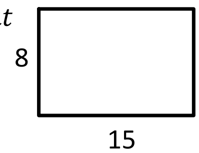
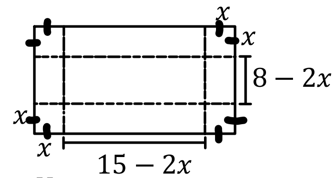
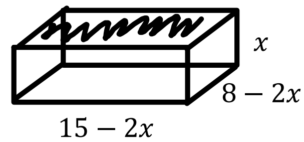
C12 - 3.6 - Prisms/Cone/Eq Max/Min Notes

An open top box* made by cutting equal lengths from each corner of a 8 x 15 cm rectangle & folding up the sides. Find length of square cut from each corner so box has a Max Volume and find it.

$let\ x = length\ to\ cut$

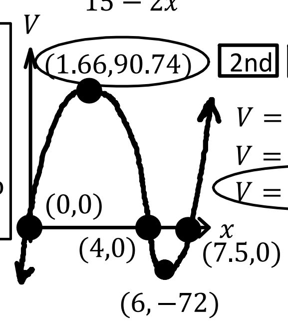






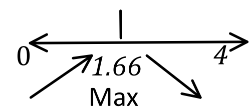
$V = lwh$
$V = (8-2x)(15-2x)x$
$V = 4x^3 - 46x^2 + 120x$
$V' = 12x^2 - 92x + 120$
$0 = 12x^2 - 92x + 120$
$0 = 3x^2 - 23x + 40$
$0 = (3x-5)(x-6)$
$x = \frac{5}{3} = 1.66$ $\quad x = 6$

$Domain:$
$0 > x > 4$
x cant be negative!
Cant cut two $4's$ off a 8!

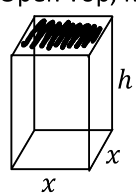


2nd Calc Max

$V = (8-2x)(15-2x)x$
$V = (8-2(1.67))(15-2(1.67)(1.67)$
$V = 90.74\ cm^3$



Open Top, Min SA?

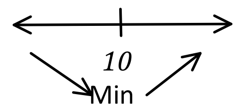


$V = 500mL$
$1mL = 1cm^3$

$V = lwh$
$500 = x^2h$
$h = \frac{500}{x^2}$
$h = \frac{500}{10^2}$
$h = 5\ cm$

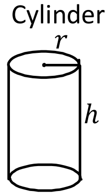
$SA = x^2 + 4xh$
$SA = x^2 + 4x\left(\frac{500}{x^2}\right)$
$SA = x^2 + 2000x^{-1}$
$SA = 10^2 + 4(10)(5)$
$SA = 300\ cm^2$

$SA' = 2x - 2000x^{-2}$
$0 = 2x - 2000x^{-2}$
$2x = \frac{2000}{x^2}$
$x^3 = 1000$
$x = 10\ cm$



Min SA is 300 cm^2.

Cylinder V=1000mL, Find dimensions for Min SA.



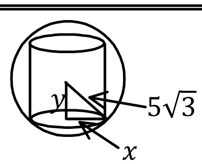
$V = \pi r^2 h$
$1000 = \pi r^2 h$
$h = \frac{1000}{\pi r^2}$
$h = \frac{1000}{\pi(5.4)^2}$
$h = 10.9\ cm$

$SA = 2\pi r^2 + 2\pi rh$
$SA = 2\pi r^2 + 2\pi r\left(\frac{1000}{\pi r^2}\right)$
$SA = 2\pi r^2 + 2000r^{-1}$

$SA' = 4\pi r - 2000r^{-2}$
$0 = 4\pi r - 2000r^{-2}$
$4\pi r = \frac{2000}{r^2}$
$r = \sqrt[3]{\frac{500}{\pi}} = 5.4\ cm$

$V = \pi r^2 h$
$V = \pi(5.4)^2(10.9)$
$V = 1000$

Max volume cylinder inscribed in sphere $r = 5\sqrt{3}$.

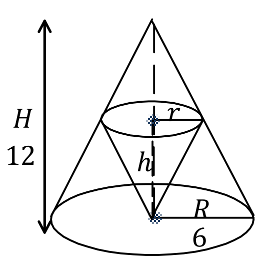


$V = \pi r^2 h$
$V = \pi x^2(2y)$
$V = \pi(75 - y^2)2y$
$V = \pi(150y - 2y^3)$

$V' = \pi(150 - 6y^2)$
$0 = \pi(150 - 6y^2)$
$y = 5$

$x^2 + y^2 = r^2$
$x^2 + y^2 = (5\sqrt{3})^2$
$x^2 + y^2 = 75$
$x^2 = 75 - y^2$

Max Inscribed Cone in a Cone.



$\frac{H-h}{r} = \frac{H}{R}$
$r = R\left(1 - \frac{h}{H}\right)$
$r = 6\left(1 - \frac{4}{12}\right)$
$r = 4$

$V = \frac{1}{3}\pi r^2 h$
$V = \frac{1}{3}\pi\left(R\left(1 - \frac{h}{H}\right)\right)^2 h$
$V = \frac{1}{3}\pi R^2\left(1 - \frac{2h}{H} + \frac{h^2}{H^2}\right)h$
$V = \frac{1}{3}\pi R^2\left(h - \frac{2h^2}{H} + \frac{h^3}{H^2}\right)$
$V = \frac{1}{3}\pi(6)^2\left(h - \frac{2h^2}{12} + \frac{h^3}{12^2}\right)$

$\frac{dV}{dt} = 12\pi\left(1 - \frac{h}{3} + \frac{h^2}{48}\right)$
$0 = \cdots Quad$
$h = 4$ $\quad$ ~~$h = 12$~~