C12 - 1.2 - Limits Inf/VA/HA Notes

$\frac{\#^*}{0^+} \approx +\infty \qquad \frac{\#^*}{0^-} \approx -\infty$

$\lim_{x \to 2} \frac{1}{x-2} = \text{DNE}$

$\lim_{x \to 2^-} \frac{1}{x-2} = \frac{1}{0^-} = -\infty$

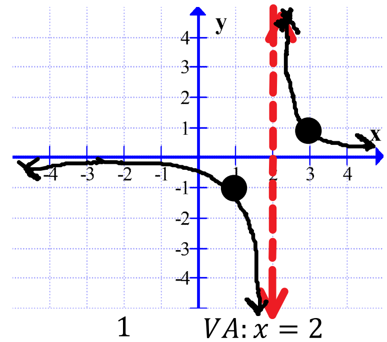
$VA: x = 2$

$\lim_{x \to 2^+} \frac{1}{x-2} = \frac{1}{0^+} = +\infty$

$2^- \sim 1.9999$	Number Line	$2^+ \sim 2.0001$

x	y
1.9	-10
1.999	-1000
2	DNE
2.001	1000
2.1	10

Fractions/Calc!

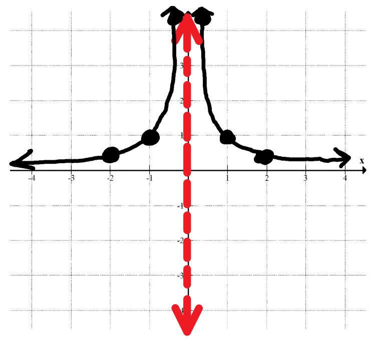


$y = \frac{1}{x-2} \qquad VA: x = 2$

$\lim_{x \to 0^+} \frac{1}{x^2}$ $\lim_{x \to 0^+} \frac{1}{(0^+)^2}$ $\frac{1}{0^+} = \infty$

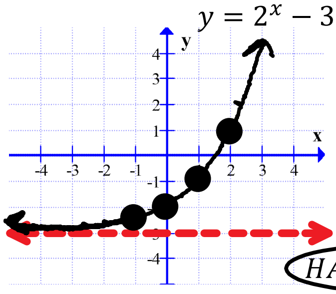
$\lim_{x \to 0^-} \frac{1}{x^2}$ $\lim_{x \to 0^-} \frac{1}{(0^-)^2}$ $\frac{1}{0^+} = \infty$

$\lim_{x \to 0} f(x) = \infty$



$y = \frac{1}{x^2}$

$VA: x = 0$



$y = 2^x - 3$

$HA: y = -3$

$\lim_{x \to \infty} 2^x - 3 = 2^\infty - 3 = \infty$

$\lim_{x \to -\infty} 2^x - 3 = 2^{-\infty} - 3 = \frac{1}{2^\infty} - 3 = 0 - 3 = -3$

$\lim_{x \to \infty} \frac{2^x}{x} = \frac{2^\infty}{\infty} = \infty$

$2^\infty \gg \infty$

$\lim_{x \to -\infty} \frac{2^x}{x} = \frac{2^{-\infty}}{\infty} = \frac{\frac{1}{2^\infty}}{\infty} = \frac{\frac{1}{\infty}}{\infty} = 0$

$\lim_{x \to \infty} e^x = e^\infty = \infty$

$e = 2.71$

$\lim_{x \to -\infty} e^x = e^{-\infty} = \frac{1}{e^\infty} = \frac{1}{\infty} = 0$

$\lim_{x \to -\infty} \frac{2^x}{x} = \frac{2^{-\infty}}{\infty} = \frac{\frac{1}{2^\infty}}{\infty} = \frac{\frac{1}{\infty}}{\infty} = 0$

$\lim_{x \to -\infty} 2^{-x} = 2^{-(-\infty)} = 2^\infty = \infty$

$\lim_{x \to \infty} \frac{2^x}{x} = \frac{2^\infty}{\infty} = \infty$

$2^\infty \gg \infty$

C12 - 1.2 - Limits HA Poly Notes

Lim x-> -inf

$$\lim_{x\to\infty} \frac{3x+1}{2-x} \qquad \frac{3x+1}{2-x}$$

$$\frac{\frac{3x}{x}+\frac{1}{x}}{\frac{2}{x}-\frac{x}{x}}$$

$$\lim_{x\to\infty} \frac{3+\frac{1}{x}}{\frac{2}{x}-1} \qquad \frac{3+\frac{1}{x}}{\frac{2}{x}-1}$$

$$\lim_{x\to\infty} \frac{3+\frac{1}{\infty}}{\frac{2}{\infty}-1}$$

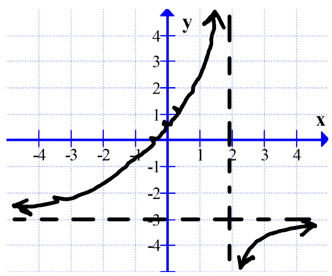
$$\frac{3+0}{0-1}$$

$$\frac{3}{-1}$$

$$-3$$

$$HA: y=-3$$

We're not really supposed to substitute infinity (In your head)



$$\lim_{x\to\infty} \frac{3x^2+4}{1x^2-1}$$

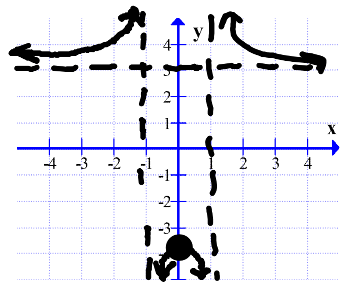
$$\lim_{x\to\infty} \frac{3x^2+4}{1x^2-1} \frac{\div x^2}{\div x^2}$$

$$\lim_{x\to\infty} \frac{3+\frac{4}{x^2}}{1-\frac{1}{x^2}}$$

$$\frac{3+\frac{4}{\infty^2}}{1-\frac{1}{\infty^2}}$$

$$\frac{3+0}{1-0}=3$$

$$HA: y=3$$



$$y=\frac{3x^2+4}{1x^2-1}$$

$$\frac{\#^*}{+\infty}\approx 0 \qquad \frac{\#^*}{-\infty}\approx 0$$

$$\lim_{x\to\infty} \frac{x^2+5}{3x^3+2x}$$

$$\lim_{x\to\infty} \frac{x^2+5}{3x^3+2x} \frac{\div x^3}{\div x^3}$$

$$\lim_{x\to\infty} \frac{\frac{1}{x}+\frac{5}{x^3}}{3+\frac{2}{x^2}}$$

$$\frac{\frac{1}{-\infty}+\frac{5}{(-\infty)^3}}{3+\frac{2}{(-\infty)^2}}$$

$$\frac{0+0}{3+0}=0$$

$$HA: y=0$$

$$\lim_{x\to\infty} \frac{x^4+3x}{2x+1}$$

$$\lim_{x\to\infty} \frac{x^4+3x}{2x+1} \frac{\div x}{\div x}$$

$$\lim_{x\to\infty} \frac{x^3+3}{2+\frac{1}{x}}$$

$$\frac{\infty+3}{2+\frac{1}{\infty}}$$

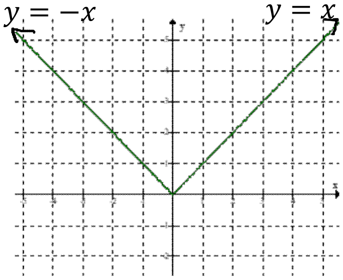
$$\frac{\infty}{2+0}=\infty$$

$$HA: None$$

C12 - 1.2 - Limits HA Root Notes

$\lim_{x\to -\infty} |x| = -x$ | $\lim_{x\to +\infty} |x| = x$

$y = -x$ $y = x$



$\lim_{x\to\infty} \frac{|x|+2}{|x|}$

$\lim_{x\to\infty} \frac{x+2}{x}$ $\begin{array}{c}\div x\\ \div x\end{array}$

$\frac{1+\frac{2}{x}}{1}$

$\lim_{x\to\infty} \frac{1+\frac{2}{\infty}}{1}$

$\frac{1+0}{1} = \boxed{1}$

$\lim_{x\to -\infty} \frac{|x|+2}{|x|}$

$\lim_{x\to -\infty} \frac{-x+2}{-x}$ $\begin{array}{c}\div x\\ \div x\end{array}$

$\frac{-1+\frac{2}{x}}{-1}$

$\lim_{x\to -\infty} \frac{-1+\frac{2}{\infty}}{-1}$

$\frac{-1+0}{-1} = \boxed{1}$

$\lim_{x\to\infty} \frac{5x}{\sqrt{x^2+5}}$

$\lim_{x\to\infty} \frac{5x}{\sqrt{x^2+5}}$ $\begin{array}{c}\div \sqrt{x^2}\\ \div \sqrt{x^2}\end{array}$

$\lim_{x\to\infty} \frac{\frac{5x}{|x|}}{\sqrt{\frac{x^2+5}{x^2}}}$

$\sqrt{x^2} = |x| = \pm x$

$\lim_{x\to\infty} \frac{\frac{5x}{x}}{\sqrt{\frac{x^2+5}{x^2}}}$

Separate Fractions

$\frac{x^2+5}{x^2}$

$\frac{x^2}{x^2} + \frac{5}{x^2}$

$1 + \frac{5}{x^2}$

$\lim_{x\to\infty} \frac{5}{\sqrt{1+\frac{5}{x^2}}}$

$\frac{5}{\sqrt{1+0}} = \boxed{5}$

$\lim_{x\to -\infty} \frac{5x}{\sqrt{x^2+5}}$

$\lim_{x\to -\infty} \frac{5x}{\sqrt{x^2+5}}$ $\begin{array}{c}\div \sqrt{x^2}\\ \div \sqrt{x^2}\end{array}$

$\lim_{x\to -\infty} \frac{\frac{5x}{|x|}}{\sqrt{\frac{x^2+5}{x^2}}}$

$\lim_{x\to\infty} \frac{\frac{5x}{-x}}{\sqrt{\frac{x^2+5}{x^2}}}$

$\lim_{x\to\infty} \frac{-5}{\sqrt{1+\frac{5}{x^2}}}$

$\frac{-5}{\sqrt{1+0}} = \boxed{-5}$

$\lim_{x\to\infty} \frac{\sqrt{x^2+3x}}{x}$

$\frac{\sqrt{x^2+3x}}{x}$

$\frac{\sqrt{x^2+3x}}{\sqrt{x^2}}$

$\sqrt{\frac{x^2+3x}{x^2}}$

$\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$

$\frac{x^2+3x}{x^2}$

$\frac{a+b}{c} = \frac{a}{c} + \frac{b}{c}$

$\frac{x^2}{x^2} + \frac{3x}{x^2}$

$1 + \frac{3}{x}$

$\lim_{x\to\infty} \sqrt{1+\frac{3}{x}}$

$\lim_{x\to\infty} \sqrt{1+\frac{3}{\infty}}$

$\sqrt{1+0}$

$\sqrt{1}$

$\boxed{1}$

$\lim_{x\to\infty} \sqrt{\frac{3x^2+4}{1x^2-1}}$

$\frac{3x^2+4}{1x^2-1}$

$\frac{\frac{3x^2}{x^2}+\frac{4}{x^2}}{\frac{x^2}{x^2}-\frac{1}{x^2}}$

$\frac{3+\frac{4}{x^2}}{1-\frac{1}{x^2}}$

$\lim_{x\to\infty} \sqrt{\frac{3+\frac{4}{x^2}}{1-\frac{1}{x^2}}}$

$\lim_{x\to\infty} \sqrt{\frac{3+\frac{4}{\infty^2}}{1-\frac{1}{\infty^2}}}$

$\sqrt{\frac{3+0}{1-0}}$

$\boxed{\sqrt{3}}$

$HA: y = \sqrt{3}$