

C12 - 1.7 - Limit Slope Notes

Find average slope over domain, equation of the secant line, and sketch only [1,2].

$$f(x) = x^2 ; [1,2], [1,1.1], [1,1.01].$$

$$f(2) = (2)^2 = 4 \quad (2,4)$$

$$f(1) = (1)^2 = 1 \quad (1,1)$$

$$f(1.1) = (1.1)^2 = 1.21 \quad (1.1, 1.21)$$

$$f(1.01) = (1.01)^2 = 1.0201 \quad (1.01, 1.0201)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{(4) - (1)}{(2) - (1)}$$

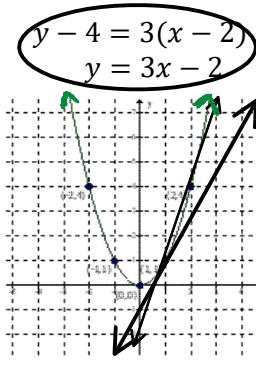
$$(m = 3)$$

$$m = \frac{1.21 - 1}{1.1 - 1}$$

$$(m = 2.1)$$

$$m = \frac{1.0201 - 1}{1.01 - 1}$$

$$(m = 2.01)$$



Find the instantaneous slope, equation of the tangent line, and sketch.

$$f(x) = x^2 ; x = 1$$

$$f(1) = 1 \quad (1,1)$$

$$\begin{aligned} y - 1 &= 2(x - 1) \\ y &= 2x - 1 \end{aligned}$$

Method 1*

$$m = f'(a) = \lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a}$$

$$\begin{aligned} m &= f'(1) = \lim_{x \rightarrow 1} \frac{x^2 - f(1)}{x - 1} \\ &\quad \text{Lim } x \rightarrow 1 \quad \frac{x^2 - 1}{x - 1} \\ &\quad \text{Lim } x \rightarrow 1 \quad \frac{(x+1)(x-1)}{(x-1)} \\ &\quad \text{Lim } x \rightarrow 1 \quad x + 1 \\ &\quad \text{Lim } x \rightarrow 1 \quad 1 + 1 \\ (m &= f'(1) = 2) \end{aligned}$$

Method 2*

$$m = f'(a) = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$$

$$\begin{aligned} m &= f'(1) = \lim_{h \rightarrow 0} \frac{f(1+h) - f(1)}{h} \\ &\quad \text{Lim } h \rightarrow 0 \quad \frac{h}{(1+h)^2 - (1)^2} \\ &\quad \text{Lim } h \rightarrow 0 \quad \frac{h}{1 + 2h + h^2 - 1} \\ &\quad \text{Lim } h \rightarrow 0 \quad \frac{h}{2h + h^2} \\ &\quad \text{Lim } h \rightarrow 0 \quad \frac{h}{h(2+h)} \\ &\quad \text{Lim } h \rightarrow 0 \quad \frac{h}{2+h} \\ &\quad \text{Lim } h \rightarrow 0 \quad 2+0 \\ (m &= f'(1) = 2) \end{aligned}$$

Find average slope over domain, equation of the secant line, and sketch only [1,2].

$$f(x) = x^3 ; [1,2], [1,1.1], [1,1.01].$$

Find the instantaneous slope, equation of the tangent line, and sketch.

$$f(x) = x^2 - 2x ; x = 3$$

$$f(x) = x^3 ; x = -1$$

$$f(x) = x^4 ; x = 1$$

$$f(x) = \frac{1}{x+2} ; x = 2$$

$$f(x) = \sqrt{x+1} ; x = 0$$

$$f(x) = \frac{1}{\sqrt{x+3}} ; x = 1$$