

C12 - 3.0 - Max/Min/Concavity Derivative Test Review

Domain:

VA, Roots, Logs $f' = \infty$ etc: None

$$y = x^3 + 12x^2 + 36x$$

Range:

$$x \in R$$

Horizontal Asymptotes (HA): None

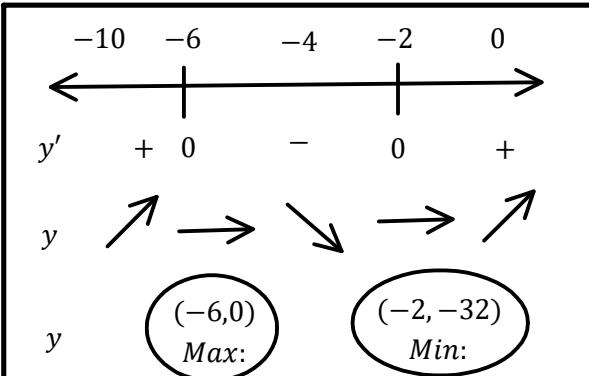
$$y \in R$$

$$\lim_{x \rightarrow \infty} x^3 + 12x^2 + 36x = \infty$$

$$\lim_{x \rightarrow -\infty} x^3 + 12x^2 + 36x = -\infty$$

$$y' = 3x^2 + 24x + 36 = 0$$

$$\begin{aligned} x^2 + 8x + 12 &= 0 \\ (x+6)(x+2) &= 0 \\ x &= -6, -2 \end{aligned}$$



Critical Points (CP):

$$\begin{aligned} y &= x^3 + 12x^2 + 36x \\ y &= (-6)^3 + 12(-6)^2 + 36(-6) \\ y &= 0 \end{aligned}$$

$$\begin{aligned} y &= x^3 + 12x^2 + 36x \\ y &= (-2)^3 + 12(-2)^2 + 36(-2) \\ y &= -32 \end{aligned}$$

Sign Analysis

$$y' = (x+6)(x+2)$$

$$\begin{aligned} y'(-10) &= (-)(-) = + \\ y'(-4) &= (+)(-) = - \\ y'(0) &= (+)(+) = + \end{aligned}$$

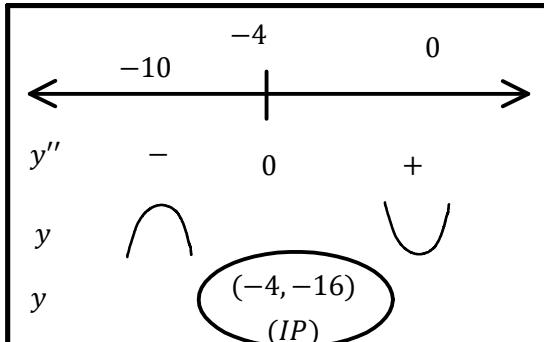
Intervals of Inc/Dec

Increasing: $(-\infty, -6), (-2, \infty)$
Decreasing: $(-6, -2)$

$$\begin{aligned} y'' &= 6x + 24 = 0 \\ 6(x+4) &= 0 \\ x &= -4 \end{aligned}$$

Inflection Point (IP):

$$\begin{aligned} y &= x^3 + 12x^2 + 36x \\ y &= (-4)^3 + 12(-4)^2 + 36(-4) \\ y &= -16 \end{aligned}$$



Sign Analysis

$$y'' = 6(x+4)$$

$$\begin{aligned} y''(-10) &= +(-) = - \\ y''(0) &= +(+) = + \end{aligned}$$

Intervals of Concavity

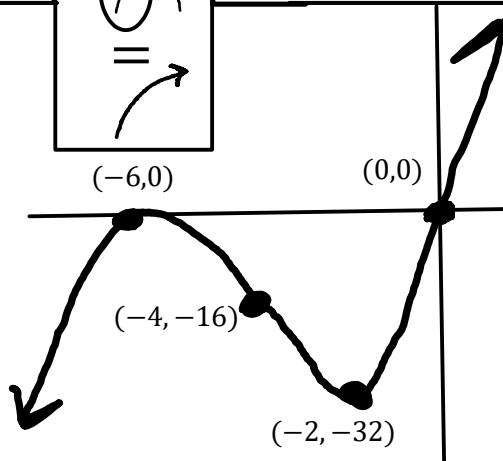
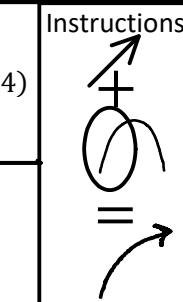
Concave Up: $(-4, \infty)$
Concave Down: $(-\infty, -4)$

x - intercepts

$$\begin{aligned} y &= x^3 + 12x^2 + 36x \\ 0 &= x^3 + 12x^2 + 36x \\ 0 &= x(x^2 + 12x + 36) \\ 0 &= x(x+6)(x+6) \end{aligned}$$

$$x = 0, -6$$

$$(0,0) (-6,0)$$



y - intercepts

$$\begin{aligned} y &= x^3 + 12x^2 + 36x \\ y &= (0)^3 + 12(0)^2 + 36(0) \\ y &= 0 \end{aligned}$$

$$(0,0)$$

C12 - 3.0 - Derivative Graphing Review

