 to Both Sides |
| $y = 2x + 1$ | Slope Intercept Form |

$y = 2x + 1$ ⟵ They are equal ⟶ $y = 2x + 1$

---

**Given two points:** $(0,1)$ and $(1,3)$
$(x_1, y_1)$ $(x_2, y_2)$

| | |
|---|---|
| $m = \dfrac{y_2 - y_1}{x_2 - x_1}$ | Slope Equation |
| $m = \dfrac{(3) - (1)}{(1) - (0)}$ | Substitute With Brackets |
| $m = \dfrac{2}{1}$ | |
| $m = 2$ | Find m |

$\boxed{\text{Repeat Beginning of page!}}$

It doesn't matter which point you use

**Slope Intercept Form to Slope Point Form**

$$y = mx + b \xrightarrow{\quad} y - y_1 = m(x - x_1)$$

(N/A)

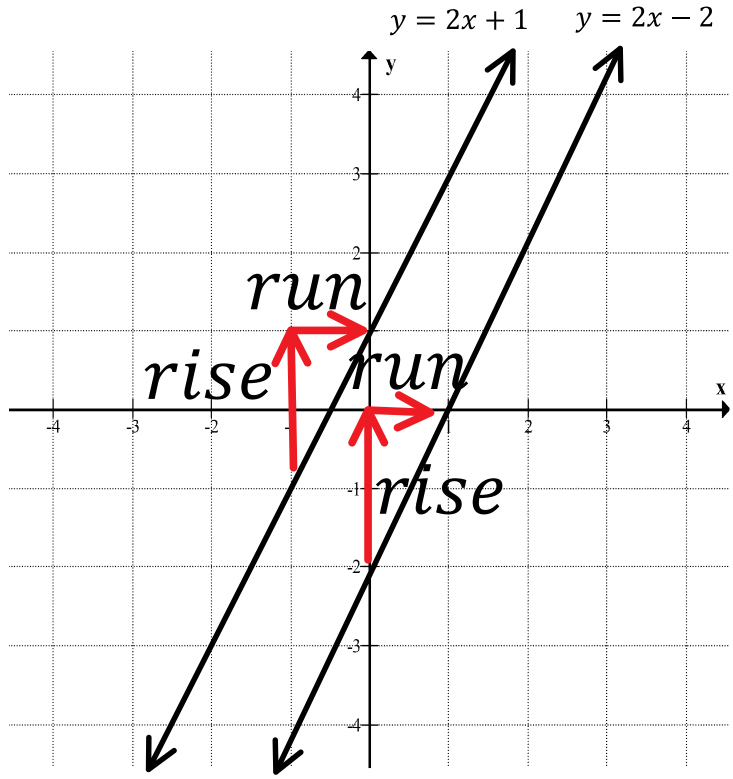
**General Form to Slope Point Form**

$$Ax + By + C = 0 \xrightarrow{\quad} y - y_1 = m(x - x_1)$$

(N/A)

# M10 - 7.5 - Parallel $m = m$/Perpendicular $m = -\frac{1}{m}$ Lines Notes

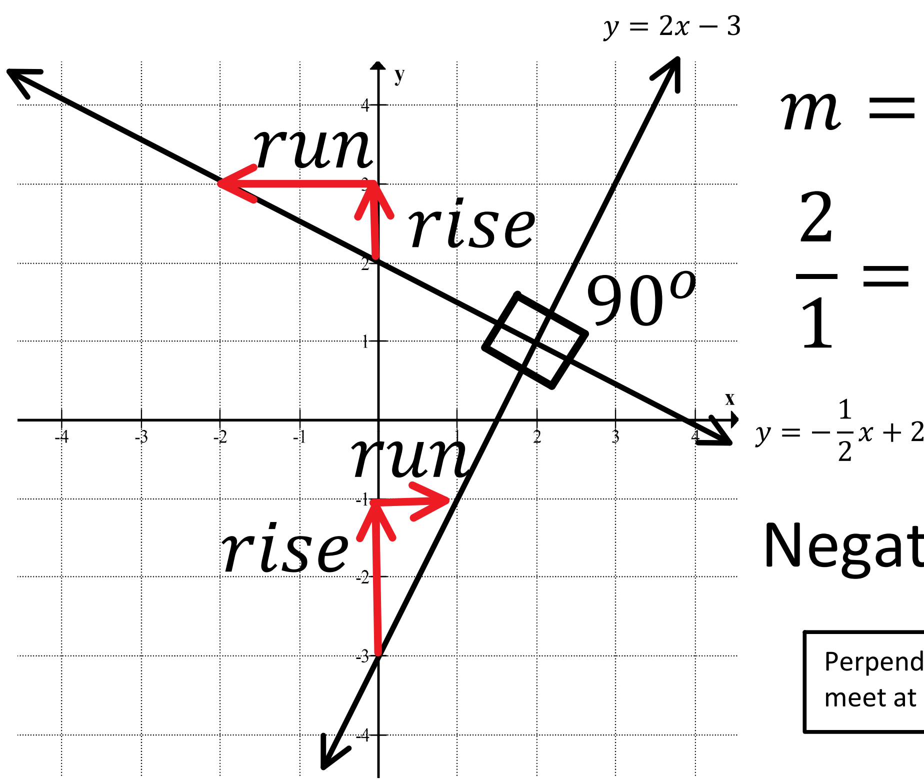
**Parallel Lines:** lines which never cross. Lines with the **<u>Same Slope</u>**. $m = m$




$$m = m$$
$$2 = 2$$
Same Slope

Notice: the graph of $y = 2x - 2$ and $y = 2x + 1$ are parallel because they have the same slope.

**Perpendicular Lines:** two lines which have N**<u>egative Reciprocal</u>** slopes and meet at 90°. $m = -\frac{1}{m}$




$$m = -\frac{1}{m}$$
$$\frac{2}{1} = -\frac{1}{2}$$
Negative Flip

Perpendicular Lines meet at a 90 degree.

Notice: The slope of the one line is the negative reciprocal of the slope of the other.

# M10 - 8.1 - Number of Intersections System Notes

**3 possible cases:**

- one solution
- no solutions
- infinite number of solutions.

**One Solution** **Different slopes**

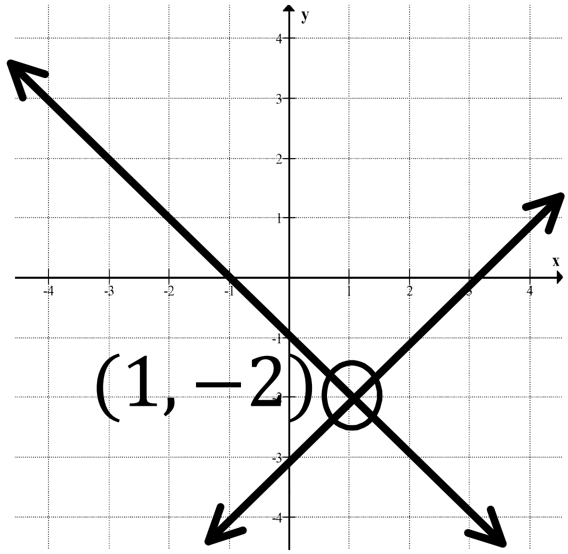
$y = x - 3$ $y = -x - 1$

$m = 1$ $m = -1$ **Different Slopes**

$$x - y - 3 = 0$$
$$+y \qquad +y$$
$$x - 3 = y$$
$$y = x - 3$$

Both to $y = mx + b$
Algebra
$+y$ to Both Sides

Mirror



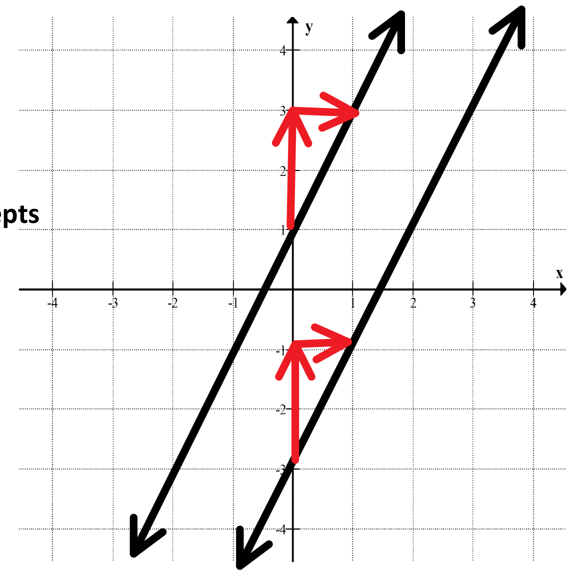

**No Solutions** **Parallel Lines**

$y = 2x - 3$ $y = 2x + 1$

$m = 2$ $m = 2$
$b = -3$ $b = 1$

**Same slope**
**Different $y$-intercepts**

**These Lines Never Intersect**



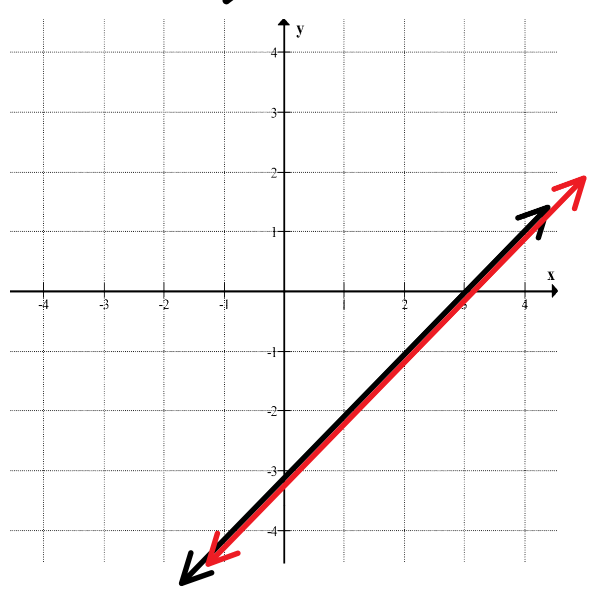

**Infinite Solutions** **Same Line**

$y = x - 3$ $y = x - 3$

$m = 1$ $m = 1$
$b = -3$ $b = -3$

**Same slope**
**Same $y$-intercept**

These Lines are on Top of Each Other



# M10 - 8.2 - Point on Line Notes

$Is\ (1,2)\ a\ point\ on\ the\ line?$

$y = x + 1$

$(1,2)$
$(x, y)$

Identify $x\ and\ y$
Substitute for $x\ and\ y$
Solve

$y = x + 1$
$(2) = (1) + 1$ ✓
$2 = 2$

If it works it's a Point on the Line

| x | y |
|---|---|
| 1 | 2 |

$(1,2)$

$Is\ (1,2)\ a\ point\ on\ the\ line?$

$y = -x + 3$

$(1,2)$
$(x, y)$

$y = -x + 3$
$(2) = -(1) + 3$ ✓
$2 = 2$

If it works it's a Point on the Line

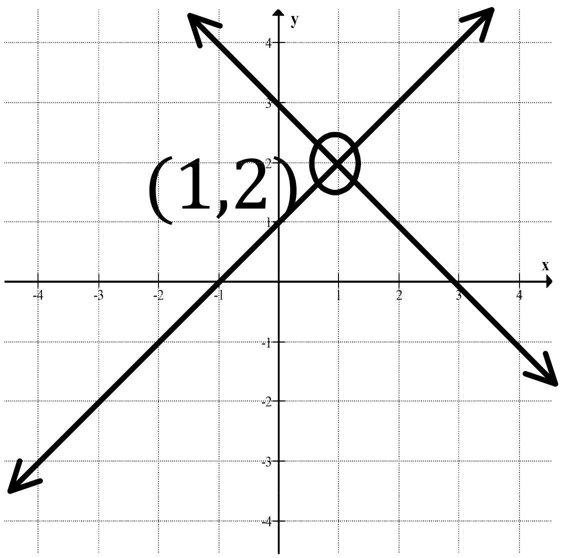
| x | y |
|---|---|
| 1 | 2 |

$(1,2)$

If it's is on both lines it must be the Intersection!

Graph both Lines:
Find Intersection

$y = mx + b$




$y = -x + 3$

$y = x + 1$

Both to $y = mx + b$
Algebra
$-x$ to Both Sides

$$\begin{aligned} x + y &= 3 \\ -x \quad &\quad -x \end{aligned}$$

$y = -x + 3$

---

$Is\ (1,3)\ a\ point\ on\ the\ line?$

$y = x + 1$

$(1,2)$
$(x, y)$

Identify $x\ and\ y$
Substitute Point for $x\ and\ y$
Solve

$y = x + 1$
$(3) \neq (1) + 1$
$3 \neq 2$ ✗

If it doesn't work it's NOT a Point on the Line.

Therefore Not the intersection!

# M10 - 9.1 - Substitution Notes

**Solve by Substitution**

1 $y = (x + 1)$ 2 $y = (-2x + 4)$

Identify equation # 1
Identify equation # 2

$$\begin{aligned} y &= y \\ x + 1 &= -2x + 4 \\ -1 \quad & \quad\quad -1 \\ x &= -2x + 3 \\ +2x \quad & \quad + 2x \\ \frac{3x}{3} &= \frac{3}{3} \end{aligned}$$

Make them equal to each other. Do it!

$a = b$ $b = c$

$a = c$

$x = 1$ Solve

1 $y = x + 1$

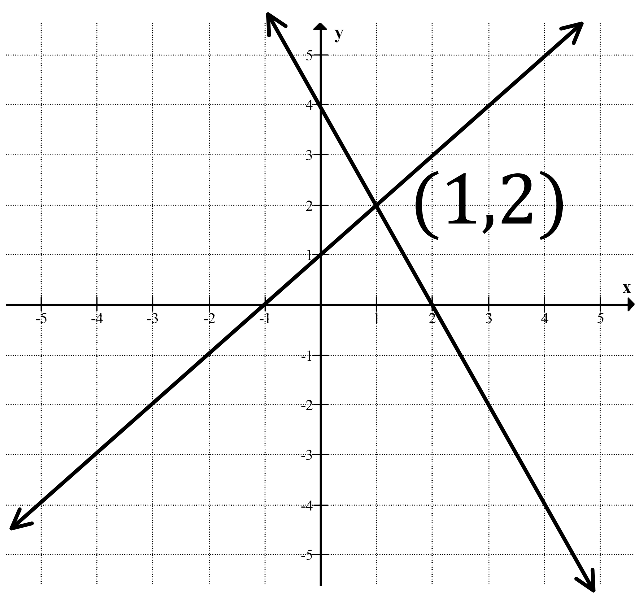
$y = (1) + 1$ Substitute

$y = 2$ Solve

(1,2) Intersection point

**Solve by Graphing**

$y = -2x + 4$ $y = x + 1$



# M10 - 9.2 - Don't/Need to Isolate Substitution Notes

**Substitution - Don't Need to Isolate**

1 $x = (3 - y)$    2 $2y - 2x = 10$

Identify equation # 1
Identify equation # 2

| Put Brackets around what $x =$ in $eq.$ #1<br>Put Brackets around $x$ $in$ $eq.$ #2 |
|---|

2
$$2y - 2(x) = 10$$
$$2y - 2(3 - y) = 10$$ Substitute
$$2y - 6 + 2y = 10$$ Distribute
$$4y - 6 = 10$$ Combine Like Terms
$$+6 \quad +6$$ Solve
$$4y = 16$$
$$y = 4$$

Substitute

1
$$x = 3 - y$$
$$x = 3 - (4)$$

Solve

$$x = -1$$

$(-1,4)$ — Intersection point

| If a variable is already isolated go ahead and substitute what that variable equals into the other equation. |
|---|

**Substitution - Need to Isolate**

1 $x + y = 11$    2 $2x - 2y = 6$

Identify equation # 1
Identify equation # 2

| Put Brackets around what $y$= in $eq.$ #1<br>Put Brackets around $y$ $in$ $eq.$ #2 |
|---|

1
$$x + y = 11$$
$$-x \quad -x$$
$$y = (11 - x)$$

Isolate

2
$$2x - 2(y) = 6$$
$$2x - 2(11 - x) = 6$$ Substitute
$$2x - 22 + 2x = 6$$
$$4x - 22 = 6$$
$$+22 \quad +22$$
$$4x = 28$$
$$\frac{4x}{4} = \frac{28}{4}$$
$$x = 7$$

Solve
Substitute
Solve

1
$$y = 11 - x$$
$$y = 11 - 7$$
$$y = 4$$

$(4,7)$

Intersection point:

# M10 - 9.3 - Elimination Notes

**Solving a system of equations using elimination**

1 $2y = x - 2$

2 $y = x - 3$

Identify equation # 1
Identify equation # 2

$$\begin{array}{r} 2y = x - 2 \\ -(y = x - 3) \\ \hline y = 0 + 1 \end{array}$$

$y = 1$

$\boxed{-2 - (-3) = 1}$

Subtract equations to eliminate $x$
Solve
Substitute
Solve
Intersection point:

2
$$\begin{array}{r} y = x - 3 \\ (1) = x - 3 \\ +3 \quad\quad +3 \\ 4 = x \end{array}$$

$x = 4$

$(4,1)$

Put brackets around what you're subtracting

---

1 $y + x = 6$

2 $y - x = 4$

Identify equation # 1
Identify equation # 2

$$\begin{array}{r} y + x = 6 \\ +(y - x = 4) \\ \hline 2y + 0x = 10 \end{array}$$

Add equations to eliminate $x$

You could have subtracted equations to eliminate y

$$2y = 10$$
$$\frac{2y}{2} = \frac{10}{2}$$

$y = 5$

Solve

1
$$\begin{array}{r} y + x = 6 \\ (5) + x = 6 \\ -5 \quad\quad -5 \end{array}$$

Substitute

$x = 1$

Solve

$(1,5)$

Intersection point:

# M10 - 9.4 - Line Up Elimination Notes

**Solving a system of equations using elimination**

1 $y = -6x + 2$ 2 $y + 4x = 0$

Identify equation # 1
Identify equation # 2

$$\begin{aligned} y &= -6x + 2 \\ +6x &\quad +6x \\ y + 6x &= 2 \end{aligned}$$

Algebra

$$\boxed{\begin{aligned} y + x &= \# \\ y + x &= \# \end{aligned}}$$

For Example

1 $y + 6x = 2$
2 $y + 4x = 0$

Line up equations
Subtract equations to eliminate $y$
Solve
Substitute
Solve
Intersection point:

$$\begin{array}{r} (y + 6x = 2) \\ -(y + 4x = 0) \\ \hline 0y + 2x = 2 \end{array}$$

$$2x = 2$$

$$\frac{2x}{2} = \frac{2}{2}$$

$$x = 1$$

1 $y = -6x + 2$

$$y = -6(1) + 2$$

$$y = -4$$

$$(1, -4)$$

# M10 - 9.5 - Multiply/Fraction/Decimal Elimination Notes

**Solving a system of equations using elimination**

1 $2x - 3y = 2$ 2 $x + 2y = 8$

Identify equation # 1
Identify equation # 2

2 $2(x + 2y = 8)$
$2x + 4y = 16$

Multiply equation #2 by 2

Line up equations

$$\begin{array}{r} 2x - 3y = 2 \\ -(2x + 4y = 16) \\ \hline 0x - 7y = -14 \end{array}$$

Subtract equations to eliminate $x$

$$-\frac{7y}{-7} = -\frac{14}{-7}$$

$y = 2$

2
$x + 2y = 8$
$x + 2(2) = 8$
$x + 4 = 8$

Solve
Substitute

$x = 4$

Solve

$(4,2)$

Intersection point:

**Solving a system of equations using elimination**

1 $3y + x = 4$ 2 $0.5y + \frac{x}{3} = 3$
$\frac{1}{2}y + \frac{x}{3} = 3$

Identify equation # 1
Identify equation # 2
Get Rid of Decimals

2 $\left(\frac{y}{2} + \frac{x}{3} = 3\right) \times 6$
$3y + 2x = 18$

Multiply equation #2 by 6 $(LCD)$

$\boxed{\text{To get rid of denominator}}$

Subtract equations to eliminate $x$
Solve

$$\begin{array}{r} 3y + x = 4 \\ \underline{-(3y + 2x = 18)} \\ -x = -14 \\ x = 14 \end{array}$$

2
$3y + 2x = 18$
$3y + 2(14) = 18$
$3y + 28 = 18$
$3y = 18 - 28$
$3y = -10$
$\frac{3y}{3} = -\frac{10}{3}$

Substitute

$y = -\frac{10}{3}$

Solve

$(14, -\frac{10}{3})$

Intersection point:

# M10 - 9.6 - Let Statement/Value of Notes

A person has 24 quarters and dimes.

$let\ q = \#\ of\ quarters$
$let\ d = \#\ of\ Dimes$

Let Statements

$q + d = 24$

Equation

A person has some Toonies. How much do they have in Toonies?

$let\ t = \#\ toonies$

$\boxed{\text{Round the bottom of your t!}}$

| $t$ | $Value\ \$$ | $Calculation$ |
|---|---|---|
| $0$ | $0$ | $0 \times 2 = 0$ |
| $1$ | $2$ | $1 \times 2 = 2$ |
| $2$ | $4$ | $2 \times 2 = 4$ |
| $t$ | $2t$ | $t \times 2 = 2t$ |

$\boxed{\#\text{ of} \times Value}$

$$2t$$

$$Value\ of\ a\ Toonie \times \#\ Toonies$$

A person has the \$2.30 in Dimes, How many Dimes do they have?

$let\ d = \#\ of\ Dimes$

| $d$ | $Value\ \$$ | $Calculation$ |
|---|---|---|
| $0$ | $0$ | $0 \times 0.1 = 0$ |
| $1$ | $0.1$ | $1 \times 0.1 = 0.1$ |
| $2$ | $0.2$ | $2 \times 0.1 = 0.2$ |
| $d$ | $0.1d$ | $d \times 0.1 = 0.1d$ |

$0.1d$

$$0.1d = 2.30$$

$$\frac{0.1d}{0.1} = \frac{2.30}{0.1}$$

$$d = 23$$

They have 23 Dimes

$$0.1 \times 23 = 2.30$$ ✓

$\boxed{\text{Check Answer}}$

An airplane is flying at a height of 400 m and descending at 5 m/s.

$let\ h = height\ (m)$
$let\ t = time\ (s)$

$h = 400 - 5t$

Jane's hair is 30 cm long and grows at 2 cm per month.

$let\ h = hair\ length\ (cm)$
$let\ t = time\ (months)$

$h = 20 + 2m$

# M10 - 9.6 - $Ax + By = C$ Coins/Mixture Notes

Jay has 12 Total Coins of Quarters and Dimes worth \$2.40. How many does he have of each?

$Let\ d = \#\ dimes$
$Let\ q = \#\ quarters$

| | | | |
|---|---|---|---|
| $d + q = 12$<br>$-q \quad -q$ | $0.1d + 0.25q = 2.40$ | 2 equations |
| $d = 12 - q$ | | Isolate |
| | $0.1(d) + 0.25q = 2.40$ | Substitute |
| | $0.1(12 - q) + 0.25q = 2.40$ | Distribute |
| | $1.2 - 0.1q + 0.25q = 2.40$ | Combne Like Terms |
| | $1.2 + 0.15q = 2.40$ | |
| | $-1.2 \qquad -1.2$ | Subract Both Sides |
| | $\frac{0.15q}{0.15} = \frac{1.20}{0.15}$ | Divide Both Sides |
| $d = 12 - q$ | | |
| $d = 12 - (8)$ | $q = 8$ | Solve<br>Substitute |
| $d = 4$ | | Solve |

$4 + 8 = 12$ ✓ $\quad 0.1 \times 4 + 0.25 \times 8 = 2.40$ ✓ — Check your answer

$Jay\ has\ 4\ dimes\ and\ 8\ quarters\ worth\ \$2.40.$ — Answer the question

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As scientist wants to make 50 L of a 40% acid solution. They mixed together a 30% acid solution with the 70% acid solution. How many litres of each solution must the scientist mix?

$let\ a = litres\ of\ 30\%\ mix$
$let\ b = litres\ of\ 70\%\ mix$

$$\boxed{\% \times Amount + \% \times Amount = \% \times Amount}$$

$a + b = 50$
$b = 50 - a$
...
$b = 12.5$

$0.3a + 0.7b = 0.4(50)$
$0.3a + 0.7(50 - b) = 20$
...
$a = 37.5$

12.5 $L\ of$ 70% Mix

37.5 $L\ of$ 30% Mix

# M10 - 9.6 - $y = mx + b$ Cell Phone Word Problems Notes

**Create Let Statements, an equation, and solve the equation.**

**A cell phone company Data Costs $40 per month plus $0.1 per Megabyte of Data.**

$Let\ c = cost$

$$c = 40 + 0.1d$$

$Let\ d = \#\ megabytes\ of\ data$

If a person uses 480 megabytes of Data what will month bill cost? $\boxed{d = 480}$

$c = 40 + 0.1d$ Formula

$c = 40 + 0.1(480)$ Substitute

$c = 40 + 4.8$

$c = \$\ 44.80$ Solve

480 megabytes of Data will cost $44.80

If a person's bill is $52.60, How many Megabytes did the use? $\boxed{c = 52.60}$

$c = 40 + 0.1d$ Formula

$52.60 = 40 + 0.1d$ Substitute

$-40 \quad -40$

$\frac{12.60}{0.1} = \frac{0.1d}{0.1}$

$126 = d$

$d = 126$

Solve

$52.60 will buy 126 megabytes of data

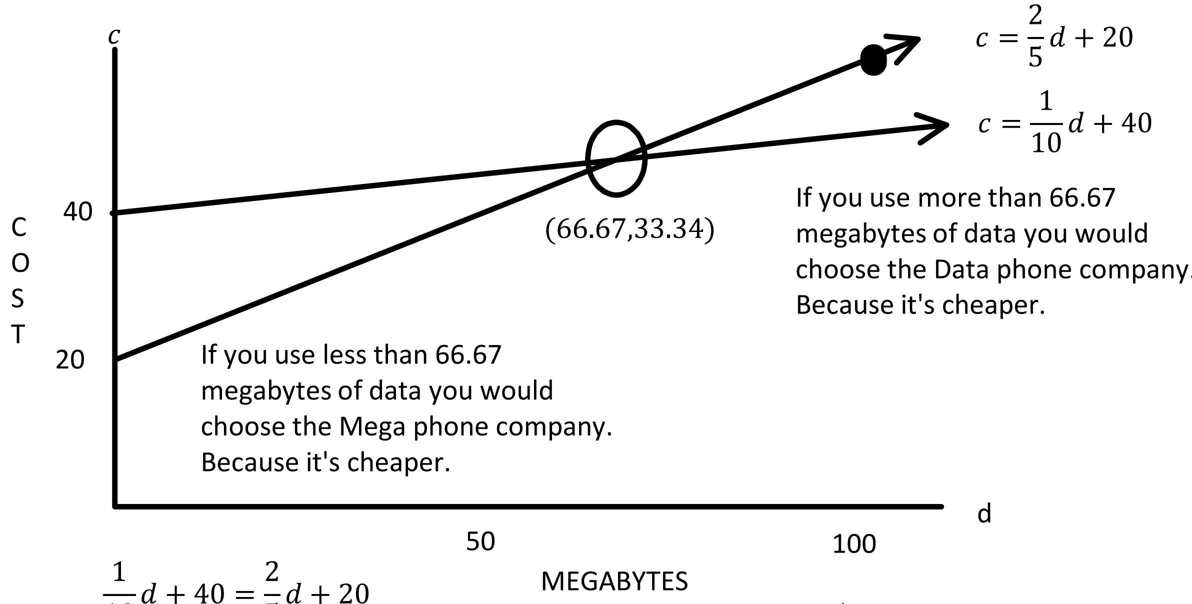
Mega Cell Phone Company charges $30 per month plus $0.2 per megabyte of data. Which company would you choose?

$Let\ c = cost$

$$c = 20 + 0.4d$$

$$\boxed{y = mx + b}$$

$Let\ d = \#\ megabytes\ of\ data$




$$\frac{1}{10}d + 40 = \frac{2}{5}d + 20$$
$$\left(\frac{1}{10}d + 40 = \frac{2}{5}d + 20\right) \times 10$$
$$d + 400 = 4d + 200$$
$$\frac{200d}{3} = \frac{3d}{3}$$
$$d = 66.67$$

$$c = \frac{1}{5}d + 20$$
$$c = \frac{1}{5}(66.67) + 20$$
$$c = 33.34$$

# M10 - 9.6 - $s = \frac{d}{t}$ Boat/Wind Word Problems Notes

A boat took 3 hours to travel 24 km with the current. On the return trip, the boat took 5 hours to travel 24 km against the current. Determine the speed of the current.

$x = speed\ of\ boat$
$c = speed\ of\ current$

$\xrightarrow{c}$ $\qquad$ $\xrightarrow{c}$

$\xrightarrow{24km \quad 3hr}$ $\qquad$ $\xleftarrow{24km \quad 5hr}$

$s = \frac{d}{t}$ $\qquad$ $s = \frac{d}{t}$

1
$$x + c = \frac{24}{3}$$
$$x + c = 8$$
$$x = 8 - c$$

2
$$(x) - c = \frac{24}{5}$$
$$(8 - c) - c = \frac{24}{5}$$
$$8 - 2c = 4.8$$
$$+2c \quad +2c$$
$$8 = 4.8 + 2c$$
$$-4.8 \quad -4.8$$
$$\frac{3.2}{2} = \frac{2c}{2}$$
$$c = 1.6$$

3

4

5
$$x = 8 - c$$
$$x = 8 - 1.6$$
$$x = 6.4$$

$$x = speed\ of\ boat = 6.4 \frac{km}{h}$$

$$c = speed\ of\ current = 1.6 \frac{km}{h}$$

# The End