

C12 - 1.3 - Difference/Sum of Cubes/Squares Notes

Examples

Difference of Squares

$$a^2 - b^2 = (a - b)(a + b)$$

$$x^2 - 4 = (x - 2)(x + 2)$$

Difference of Cubes

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

$$x^3 - 27 = (x - 3)(x^2 + 3x + 3^2)$$

Sum of Cubes

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

$$x^3 + 1 = (x + 1)(x^2 - 1x + 1^2)$$

SOAP - Same sign, Opposite sign, Always Plus

Check by Long/Synthetic Division/FOIL

$$4x^2 - 49$$

$$(2x)^2 - 7^2$$

$$(2x + 7)(2x - 7)$$

Change of base

(What is being Squared)

$$(x + 1)^2 - 16$$

$$((x + 1) + 4)((x + 1) - 4)$$

$$(x + 5)(x - 3)$$

OR

$$(x + 1)^2 - 16$$

$$x^2 + 2x + 1 - 16$$

$$x^2 + 2x - 15$$

$$(x + 5)(x - 3)$$

$$(x + 2)^4 - 16$$

$$((x + 2)^2)^2 - 4^2$$

$$((x + 2)^2 + 4)((x + 2)^2 - 4)$$

$$(x^2 + 4x + 4 + 4)(x^2 + 4x + 4 - 4)$$

$$(x^2 + 4x + 8)(x^2 + 4x)$$

$$x(x^2 + 4x + 8)(x + 4)$$

$$x^3 - 8 = (x - 2)(x^2 + 2x + 2^2)$$

$$\pm 2 \quad f(2) = 0$$

$(x - 2)$ Is a factor

$$\begin{array}{r|rrrr} 2 & 1 & 0 & 0 & -8 \\ & & 2 & 4 & 8 \\ \hline & 1 & 2 & 4 & 0 \end{array}$$

List Factors

Inspection

Synthetic

Check by FOIL

$$(x - 2)(x^2 + 2x + 4)$$

$$x^3 + 2x^2 + 4x - 2x^2 - 4x - 8$$

$$x^3 - 8$$

$$8x^3 - 1$$

$$(2x)^3 - (1)^3$$

$$(2x - 1)((2x)^2 + (2x)(1) + (1)^2)$$

$$(2x - 1)(4x^2 + 2x + 1)$$

Change of Base

(What is being Cubed)

Formula/Simplify

$$27x^3 + 8$$

$$(3x)^3 + (2)^3$$

$$(3x + 2)((3x)^2 + (3x)(2) + (2)^2)$$

$$(3x + 2)(9x^2 + 6x + 4)$$

$$(First\ Thing)^3 \pm (Second\ Thing)^3 = (a)^3 - (b)^3$$

$$(First\ Thing \pm Second\ Thing)(First\ Thing\ Squared \pm First\ Thing \times Second\ Thing \pm Second\ Thing\ Squared)$$

$$(x + 1)^3 - 27$$

$$(x + 1)^3 - 3^3$$

$$((x + 1) - (3))((x + 1)^2 + 3(x + 1) + (3)^2)$$

OR

$$(x - 2)(x^2 + 2x + 1 + 3x + 3 + 9)$$

$$(x - 2)(x^2 + 5x + 13)$$

$$27 - (x + 1)^3$$

$$(3 - (x + 1))(3^2 - 3(x + 1) + (x + 1)^2)$$

FOIL/Binomial Expansion

$$(x + 1)^3 =$$

$$(x + 1)(x + 1)(x + 1)$$

$$(x^2 + 2x + 1)(x + 1)$$

$$x^3 + 3x^2 + 3x + 1 \dots$$