C12 - 5.12 - Int by Parts Notes

$$\int_a^b u\,dv = uv\Big|_a^b - \int_a^b v\,du \qquad Definite\ Integrals$$

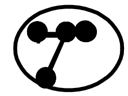
$$\int uv' = uv - \int vu'$$

Set it up

$u =$ $u' =$ ↓ $v =$ $v' =$ ↑

1	4
3	2

1. u 4. v 3. u' 2. v' Steps



Seven

$$\int xe^x dx = xe^x - \int e^x dx$$

$$= xe^x - e^x + C$$

$u = x$, $u' = 1dx$ ↓ $v = e^x$, $v' = e^x dx$ ↑

u:LIPET:v

u: Logs-Inverse Trig-Poly-Exp-Trig:v

$u = x$

$\frac{du}{dx} = 1$

$du = 1dx$

$y' = e^x + xe^x - e^x$

$y' = xe^x$

Check : Take the derivative

$$\int lnx\,dx$$

$u = lnx$, $du = \frac{1}{x}dx$ $\quad v = x$, $dv = dx$

$$xlnx - \int x\frac{1}{x}dx$$

$$xlnx - \int 1\,dx$$

$$xlnx - x + C$$

$y = xlnx - x$

$y' = lnx + 1 - 1$

$y' = lnx$

Proof

$(uv)' = u'v + uv'$ *Product Rule*

$\int (uv)' = \int (u'v + uv')dx$ *Integrate Both Sides*

$\int (uv)' = \int u'v\,dx + \int uv'dx$ *Sum Rule*

$uv = \int u'v\,dx + \int uv'dx$ $\quad \int (uv)' = uv$

$\int uv'dx = uv - \int u'v\,dx$ *Rearrange*

$$\int xlnx\,dx$$

$u = lnx$, $du = \frac{1}{x}dx$ $\quad v = \frac{x^2}{2}$, $dv = xdx$

$$\frac{x^2}{2}lnx - \int \frac{x^2}{2}\frac{1}{x}dx$$

$$\frac{x^2}{2}lnx - \int \frac{x}{2}dx$$

$$\frac{x^2}{2}lnx - \frac{x^2}{4} + C$$

$\int x\sqrt{x-1}\,dx$ $\quad u = x$

$v' = \sqrt{x-1}dx$

$$\int cos^4x\,dx$$

$u = \cos^3 x$, $du = -3\cos^2 x\,sinx\,dx$ $\quad v = sinx$, $v' = cosxdx$

$$\int cos^3xcosx\,dx$$

$$\cos^3 xsinx - \int -3cos^2x\,sin^2x\ dx$$

$$\cos^3 xsinx + \frac{3}{8}\left(x - \frac{sin4x}{4}\right) + C$$

$$\int x^2sinx\,dx = -x^2cosx + \int 2xcosx\,dx$$

$u = x^2$, $du = 2xdx$ $\quad v = -cosx$, $dv = \sin x\,dx$

$$= -x^2cosx + 2\int xcosx\,dx$$

$u = x$, $du = dx$ $\quad v = sinx$, $dv = cosxdx$

$$= -x^2cosx + 2[xsinx - \int sinx\,dx]$$

Repeat Parts

$$= -x^2cosx + 2[xsinx + cosx] + C$$

$$\int e^xsinx\,dx = -e^xcosx + \int e^xcosx\,dx$$

$u = e^x$, $u' = e^xdx$ $\quad v = -cosx$, $v' = sinxdx$

$$= -e^xcosx + e^xsinx - \int e^xsinx\,dx$$

$u = e^x$, $u' = e^xdx$ $\quad v = sinx$, $v' = cosxdx$

$m = -e^xcosx + e^xsinx - m$

$2m = -e^xcosx + e^xsinx$

$m = \frac{e^x(simx - cosx)}{2}$

let $m = \int e^xsinx\,dx$