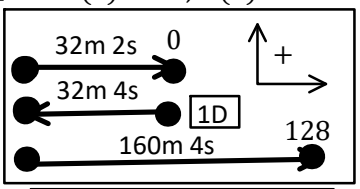
C12 - 3.11 - Part* Mot

$t\,;s$ $\quad v(t)\,;\frac{m}{s}$ $\quad a^*(t)\,;\frac{m}{s^2}$ $\quad s'(t)=v(t)$ $\quad \int v(t)\,dt=s(t)$

$s(t)\,;m$ $\quad v'(t)=a(t)$ $\quad \int a(t)\,dt=v(t)$

A Particle moving East along a straight line, by the function $s(t)\ ;\ 0\le t\le 10.\ v(0)=36,\ s(0)=-32.$

$s(t)=t^3-12t^2+36t-32$



$y=position\ (s)$
$y'=velocity\ (v)$
$y''=acceleration\ (a)$

? s, t = 4.

$s(t)=t^3-12t^2+36t-32$

$s(4)=(4)^3-12(4)^2+36(4)-32$

$s(4)=-16$

? t, Origin. $s(t)=0$

$s(t)=t^3-12t^2+36t-32$

$0=t^3-12t^2+36t-32$

$0=(t-2)^2(t-8)$

$t=2,8$

PC 12 Cubic Factoring

Find $v(t)$.

$s(t)=t^3-12t^2+36t-32$

$v(t)=s'(t)=3t^2-12t+36$

Check End Points.

? v, t = 4.

$v(t)=3t^2-24t+36$

$v(4)=3(4)^2-24(4)+36$

$v(4)=-12$

? t, Rest? $v(t)=0.$

$v(t)=3t^2-24t+36$

$0=3t^2-24t+36$

$0=t^2-8t+12$

$0=(t-6)(t-2)$

$t=2,6$

Change Direction. $v(t)=0\ \&\ v(t)+\to-$ $\quad v(t)-\to+$

? t, Positive Velocity. $v(t)>0\ ?$ $\quad (0,2)\ U\ (6,10)$

? t, Negative Velocity. $v(t)<0\ ?$ $\quad (2,6)$

? t, Farthest from the origin. $v(t)=0$

$s(t)=t^3-12t^2+36t-32$

$s(6)=(6)^3-12(6)^2+36(6)-32$

$s(6)=-32$ Check End Points.

$s(2)=0^*$ $\quad s(4)=-16^*$

$s(10)=(10)^3-12(10)^2+36(10)-32$

$s(10)=128$

$v(t)=3t^2-24t+36$

$v(1)=3(1)^2-24(1)+36$

$v(1)=+15$

$v(3)=3(3)^2-24(3)+36$

$v(3)=-9$

$v(7)=3(7)^2-24(7)+36$

$v(7)=+15$

Find $a(t)$.

$v(t)=3t^2-24t+36$

$a(t)=v'(t)=6t-24$

? a, t = 5.

$a(t)=6t-24$

$a(5)=6(5)-24$

$a(5)=6$

? t, a = 6.

$a(t)=6t-24$

$6=6t-24$

$t=5$

? t, Greatest v. $v'(t)=6t-24$

? t, Least v. $v'(t)=0$

$0=6t-24$

$t=4$

$v(4)=-12^*$ Min

$v(t)=3t^2-24t+36$

$v(0)=3(0)^2-24(0)+36$

$v(0)=36$

$v(10)=3(10)^2-24(10)+36$

$v(10)=96$ Max

? t, Greatest a. $a(t)=6t-24$

? t, Least a. $a'(t)=6$

$a'(t)=0$ Jerk

Snap Crackle Pop

$a(t)=6t-24$

$a(0)=6t-24$

$a(0)=-24$ Min

$a(10)=6(10)-24$

$a(10)=36$ Max

? t, Speeding Up? $(2,4)\ U\ (6,10)$

$a\ \&\ v\ Same\ Sign.$

? t, Slowing Down? $(0,2)\ U\ (4,6)$

$a\ \&\ v\ Different\ Signs.$

$v(1)=+15^*$ $\quad v(3)=-9^*$

$a(t)=6t-24$

$a(1)=6(1)-24$

$a(1)=-18$

$a(3)=6(3)-24$

$a(3)=-8$

$v(t)=3t^2-24t+36$

$v(5)=3(5)^2-24(5)+36$

$v(5)=-9$

$a(5)=+6$

$v(7)=15^*$

$a(7)=6(7)-24$

$a(7)=+18$

? $\vec{d}$, t = 4. $s(0)=-32$

$-32\to-16=+16$

$\vec{d}=16$ $\quad s(4)=-16^*$

? d, t = 4. Need $v(t)=0, s(t)^*$.

$s(0)=-32$ $\quad s(4)=-16^*$

$s(t)=t^3-12t^2+36t-32$

$s(2)=(2)^3-12(2)^2+36(2)-32$

$s(2)=0$

$-32\to0=32$ $\quad d=32+16$

$0\to-16=16$ $\quad d=48$

Find $v(t)$. $\quad a(t)=6t-24$ $\quad$ Find $s(t)$.

$v(t)=\int(6t-24)\,dt$ $\qquad s(t)=\int(3t^2-24t+36)\,dt$

$v(t)=3t^2-24t+C$ $\qquad s(t)=t^3-12t^2+36t+C$

$36=3(0)^2-24(0)+C$ $\qquad -32=(0)^3-12(0)^2+36(0)+C$

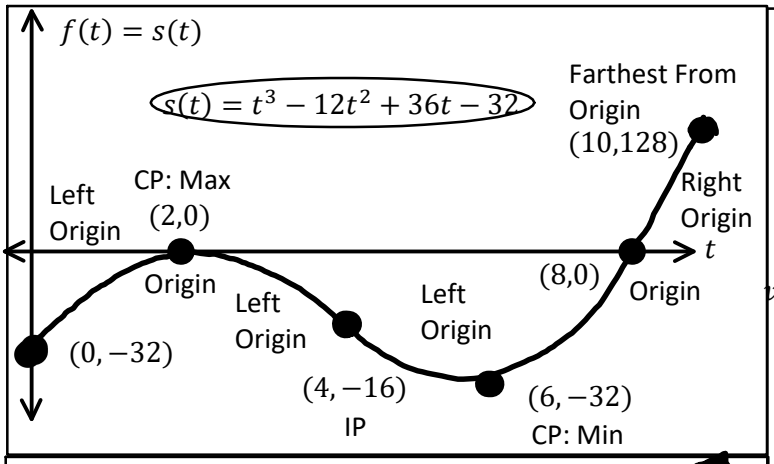
$C=36$ $\qquad C=-32$

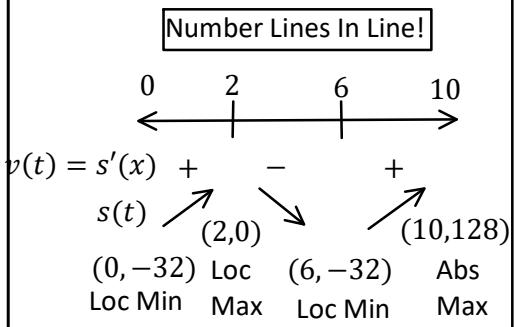
$v(t)=3t^2-24t+36$ $\qquad s(t)=t^3-12t^2+36t-32$

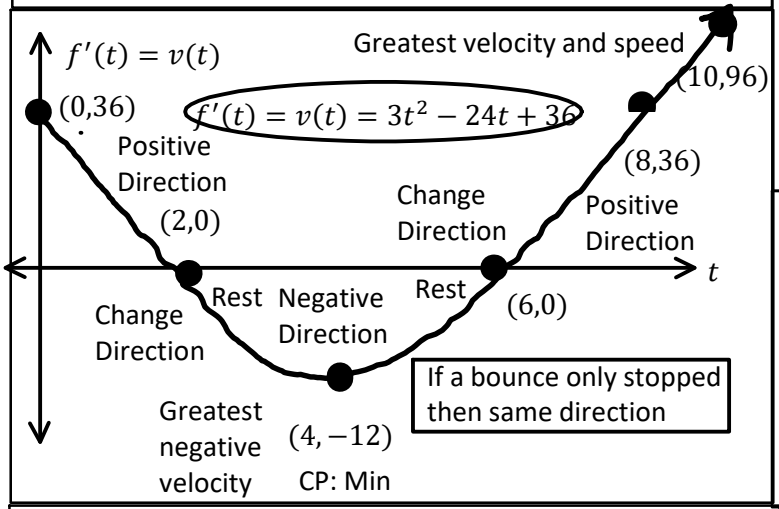
$v(t)=v(0)+\int_0^t a(t)\,dt$ $\qquad s(t)=s(0)+\int_0^t v(t)\,dt$

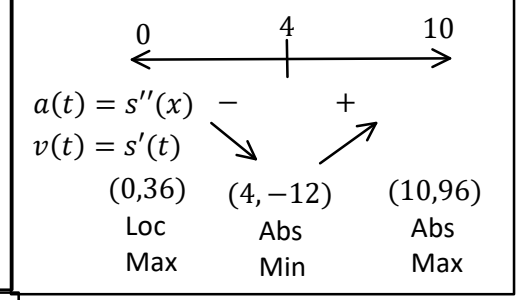
Def. Int. Net Change.

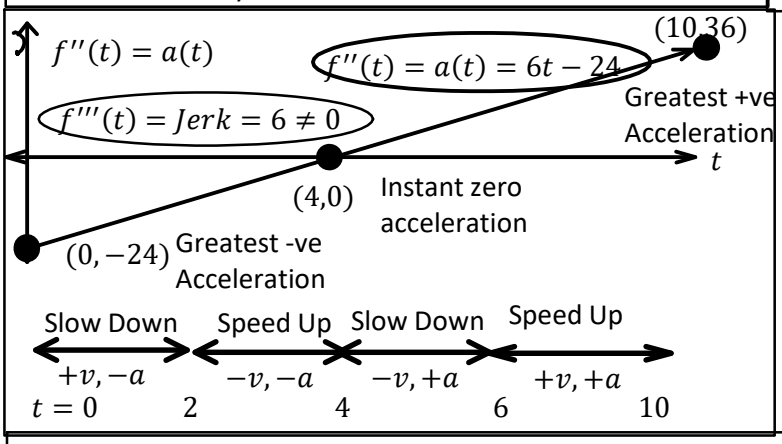
C12 - 3.11 - Particle Motion/Int Notes











? $\vec{d}$, t = 4.

$$\int_0^4 v(t)\,dt$$

$$\int_0^4 (3t^2 - 24t + 36)\,dt$$

$$t^3 - 12t^2 + 36t - 32\Big|_0^4$$

$$(4)^3 - 12(4)^2 + 36(4) - 32 - ((0)^3 - 12(0)^2 + 36(0) - 32)$$

$$-16 - (-32) = \boxed{16}$$

? d, t = 4. Need $v(t) = 0, s(t)^*$.

$$\int_0^4 |v(t)|\,dt = \int_0^2 v(t)dt + -\int_2^4 v(t)dt$$

$$\int_0^2 (3t^2 - 24t + 36)\,dt - \int_2^4 (3t^2 - 24t + 36)\,dt$$

$$t^3 - 12t^2 + 36t - 32\Big|_0^2 - t^3 - 12t^2 + 36t - 32\Big|_2^4$$

$$(2)^3 - 12(2)^2 + 36(2) - 32 - (-32^*) -$$

$$((4)^3 - 12(4)^2 + 36(4) - 32 - ((0)^3 - 12(0)^2 + 36(0) - 32)$$

$$0 - (-32^*) - (-16^* - (0^*) = \boxed{48}$$