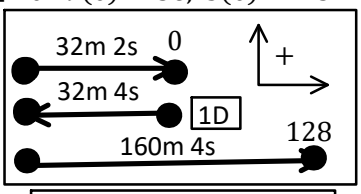
C12 - 5.7 - Part* Mot

$t\,;s$ $\quad s(t)\,;m$ $\quad v(t)\,;\frac{m}{s}$ $\quad a^*(t)\,;\frac{m}{s^2}$ $\quad s'(t)=v(t)$ $\quad v'(t)=a(t)$ $\quad \int v(t)\,dt=s(t)$ $\quad \int a(t)\,dt=v(t)$

A Particle moving East along a straight line, by the function $s(t)\ ;\ 0\le t\le 10.\ v(0)=36,\ s(0)=-32.$

$s(t)=t^3-12t^2+36t-32$



$y=position\ (s)$
$y'=velocity\ (v)$
$y''=acceleration\ (a)$

? s, t = 4.

$s(t)=t^3-12t^2+36t-32$

$s(4)=(4)^3-12(4)^2+36(4)-32$

$s(4)=-16$

? t, Origin. $s(t)=0$

$s(t)=t^3-12t^2+36t-32$

$0=t^3-12t^2+36t-32$

$0=(t-2)^2(t-8)$

$t=2,8$

PC 12 Cubic Factoring

Find $v(t)$.

$s(t)=t^3-12t^2+36t-32$

$v(t)=s'(t)=3t^2-12t+36$

Check End Points.

? v, t = 4.

$v(t)=3t^2-24t+36$

$v(4)=3(4)^2-24(4)+36$

$v(4)=-12$

? t, Rest? $v(t)=0.$

$v(t)=3t^2-24t+36$

$0=3t^2-24t+36$

$0=t^2-8t+12$

$0=(t-6)(t-2)$

$t=2,6$

Change Direction. $v(t)=0\ \&\ v(t)\ +\to-\ \ -\to+$

? t, Positive Velocity. $v(t)>0\ ?$

$(0,2)\ U\ (6,10)$

? t, Negative Velocity. $v(t)<0\ ?$

$(2,6)$

? t, Farthest from the origin. $v(t)=0$

$s(t)=t^3-12t^2+36t-32$

$s(6)=(6)^3-12(6)^2+36(6)-32$

$s(6)=-32$ Check End Points.

$s(2)=0^*$ $\quad s(4)=-16^*$

$s(10)=(10)^3-12(10)^2+36(10)-32$

$s(10)=128$

$v(t)=3t^2-24t+36$

$v(1)=3(1)^2-24(1)+36$

$v(1)=+15$

$v(3)=3(3)^2-24(3)+36$

$v(3)=-9$

$v(7)=3(7)^2-24(7)+36$

$v(7)=+15$

Find $a(t)$.

$v(t)=3t^2-24t+36$

$a(t)=v'(t)=6t-24$

? a, t = 5.

$a(t)=6t-24$

$a(5)=6(5)-24$

$a(5)=6$

? t, a = 6.

$a(t)=6t-24$

$6=6t-24$

$t=5$

? t, Greatest v. $v'(t)=6t-24$

? t, Least v.

$v'(t)=0$

$0=6t-24$

$t=4$

$v(4)=-12^*$ Min

$v(t)=3t^2-24t+36$

$v(0)=3(0)^2-24(0)+36$

$v(0)=36$

$v(10)=3(10)^2-24(10)+36$

$v(10)=96$ Max

? t, Greatest a. $a(t)=6t-24$

? t, Least a. $a'(t)=6$

$a'(t)=0$ Jerk

$a(t)=6t-24$

$a(0)=6t-24$

$a(0)=-24$ Min

$a(10)=6(10)-24$

$a(10)=36$ Max

Snap Crackle Pop

? t, Speeding Up? $(2,4)\ U\ (6,10)$

$a\ \&\ v\ Same\ Sign.$

? t, Slowing Down? $(0,2)\ U\ (4,6)$

$a\ \&\ v\ Different\ Signs.$

$v(1)=+15^*$ $\quad v(3)=-9^*$

$a(t)=6t-24$

$a(1)=6(1)-24$

$a(1)=-18$

$a(3)=6(3)-24$

$a(3)=-8$

$v(t)=3t^2-24t+36$

$v(5)=3(5)^2-24(5)+36$

$v(5)=-9$

$a(5)=+6$

$v(7)=15^*$

$a(7)=6(7)-24$

$a(7)=+18$

? $\vec{d}$, t = 4. $s(0)=-32$

$-32\to-16=+16$

$\vec{d}=16$ $\quad s(4)=-16^*$

? d, t = 4. Need $v(t)=0, s(t)^*$.

$s(0)=-32$ $\quad s(4)=-16^*$

$s(t)=t^3-12t^2+36t-32$

$s(2)=(2)^3-12(2)^2+36(2)-32$

$s(2)=0$

$-32\to0=32$ $\quad d=32+16$

$0\to-16=16$ $\quad d=48$

Find $v(t)$. $\quad a(t)=6t-24$

$v(t)=\int(6t-24)\,dt$

$v(t)=3t^2-24t+C$

$36=3(0)^2-24(0)+C$

$C=36$

$v(t)=3t^2-24t+36$

Find $s(t)$.

$s(t)=\int(3t^2-24t+36)\,dt$

$s(t)=t^3-12t^2+36t+C$

$-32=(0)^3-12(0)^2+36(0)+C$

$C=-32$

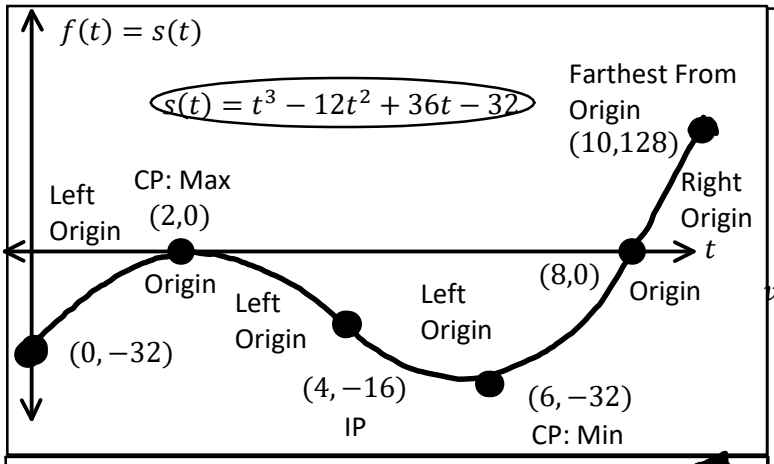
$s(t)=t^3-12t^2+36t-32$

$v(t)=v(0)+\int_0^t a(t)\,dt$

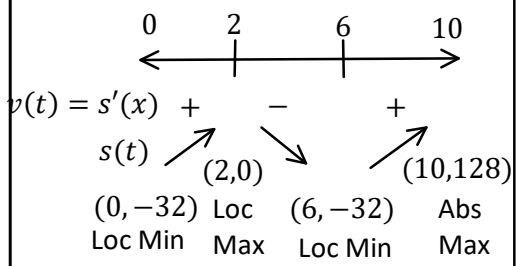
$s(t)=s(0)+\int_0^t v(t)\,dt$

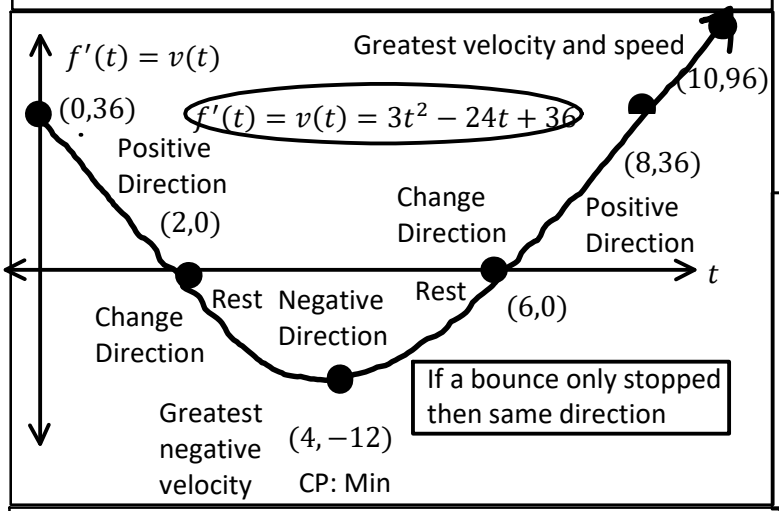
Def. Int. Net Change.

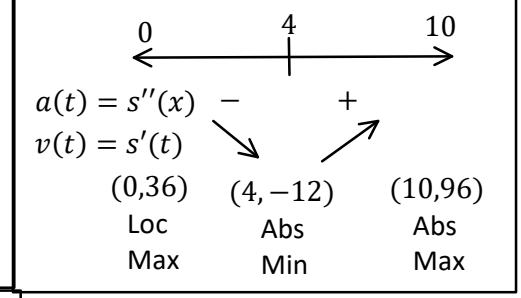
C12 - 5.7 - Particle Motion/Int Notes

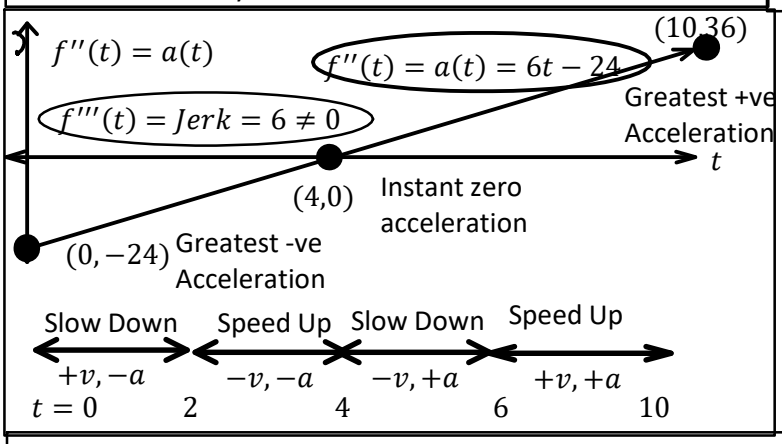


Number Lines In Line!









? $\vec{d}$, t = 4.

$$\int_0^4 v(t)\,dt$$

$$\int_0^4 (3t^2 - 24t + 36)\,dt$$

$$t^3 - 12t^2 + 36t - 32\Big|_0^4$$

$$(4)^3 - 12(4)^2 + 36(4) - 32 - ((0)^3 - 12(0)^2 + 36(0) - 32)$$

$$-16 - (-32) = \boxed{16}$$

? d, t = 4. Need $v(t) = 0, s(t)^*$.

$$\int_0^4 |v(t)|\,dt = \int_0^2 v(t)dt + -\int_2^4 v(t)dt$$

$$\int_0^2 (3t^2 - 24t + 36)\,dt - \int_2^4 (3t^2 - 24t + 36)\,dt$$

$$t^3 - 12t^2 + 36t - 32\Big|_0^2 - t^3 - 12t^2 + 36t - 32\Big|_2^4$$

$$(2)^3 - 12(2)^2 + 36(2) - 32 - (-32^*) -$$

$$((4)^3 - 12(4)^2 + 36(4) - 32 - ((0)^3 - 12(0)^2 + 36(0) - 32)$$

$$0 - (-32^*) - (-16^* - (0^*) = \boxed{48}$$

C12 - 5.7 - Particle Motion Int Notes

$$F(b) = F(a) + \int_a^b f(x)\,dx$$

N ↑ → E

A particle starts moving along a straight line East... $a(t) = 4cost \quad v(0) = 1 \quad s(0) = -2 \quad [0,7]$

$a(3) = ?$
$a(5) = ?$
$v(3) = ?$
$v(5) = ?$
$v(t) = ?$
$s(2) = ?$
$s(t) = ?$

$a(t) = 4cost$
$a(3) = 4cos3$
$a(3) = -3.96 \frac{m}{s^2}$

$a(t) = 4cost$
$a(5) = 4cos5$
$a(5) = 1.13 \frac{m}{s^2}$

$v(3) = v(0) + \int_0^3 a(t)\,dt$
$v(3) = 1 + \int_0^3 4cost\,dt$
$v(3) = 1 + (4\sin(3) - 4\sin(0))$
$v(3) = 1.56 \frac{m}{s}$

$v(t) = v(0) + \int_0^t a(t)\,dt$
$v(t) = 1 + \int_0^t 4cost\,dt$
$v(t) = 1 + (4\sin(t) - 4\sin(0))$
$v(t) = 4sint + 1$

$v(t) = 4sint + 1$
$v(3) = 4sin3 + 1$
$v(3) = 1.56 \frac{m}{s}$
Moving Right

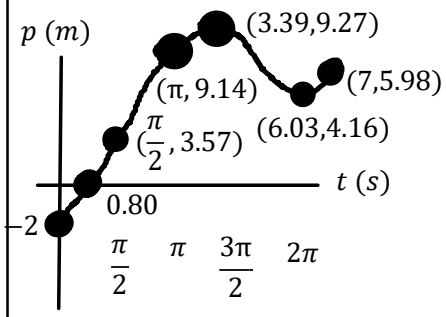
$v(t) = 4sint + 1$
$v(5) = 4sin5 + 1$
$v(5) = -2.84 \frac{m}{s}$
Moving Left

$p(t) = p(0) + \int_0^t v(t)\,dt$
$p(2) = p(0) + \int_0^2 v(t)\,dt$
$p(2) = -2 + \int_0^2 (4sint + 1)\,dt$
$p(2) = -2 + \left((-4\cos(t) + 1t)\right)\Big|_0^2$
$p(2) = -2 + \begin{pmatrix} (-4\cos(2) + 1(2)) \\ -(-4\cos(0) + 1(0)) \end{pmatrix}$
$p(2) = 4 - 4cos2$
$p(2) = 5.66\ m$

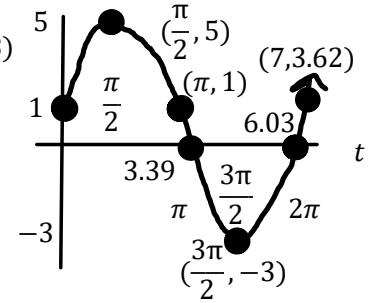
$p(t) = p(0) + \int_0^t v(t)\,dt$
$p(t) = p(0) + \int_0^t (4sint + 1)\,dt$
$p(t) = -2 + (-4cost + 1t)\Big|_0^t$
$p(t) = -2 + ((-4cost + 1t) - (-4cos0 + 1(0)))$
$p(t) = -4cost + 1t + 2$

$p(t) = -4cost + 1t + 2$
$p(2) = -4cos2 + 1(2) + 2$
$p(2) = 5.66\ m$

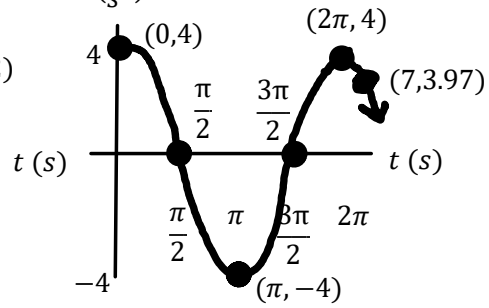
$p(t) = -4cost + 1t + 2$



$v\left(\frac{m}{s}\right)$ $v(t) = 4sint + 1$



$a\left(\frac{m}{s^2}\right)$ $a(t) = 4cost$



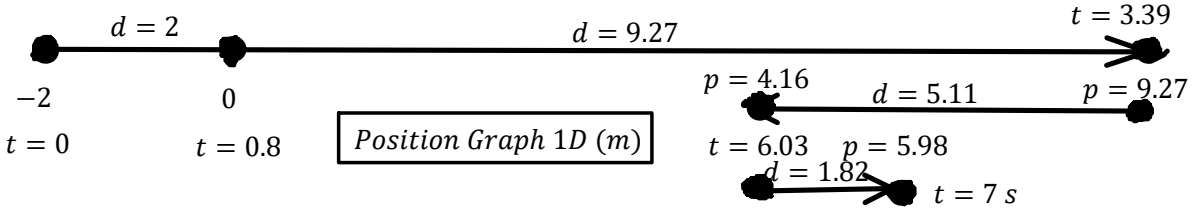
$p(t) = -4cost + 1t + 2$
$s'(t) = 4sint + 1$
$v(t) = 4sint + 1$
$0 = 4sint + 1$
$sint = -\frac{1}{4}$ *QIII, IV*
$t_r = \sin^{-1}(+\frac{1}{4})$
$t_r = 0.25$
$t = 3.39\ s\ ,\ 6.03\ s$

t	$p(t)$
0	−2
0.8	0
3.39	9.27
6.03	4.16
7	5.98

$v(t) = 4sint + 1$
$v'(t) = 4cost$
$a(t) = 4cost$
$0 = 4cost$
$cost = 0$
$t = \frac{\pi}{2}\ s\ ,\ \frac{3\pi}{2}\ s$

t	$v(t)$
0	1
$\frac{\pi}{2}$	5
3.39	0
$3\pi/2$	−3
6.03	0
7	3.62

t	$a(t)$
0	4
$\pi/2$	0
π	−4
$3\pi/2$	0
2π	4
7	3.97



Position Graph 1D (m)

C12 - 5.7 - Particle Motion Int Notes

When is the particle at rest?

$v = 0$ $t = 3.39\ s\ , 6.03\ s$

When is the particle moving in the positive direction?

$v > 0$ $(0,3.39) U (6.03,7)$

When is the particle moving in the negative direction?

$v < 0$ $(3.39,6.03)$

When is the particle at the origin?

$s(t) = 0$ $t = 0.80\ s$

$s(t) = -4cost + 1t + 2$

$0 = -4cost + 1t + 2$

$TI - 84$

When does the particle change direction?

$s' = v$ $+$ 3.39 $-$ 6.03 $+$

$v = 0\ \&\ v + \to - \ or\ - \to +$

$t = 3.39\ s\ , 6.03\ s$

$(3.39,9.27)$ Abs Max $(6.03,4.16)$ Loc Min

$(0,-2)$ Loc/Abs Min* $(7,5.98)$ Loc Max

When is the particle farthest from the origin?

$v = 0$ & $s(t)$ Abs Max or Min

$s'(t) = 0\ or\ DNE$

$s'(t) + \to -, or - \to +$

$v(t) = 0$ $t = 3.39\ s$

$p(t) = -4cost + 1t + 2$

$p(3.39) = -4\cos(3.39) + 1t + 2$

$p(3.39) = 9.27\ m\ [W]$

Check Endpoints

~~$p(0) = -2$~~

~~$p(6.03) = 4.16$~~

~~$p(7) = 3.62)$~~

When is the velocity the greatest.

$v' = 0$ $t = \frac{\pi}{2}\ s\ , \frac{3\pi}{2}\ s$

v' $+$ $\frac{\pi}{2}$ $-$ $\frac{3\pi}{2}$ $+$

$(\frac{\pi}{2}, 5)$ Abs Max $(\frac{3\pi}{2}, -1)$ Abs Min

$(0,1)$ Loc Min $(7,3.62)$ Loc Max

When is the particles acceleration :

Greatest? $a = max$ Least? $a = min$

a' 0 $-$ π $+$ 2π $-$

$a(t) = 4cost$

$a'(t) = -4sint$

$0 = -4sint$

$sint = 0$

$a = 4\frac{m}{s^2}$

$t = 0, \pi, 2\pi\ s$

$a(0) = 4\cos(0) = 4$

$a(\pi) = 4\cos(\pi) = -4$ $a = -4\frac{m}{s^2}$

$a(2\pi) = 4\cos(2\pi) = 4$

When is the particle speeding up?

$v, a\ same\ sign$

$v = +, a = +$

$v = -\ a = -$

Speeding up in the negative direction →

$\left(0, \frac{\pi}{2}\right) U \left(3.39, \frac{3\pi}{2}\right)$

When is the particle slowing down?

$v, a\ different\ signs$

$v = +, a = -$

$v = -\ a = +$

$\left(\frac{\pi}{2}, 3.39\right) U \left(\frac{3\pi}{2}, 7\right)$

Going forward Slowing Down

Going backward Slowing Down

What is the displacement : $\vec{d}$ travelled by the particle (0,6.03)?

$\vec{d} = \int_a^b v(t)dt$

$\vec{d} = \int_0^{6.03} (4sint + 1)dt$

$\vec{d} = (-4cost + t)\Big|_0^{6.03}$

$\vec{d} = (-4\cos 6.03 + 6.03) - (-4cos0 + 0)$

$\vec{d} = 2.16 + 4$

$\vec{d} = 6.16\ m\ [E]$ $2 + 9.27 - (9.27 - 4.16) = 6.16$

$s(t) = -4cost + 1t + 2$

$s(6.03) = -4\cos(6.03) + 1(6.03) + 2$

$s(6.03) = 4.16\ m\ [E]$

$s(0) = -2\ or\ 2[W]$ $2 + 4.16 = 6.16$

$s(3.39) = 9.27\ [E]$

What is the distance : d travelled by the particle (0,6.03)?

$d = \int_a^b |v(t)|dt$

$d = \int_0^{6.03} |4sint + 1|dt$ Negative Area so Minus (3.39,6.03)

$d = \int_0^{3.39} (4sint + 1)dt + \int_{3.39}^{6.03} |4sint + t|dt$

$d = \int_0^{3.39} (4sint + 1)dt - \int_{3.39}^{6.03} (4sint + 1)dt$

$d = (-4cost + t)\Big|_0^{3.39} + (-4cost + t)\Big|_{3.39}^{6.03}$

$d = \big((-4cos3.39 + 3.39) - (-4cos0 + 0)\big) -$

$\big((-4cos6.03 + 6.03) - (-4cos3.39 + 3.39)\big) +$

$d = (11.27) - (-5.11)$

$d = 16.38\ m\ [E]$ $= 2 + 9.27 + (9.27 - 4.16) = 16.38$