C12 - 5.18 - Exp/Prop Growth

A Substance has a Half-Life of 50 Years. How Long to Decay 400 g to 50 g?

$$\frac{dy}{dt} = ky$$
$$\frac{dy}{y} = kdt$$
$$\int \frac{1}{y}dy = \int kdt$$
$$lny = kt + C$$
$$y = e^{kt+C}$$
$$y = e^{kt} \times e^{C}$$
$$y = e^{C}e^{kt}$$
$$\boxed{y = Ae^{kt}}$$

Separate Variables. Integrate Both Sides. Initial Condition. Change Forms. Separate Exponents.

$$\boxed{A = e^{C}}$$

$$y = Ae^{kt}$$
$$400 = Ae^{k(0)} \quad \boxed{f(0) = 400}$$
$$400 = Ae^{0}$$
$$A = 400$$

$$\boxed{f(50) = 200}$$

Half, 50 yrs

$$y = 400e^{kt}$$

$A = Present\ Value$
$y = Future\ Value$
$k = Constant\ Rate$
$t = time$

t	y
0	400
50	200
100	100
150	50
200	25

$$y = 400e^{kt}$$
$$200 = 400e^{50k}$$
$$0.5 = e^{50k}$$
$$ln0.5 = 50klne$$
$$ln0.5 = 50k$$
$$k = \frac{ln0.5}{50}$$
$$k = \frac{ln2^{-1}}{50}$$
$$k = -\frac{ln2}{50} = -0.01386$$

$$y = 400e^{-\frac{ln2}{50}t}$$
$$50 = 400e^{-\frac{ln2}{50}t}$$
$$0.125 = e^{-\frac{ln2}{50}t}$$
$$ln0.125 = -\frac{ln2}{50}t$$
$$t = 150\ yrs$$

Doubles every 4 hours. Start at 1. Find k.

$$\boxed{y = y_0e^{kt}}$$
$$2 = 1e^{4k}$$
$$2 = e^{4k}$$
$$ln2 = lne^{4k}$$
$$ln2 = 4klne$$
$$\ln_e 2 = 4k$$
$$k = \frac{ln\,2}{4}$$
$$k = 0.173$$

Formula. Sub/Simplify. Change Forms. OR In Both Sides. Exponent In Front. $\ln e = 1$. Divide.

Start 4. 10 After 2 Days. How much After 5 Days.

$$y = y_0e^{kt}$$
$$10 = 4e^{2k}$$
$$\frac{5}{2} = e^{2k}$$
$$\ln_e\left(\frac{5}{2}\right) = 2k$$
$$k = \frac{\ln_e\left(\frac{5}{2}\right)}{2}$$
$$k = 0.458$$

$$y = 4e^{\left(\frac{\ln_e\left(\frac{5}{2}\right)}{2}\right)(5)}$$
$$y = 39.53$$

t	y
0	4
1	
2	10
3	
4	
5	39.5

Doubles every 5 hours. Start at 1. How Long to 5.

$$y = y_0e^{kt}$$
$$2 = 1e^{5k}$$
$$k = \frac{\ln_e 2}{5}$$

t	y
0	1
5	2
10	4
15	8

$$5 = 1e^{\left(\frac{ln_e(2)}{5}\right)t}$$
$$\ln_e 5 = \left(\frac{ln_e(2)}{5}\right)t$$
$$t = \frac{5\ln_e 5}{\ln_e 2}$$
$$t = 11.61hrs$$

Half-Life is 5730 yrs. How Long to 1%.

$$y = y_0e^{kt}$$
$$\frac{1}{2} = 1e^{5730k}$$
$$\ln_e\left(\frac{1}{2}\right) = 5730k$$
$$k = \frac{\ln_e\left(\frac{1}{2}\right)}{5730}$$

$$y = y_0e^{kt}$$
$$0.01 = 1e^{\frac{\ln_e\left(\frac{1}{2}\right)}{5730}t}$$
$$\ln_2 0.01 = \frac{\ln_e\left(\frac{1}{2}\right)}{5730}tlne$$
$$t = 5730\frac{\ln_e 0.01}{\ln_e\left(\frac{1}{2}\right)}yrs$$
$$t = 38069.30\ yrs$$

$$\frac{dP}{dt} = 0.025P$$
$$P(0) = 75000$$
$$y = Ae^{kt}$$
$$P = 75000e^{0.025t}$$

$$\frac{dP}{dt} = 40t^3 - 70t^{\frac{1}{3}}$$
$$dP = (40t^3 - 70t^{\frac{1}{3}})dt$$
$$\int dP = \int (40t^3 - 70t^{\frac{1}{3}})dt$$
$$P = 10t^4 - \frac{105t^{\frac{4}{3}}}{2} + C$$

$$P = 10t^4 - \frac{105t^{\frac{4}{3}}}{2} + C$$
$$\boxed{P(0) = 16000}$$
$$16000 = 10(0)^4 - \frac{105(0)^{\frac{4}{3}}}{2} + C$$
$$c = 16000$$
$$P = 10t^4 - \frac{105t^{\frac{4}{3}}}{2} + 16000$$

$$P = 10t^4 - \frac{105t^{\frac{4}{3}}}{2} + 16000$$
$$\boxed{p(8) = ?}$$
$$P(8) = 10(8)^4 - \frac{105(8)^{\frac{4}{3}}}{2} + 16000$$
$$P(8) = 56120\ pop$$

$$P = 10t^4 - \frac{105t^{\frac{4}{3}}}{2} + 16000$$
$$\boxed{P(t) = 20000, t = ?}$$
$$20000 = 10t^4 - \frac{105t^{\frac{4}{3}}}{2} + 16000$$
$$t = 4.58\ years$$ Calc