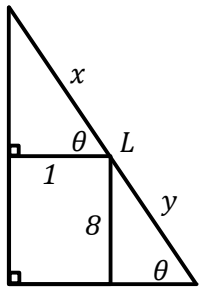
C12 - 3.15 - Trig/Pythag Max Min Notes

Shortest Ladder.



$L = x + y \quad 0 < \theta < \frac{\pi}{2}$

$L = 1sec\theta + 8csc\theta$

$L' = 1sec\theta tan\theta - 8cscxcotx$

$L' = 1\frac{1}{cos\theta}\frac{sin\theta}{cos\theta} - 8\frac{1}{sin\theta}\frac{cos\theta}{sin\theta}$

$L' = \frac{1sin\theta}{\cos^2\theta} - \frac{8cos\theta}{\sin^2\theta}$

$0 = \frac{1\sin^3\theta - 8\cos^2\theta}{\sin^2\theta\cos^2\theta}$

$0 = 1\sin^3\theta - 8\cos^2\theta$

$8\cos^2\theta = 1\sin^3\theta$

$8 = \frac{\sin^3\theta}{\cos^3\theta}$

$8 = \tan^3\theta$

$\tan\theta = 2$

$\theta = \tan^1(2)$

$\theta = 1.1$

$cos\theta = \frac{1}{x}$

$x = \frac{1}{cos\theta}$

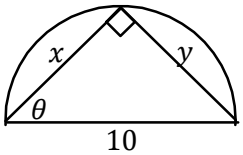
$x = 1sec\theta$

$sin\theta = \frac{8}{y}$

$y = \frac{8}{sin\theta}$

$y = 8csc\theta$

Max Inscribed Triangle Area



Inscribed ∠ in a Semi-Circle $= 90^o$.

$A = \frac{xy}{2}$

$A = \frac{1}{2}(10cos\theta)(10sin\theta)$

$A = 50sin\theta cos\theta$

$A = 25sin2\theta$

$A' = 25cos2\theta \times 2$

$0 = cos2\theta$

…

$\theta = \frac{\pi}{4}$

$cos\theta = \frac{x}{10}$

$x = 10cos\theta$

$x = 10\cos\frac{\pi}{4}$

$x = 5\sqrt{2}$

$sin\theta = \frac{y}{10}$

$y = 10sin\theta$

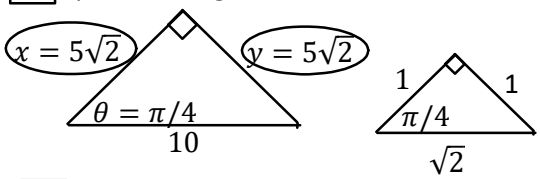
$y = 10\sin\frac{\pi}{4}$

$y = 5\sqrt{2}$

$A = \frac{(5\sqrt{2})(5\sqrt{2})}{2}$

$A = 25$

OR Special Triangles



OR Pythag/Root Derivatives

$y = \sqrt{100 - x^2}$

$y = \sqrt{100 - (5\sqrt{2})^2}$

$y = 5\sqrt{2}$

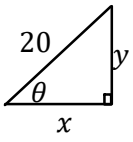
$A = \frac{xy}{2}$

$A = \frac{x\sqrt{100 - x^2}}{2}$

$A' = \frac{\dots}{\dots}$

$x = \frac{10}{\sqrt{2}} = 5\sqrt{2}$

Max Perimeter



$P = 20cos\theta + 20sin\theta + 20$

$P' = -20sin\theta + 20cos\theta$

$20sin\theta = 20cos\theta$

$tan\theta = 1$

$\theta = \frac{\pi}{4}$

$cos\theta = \frac{x}{20}$

$x = 20cos\theta$

$x = 20\cos\frac{\pi}{4}$

$x = 10\sqrt{2}$

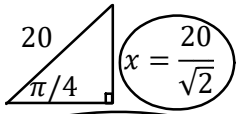
$sin\theta = \frac{y}{20}$

$y = 20sin\theta$

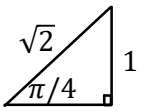
$y = 20\sin\frac{\pi}{4}$

$y = 10\sqrt{2}$

OR Special Triangles



$x = \frac{20}{\sqrt{2}}$



$y = \frac{20}{\sqrt{2}} = 10\sqrt{2}$

OR Pythag/Root Derivatives

$x^2 + y^2 = 400$

$y = \sqrt{400 - x^2}$

$y = 10\sqrt{2}$

$P = x + y + 20$

$P = x + \sqrt{400 - x^2} + 20$

$P' = 1 - \frac{x}{\sqrt{400 - x^2}}$

$0 = \dots$

$1 = \frac{x}{\sqrt{400 - x^2}}$

$x = \sqrt{400 - x^2}$

$x^2 = 400 - x^2$

$2x^2 = 400$

$x = 10\sqrt{2}$

$P = 10\sqrt{2} + 10\sqrt{2} + 20$

$P = 20 + \sqrt{2}$