C12 - 3.14 - Revenue/Profit Max

You sell candy 10 candies for 6 dollars each. If you increase the price by 1 dollar, 1 less friend buy the candy. What is the price and quantity that will max revenue?

p	q
6	10
7	9

$let\ x = quantity$
$let\ p = demand$
$let\ R = Revenue$

Demand Function is Price

$m = \frac{y_2 - x_1}{x_2 - x_1}$
$m = \frac{10 - 9}{6 - 7}$
$m = -1$

$y = mx + b$
$p = -1x + b$
$6 = -1(10) + b$
$b = 16$
$p = -x + 16$

$R = pq$
$R = (-x + 16)x$
$R = -x^2 + 16x$
$V: (8,64)$ Calc/
(x, R) Complete Square

$\boxed{\text{OR}}$ $R = -x^2 + 16x$
$R' = -2x + 16$
$0 = -2x + 16$
$x = 8\ units$

$\boxed{\text{OR}}$ $\boxed{\text{PC11}}$ $Let\ p = price, Let\ q = quantity, Let\ R = revenue$ $\boxed{Let\ x = \#\ of\ price\ increases}$

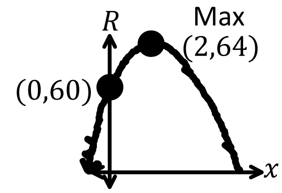
$p = 6 + 1t$ $\quad q = 10 - 1x$

$R = p \times q$
$R = (6 + 1x)(10 - 1x)$
$R = 60 - 6x + 10x - x^2$
$R = -x^2 + 4x + 60$

$R' = -2x + 4$
$0 = -2x + 4$
$x = 2$

$p = 6 + 1(2)$
$p = 6 + 2$
$p = \$8$
$q = 10 - 1(2)$
$q = 10 - 2$
$q = 8\ units$

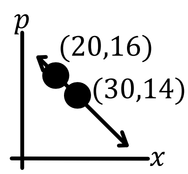
$R = pq$
$R = 8 \times 8$
$R = \$64$



16\$ units, sell 20 units.
14\$ units, sell 30 units.

$\boxed{R = px}$
$\boxed{P = R - C}$
$C = 4x + 140$

$p = price$
$x = quantity$
$R = Revenue$
$C = Cost$
$P = Profit$



x	p	R	C	P
0	20	0	140	− 140
10	18	180	180	0
20	16	320	240	120
30	14	420	260	160
40	12	480	300	180
50	10	500	340	160
60	8	480	380	100
70	6	420	420	0

Max Profit

Find x to Max P & Break Even P=0.

$m = \frac{y_2 - y_1}{x_2 - x_1}$
$m = \frac{16 - 15}{20 - 30}$
$m = -\frac{1}{5}$
Down \$1
Sell 5 more

$y - y_1 = m(x - x_1)$
$p - 16 = -\frac{1}{5}(x - 20)$
$p = -\frac{1}{5}x + 20$
Demand Function

$p = -\frac{1}{5}x + 20$
$p = -\frac{1}{5}(40) + 20$
$p = \$\,12$

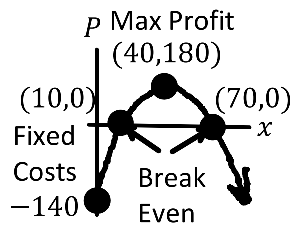
$R = px$
$R = \left(-\frac{1}{5x} + 20\right)x$
$R = -\frac{1}{5}x^2 + 20x$
Revenue Function

$P = R - C$
$P = -\frac{1}{5}x^2 + 20x - (4x + 140)$
$P = -\frac{1}{5}x^2 + 16x - 140$
Profit Function

$\left(\frac{-b}{2a}, P\right)$
$\left(\frac{-16}{2\left(\frac{-1}{5}\right)}, P\right)$
$(40,180)$

Max Profit $\boxed{\frac{dP}{dx} = 0}$
$\frac{dP}{dx} = -\frac{2}{5}x + 16$
$0 = -\frac{2}{5}x + 16$
$x = 40\ units$

$P = -\frac{1}{5}(40)^2 + 16(40) - 140$
$P = \$\,180$



$\boxed{\text{Elasticity*}}$

$\boxed{\text{OR}}$ Max Profit
$\boxed{MR = MC}$
$\frac{dR}{dx} = \frac{dC}{dx}$
$-\frac{2}{5}x + 20 = 4$
$x = 40$

$R = -\frac{1}{5}x^2 + 20x$
$\frac{dR}{dx} = -\frac{2}{5}x + 20$
$C = 4x + 140$
$\frac{dC}{dx} = 4$