C12 - 5.13 - Int Partial Frac Notes

$A + B = 0^*\,; 0x$

$A^* = \frac{1}{2} \quad \frac{1}{2(x-\#)}$

$a \neq b$

$$\frac{qx+r}{(x-a)(x-b)} = \frac{A}{x-a} + \frac{B}{x-b}$$

$$\int \frac{5x+8}{(x+3)(x+2)}\,dx = ?$$

$$\frac{A}{x+3} + \frac{B}{x+2} = \frac{5x+8}{(x+3)(x+2)}$$

$$A(x+2) + B(x+3) = 5x+8$$

$$Ax + Bx + 2A + 3B = 5x + 8$$

$$(A+B)x + (2A+3B) = 5x+8$$

Multiply both sides by the LCD. Group/Factor

$$A + B = 5 \qquad 2A + 3B = 8$$

$$A = 5 - B \qquad 2(5-B) + 3B = 8$$

$$A = 5 - (-2) \qquad 10 - 2B + 3B = 8$$

$$A = 7 \qquad B = -2$$

$$\int \left(\frac{7}{x+3} - \frac{2}{x+2}\right) dx$$

$$7\ln(x+3) - 2\ln(x+2) + C$$

$$y' = \frac{7}{x+3} - \frac{2}{x+2}$$

Check: Take Derivative

$$\frac{7}{x+3} - \frac{2}{x+2} = \frac{7(x+2) - 2(x+3)}{(x+3)(x+2)}$$

Add Fractions

$$\frac{5x+8}{(x+3)(x+2)}$$

OR Convenient Sub

$$\int \frac{1}{2x^2+3x+1}\,dx$$

$$\frac{A}{2x+1} + \frac{B}{x+1} = \frac{1}{(2x+1)(x+1)}$$

$$A(x+1) + B(2x+1) = 1$$

$$Ax + 2Bx + A + B = 1$$

$$\int \frac{1}{(2x+1)(x+1)}\,dx$$

$$\int \frac{2}{2x+1} - \frac{2}{x+1}\,dx$$

$$Ax + 2Bx = 0x \qquad A + B = 1$$

$$A + 2B = 0 \qquad B = 1 - A$$

$$\ln|2x+1| - 2\ln|x+1| + C$$

$$A + 2(1-A) = 0 \qquad B = 1 - 2$$

$$A + 2 - 2A = 0 \qquad B = -2$$

$$-A = -2$$

$$A = 2$$

$$y' = \frac{2}{2x+1} - \frac{2}{x+1}$$

$$\int \frac{x^2 - x - 1}{x-2}\,dx = ?$$

Higher Degree Numerator

Long Division

$$\int x + \frac{x-1}{x-2}\,dx$$

$$\int x + 1 + \frac{1}{x-2}\,dx$$

$$\frac{x^2}{2} + x + \ln|x-2| + C$$

$$y' = x + 1 + \frac{1}{x-2}$$

$$x - 2\overline{)\,x^2 - x - 1}$$

$$-(x^2 - 2x)$$

$$x - 1$$

$$x - 2\overline{)\,x - 1}$$

$$-(x - 2)$$

$$1$$

$$\int \frac{2x^2 - 7x - 6}{x(x+1)(x-2)}\,dx = ?$$

$$\frac{A}{x} + \frac{B}{x+1} + \frac{C}{x-2} = \frac{2x^2-7x-6}{x(x+1)(x-2)}$$

$$\int \left(\frac{3}{x} + \frac{1}{x+1} - \frac{2}{x-2}\right) dx$$

$$\frac{A(x+1)(x-2) + Bx(x-2) + Cx(x+1)}{x(x+1)(x-2)} = \frac{2x^2-7x-6}{x(x+1)(x-2)}$$

$$3\ln x + \ln(x+1) - 2\ln(x-2) + C$$

$$Ax^2 - Ax - 2A + Bx^2 - 2Bx + Cx^2 + Cx = 2x^2 - 7x - 6$$

$$(A+B+C)x^2 + (-A-2B+C)x - 2A = 2x^2 - 7x - 6$$

$$y = 3\ln x + \ln(x+1) - 2\ln(x-2)$$

$$y' = \frac{3}{x} + \frac{1}{x+1} - \frac{2}{x-2}$$

$$A + B + C = 2 \qquad -A - 2B + C = -7 \qquad -2A = -6$$

$$(3) + B + C = 2 \qquad -(3) - 2B + C = -7 \qquad A = 3$$

$$-2B + C = -4$$

$$B + C = -1$$

$$(1) + C = -1$$

$$C = -2$$

$$B + C = -1$$

$$-(-2B + C = -4)$$

$$3B = 3$$

$$B = 1$$

$$\frac{px^{2^*} + qx + r}{(x-a)(x-b)(x-c)} = \frac{A}{x-a} + \frac{B}{x-b} + \frac{C}{x-c}$$

$$\int \frac{x+1}{(x-1)^2}\,dx = ?$$

$$\frac{A}{x-1} + \frac{B}{(x-1)^2} = \frac{x+1}{(x-1)^2}$$

$$\int \frac{1}{x-1} + \frac{2}{(x-1)^2}\,dx$$

$$A(x-1) + B = x + 1$$

$$Ax - A + B = x + 1$$

$$\int \frac{1}{x-1} + 2(x-1)^{-2}\,dx$$

$$A = 1 \qquad -A + B = 1$$

$$-(1) + B = 1$$

$$B = 2$$

$$\ln|x-1| - \frac{2}{x-1} + C$$

$$y = \ln|x-1| - 2(x-1)^{-1} + C$$

$$y' = \frac{1}{x-1} + \frac{2}{(x-1)^2}$$

$$\frac{px+q}{(x-a)^2} = \frac{A}{x-a} + \frac{B}{(x-a)^2}$$

$$\int \frac{1}{x(x^2+1)}\,dx = ?$$

$$\frac{A}{x} + \frac{Bx+C}{x^2+1} = \frac{1}{x(x^2+1)}$$

$$\int \frac{1}{x} - \frac{x}{x^2+1}\,dx$$

U Sub

$$A(x^2+1) + (Bx+C)x = 1$$

$$Ax^2 + Bx^2 + Cx + A = 1$$

$$(A+B)x^2 + Cx + A = 1$$

$$\ln|x| - \frac{1}{2}\ln|x^2+1| + C$$

$$A + B = 0 \qquad C = 0 \qquad A = 1$$

$$(1) + B = 0$$

$$B = -1$$

$$y' = \frac{1}{x} - \frac{2x}{2(x^2+1)}$$

$$y' = \frac{1}{x} - \frac{x}{x^2+1}$$

$$\frac{px^2+qx+r}{(x-a)(x^2+bx+c)} = \frac{A}{x-a} + \frac{Bx+C}{x^2+bx+c}$$