

C12 - 5.8 - ln/e +1 - 1 + Long Div/Sep Frac Int Notes

$$\int \frac{1}{x} dx \quad \int \frac{1}{x+1} dx \quad \int \frac{x}{x+1} dx \quad \boxed{+1-1}$$

$$\ln|x| + C \quad \ln|x+1| + C$$

$$\int \frac{x+1-1}{x+1} dx$$

$$\int \frac{x+1}{x+1} - \frac{1}{x+1} dx \quad \boxed{\text{Separate Fractions}}$$

$$\int 1 - \frac{1}{x+1} dx \quad \frac{a+b}{c} = \frac{a}{c} + \frac{b}{c}$$

$$x - \ln(x+1) + C$$

$$\int \frac{2}{x-2} dx$$

$$2 \int \frac{1}{x-2} dx \quad \boxed{2\ln|x-2| + C}$$

$$y = 2\ln|x-2|$$

$$y' = \frac{2}{x-2}$$

$$\int \frac{1}{2x+1} dx \quad \ln|2x+1| + C$$

$$\int \frac{x}{x-2} dx = ?$$

$$\int (1 + \frac{2}{x-2}) dx = ? \quad \boxed{x + 2\ln(x-2) + C}$$

$$\begin{array}{r} 1 \\ x-2 \overline{) x+0} \\ \underline{x-2} \\ 2 \end{array}$$

Long Division

$$\frac{x}{x-2} = 1 + \frac{2}{x-2}$$

$$\frac{x}{x-2} = \frac{x-2}{x-2} + \frac{2}{x-2} \quad \boxed{+2-2}$$

$$\frac{x-2}{x-2} + \frac{2}{x-2} \quad \boxed{\text{Separate Fractions}}$$

$$\int e^x = e^x + C \quad \int e^{2x} = \frac{e^{2x}}{2} + C$$

Think: What would you have to do/divide by to reverse chain rule?

$$\int 5^x = \frac{5^x}{\ln 5} + C \quad \int 7^{2x} = \frac{7^{2x}}{2\ln 7} + C$$

$$\int e^{-x} = -e^{-x} + C$$

$$\int \frac{1}{1+e^x} dx$$

$$\int 1 - \frac{e^x}{1+e^x} dx$$

$$\int 1 dx - \int \frac{e^x}{1+e^x} dx \quad \boxed{x - \ln(1+e^x) + C}$$

$$\frac{1}{1+e^x} \quad \boxed{+e^x - e^x}$$

$$\frac{1+e^x - e^x}{1+e^x} = \frac{1}{1+e^x}$$

$$\frac{1}{1+e^x} - \frac{e^x}{1+e^x}$$

$$1 - \frac{e^x}{1+e^x} \quad \boxed{\text{Separate Fractions}}$$

$$\int \frac{e^x}{1+e^x} dx$$

$$\int \frac{e^x du}{1+u} \quad \frac{du}{dx} = e^x$$

$$\int \frac{1}{1+u} du \quad \frac{dx}{du} = \frac{1}{e^x}$$

$$\ln(1+u) + C \quad \boxed{\ln(1+e^x) + C}$$

$$\int \frac{e^x}{1+e^x} dx \quad u = 1+e^x$$

$$\int \frac{e^x du}{u} \quad \frac{du}{dx} = e^x$$

$$\int \frac{1}{u} du \quad \boxed{\text{OR}} \quad dx = \frac{du}{e^x}$$

$$\ln u + C \quad \boxed{\ln(1+e^x) + C}$$

$$\int \frac{2}{2x+1} dx$$

$$2 \int \frac{1}{2x+1} dx$$

$$\frac{\ln|2x+1|}{2} + C$$

$$\boxed{\ln|2x+1| + C}$$

$$y = \ln(2x+1)$$

$$y' = \frac{1}{2x+1} \times 2$$

$$y' = \frac{2}{2x+1}$$

$$\int \frac{2}{2x+1} dx$$

$$\int \frac{2 du}{u}$$

$$\int \frac{1}{u} du$$

$$\ln u + C \quad \boxed{\ln|2x+1| + C}$$

$$u = 2x+1$$

$$\frac{du}{dx} = 2$$

$$dx = \frac{du}{2}$$