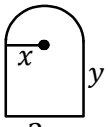
C12 - 3.16 - Norman/Track/Don't Cut Max Min Notes

Norman Window Max Area Perimeter of 8m. (Rectangle/Semi-Circle.)



$P = 2x + 2y + \frac{2\pi r}{2}$

$8 = 2x + 2y + \pi x$ $\boxed{x = r}$

$y = 4 - x - \frac{\pi x}{2}$

$y = 4 - (0.71) - \frac{\pi(0.71)}{2}$

$y = 2.16m$

$A = 2x(y) + \frac{\pi r^2}{2}$

$A = x(y) + \pi x^2$

$A = x\left(4 - x - \frac{\pi x}{2}\right) + \pi x^2$

$A = 4x - x^2 - \frac{\pi^2 x^2}{2} + \pi x^2$

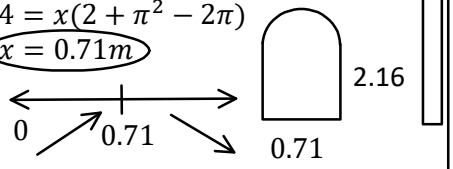
$A' = 4 - 2x - \pi^2 x + 2\pi x$

$0 = 4 - 2x - \pi^2 x + 2\pi x$

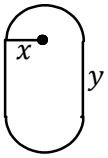
$4 = x(2 + \pi^2 - 2\pi)$

$x = 0.71m$

$0 \le x \le ?^*$



Ant Olympic Track Perimeter of 1m. (Rectangle*/Semi-Circles.)



$P = \pi x^2 + 2y$

$1 = \pi x^2 + 2y$

$y = \frac{1 - \pi x^2}{2}$

$y = \frac{1 - \pi(0.8)^2}{2}$

$y = -0.51$

$A = \pi x^2 + 2xy$

$A = \pi x^2 + 2x\left(\frac{1 - \pi x^2}{2}\right)$

$A = \pi x^2 + x - \pi x^3$

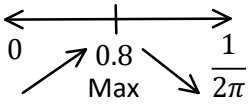
$A' = 2\pi x + 1 - 3\pi x^2$

$0 = 3\pi x^2 - 2\pi x - 1$

$x = 0.8, -0.13$ Quad

Check End Points

All Circle $y = 0$



$P = 2\pi r$

$1 = 2\pi x$

$x = \frac{1}{2\pi}$

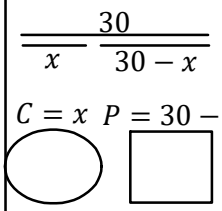
$x = 0.159$

$A = \pi x^2$

$A = \pi(0.159)^2$

$A = 0.226\ m^2$

A 30 cm string cut to make a square and a circle of max total area.



$0 \le x \le 30$

$C = x \quad P = 30 - x$

$C = 2\pi r \quad \frac{30 - x}{4}$

$x = 2\pi r$

$r = \frac{x}{2\pi}$

$A = \pi r^2$

$A = \pi\left(\frac{x}{2\pi}\right)^2$

$A = \frac{x^2}{4\pi}$

$A = lw$

$A = \left(\frac{30 - x}{4}\right)\left(\frac{30 - x}{4}\right)$

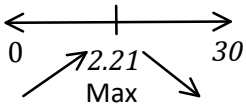
$A = \frac{900 - 60x + x^2}{16}$

$A_T = \frac{x^2}{4\pi} + \frac{1}{16}(900 - 60x + x^2)$

$A_T' = \frac{1}{2\pi}x + \frac{1}{16}(-60 + 2x)$

$0 = \frac{x}{2\pi} - \frac{15}{4} + \frac{1}{8}x$

$x = 2.21\ cm$



$A_T = \frac{x^2}{4\pi} + \frac{1}{16}(900 - 60x + x^2)$

$A_T = \frac{2.21^2}{4\pi} + \frac{1}{16}(900 - 60(2.21) + (2.21)^2)$

$A_T = 16.2\ cm^2$

All Circle

$C = 2\pi r \quad A = \pi r^2$

$30 = 2\pi r$

$r = \frac{30}{2\pi}$

$r = \frac{15}{\pi}$

$A = \pi\left(\frac{15}{\pi}\right)^2$

$A = \frac{225}{\pi} = 71.7\ cm^2$

All Square

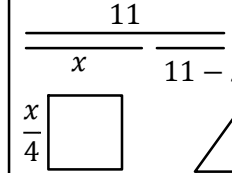
$A = lw$

$A = \left(\frac{30}{4}\right)\left(\frac{30}{4}\right)$

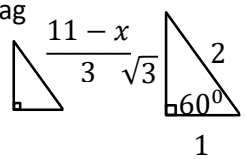
$A = 56.25\ cm^2$

Don't cut it. All circle.
Check your endpoints.

Cut 11m to make Square & Equilateral Triangle Max Area.



OR Pythag



Square

$A = \left(\frac{x}{4}\right)^2$

$A = \frac{x^2}{16}$

Eq Tri

$A = \frac{\frac{11 - x}{3}\left(\frac{\sqrt{3}}{2}\frac{11 - x}{3}\right)}{2}$

$A = \frac{\sqrt{3}}{36}(11 - x)^2$

$A_T = \frac{(4.78)^2}{16} + \frac{\sqrt{3}}{36}(11 - 4.78)^2$

$A_T = 3.28$

$A_T = \frac{x^2}{16} + \frac{\sqrt{3}}{36}(11 - x)^2$

$A_T' = \frac{x}{8} + \frac{\sqrt{3}}{18}(11 - x)(-1)$

$A_T' = \frac{x}{8} - \frac{\sqrt{3}(11 - x)}{18}$

$0 = \cdots$

$\frac{x}{8} = \frac{\sqrt{3}(11 - x)}{18}$

$18x = 88\sqrt{3} - 8\sqrt{3}x$

$x = 4.78m$

All Square

$x = \frac{11}{4}$

$x = 2.75$

$A = x^2$

$A = (2.75)^2$

$A = 7.56$

All Triangle

$A = \frac{\sqrt{3}}{36}(11 - x)^2$

$A = \frac{\sqrt{3}}{36}(11 - 4.78)^2$

$A = 1.86$