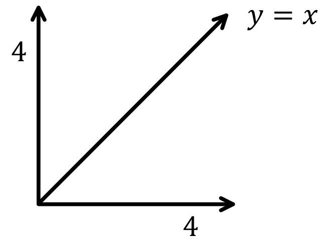
C12 - 5.15 - Arc Length/Surface Area Int Notes

$L = \int_a^b ds \qquad ds = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx \qquad y = f(x) \qquad ds = \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy \qquad x = f(y)$



$L = \int_a^b ds$

$L = \int_0^4 \sqrt{2}\, dx$

$L = \sqrt{2}\, x \Big|_0^4$

$L = \sqrt{2}(4 - 0)$

$L = 4\sqrt{2}$

$ds = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$

$ds = \sqrt{1 + (1)^2}\, dx$

$ds = \sqrt{2}\, dx$

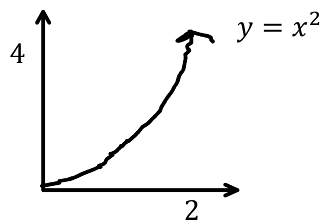
$y = x$

$\frac{dy}{dx} = 1$

$a^2 + b^2 = c^2$

$4^2 + 4^2 = c^2$

$c = 4\sqrt{2}$



$L = \int_0^2 \sqrt{1 + 4x^2}\, dx$

$L = 4.65$

$ds = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$

$ds = \sqrt{1 + (2x)^2}\, dx$

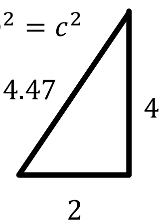
$ds = \sqrt{1 + 4x^2}\, dx$

$y = x^2$

$\frac{dy}{dx} = 2x$

$a^2 + b^2 = c^2$

$2\sqrt{5} = 4.47$



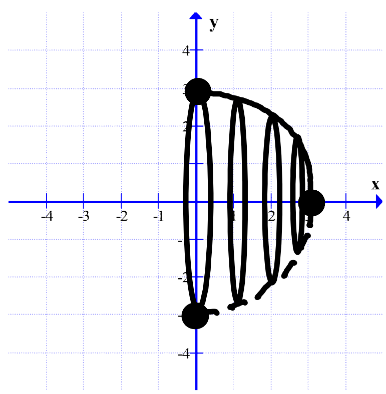
$4.65 > 4.47$

Obviously!

Rotation around x-axis

$SA = \int_a^b 2\pi y\, ds$

$y = \sqrt{9 - x^2} \qquad 0 \le x \le 3$



$SA = \int_0^3 2\pi y\, ds$

$SA = \int_0^3 2\pi\sqrt{9 - x^2}\, \frac{3}{\sqrt{9 - x^2}}\, dx$

$SA = \int_0^3 6\pi\, dx$

$SA = 6\pi x \Big|_0^3$

$SA = 6\pi(3 - (0))$

$SA = 18\pi$

$ds = \sqrt{1 + \left(\frac{-x}{\sqrt{9 - x^2}}\right)^2}\, dx$

$ds = \sqrt{1 + \frac{x^2}{9 - x^2}}\, dx$

$ds = \sqrt{\frac{9}{9 - x^2}}\, dx$

$ds = \frac{3}{\sqrt{9 - x^2}}\, dx$

$y = \sqrt{9 - x^2}$

$\frac{dy}{dx} = \frac{-x}{\sqrt{9 - x^2}}$

Geometry Check

$SA = 4\pi r^2$

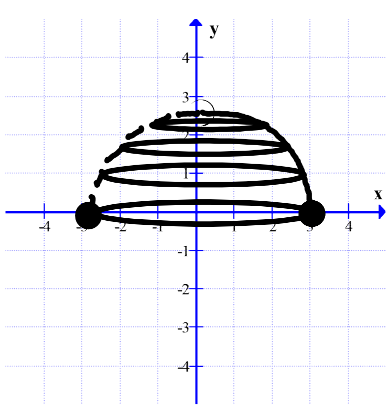
$SA = 4\pi(3)^2$

$SA = 36\pi \div 2$

$SA = 18\pi$

Rotation around y-axis

$SA = \int_a^b 2\pi x\, ds$



$SA = \int_0^3 2\pi\sqrt{9 - y^2}\, \frac{3}{\sqrt{9 - y^2}}\, dy$

...

SA = 18π

$ds = \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$

$ds = \sqrt{1 + \left(\frac{-y}{\sqrt{9 - y^2}}\right)^2}\, dy$

$ds = \sqrt{1 + \frac{y^2}{9 - y^2}}\, dx$

$ds = \sqrt{\frac{9}{9 - y^2}}\, dx$

$ds = \frac{3}{\sqrt{9 - y^2}}\, dx$

$x = \sqrt{9 - y^2}$

$\frac{dx}{dy} = \frac{-y}{\sqrt{9 - y^2}}$