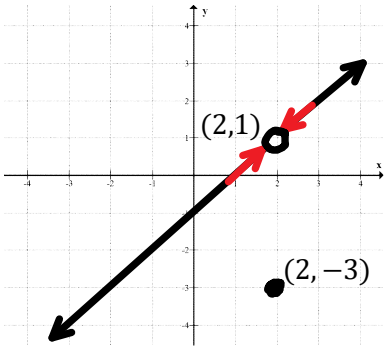
C12 - 1.1 - Limits Intro Notes

Limit : What y is approaching*



$$f(x)=\begin{cases} x-1 & ;\ x\neq 2 \\ -3 & ;\ x=2 \end{cases}$$

Point $(2,-3)$

$$f(x)=\frac{(x-1)(x-2)}{(x-2)}$$

Hole: $x-2=0$, $x=2$

$(2,1)$

$f(2)=1$

Remove Discontinuity $f(x)=1, x=2$

Not a function*

Domain : $(-\infty,2)U(2,\infty)$

x	y
1.9	.9
1.999	.999
2	−3
2.001	1.001
2.1	1.1

$\lim_{x\to 2} f(x)=?$

$\lim_{x\to 2} f(x)=1$

What is y approaching as x approaches 2?

The Limit of $f(x)$, as x approaches 2, equals 1.

y approaches 1 as x approaches 2.

$\lim_{x\to 2^-} f(x)=1$ *and* $\lim_{x\to 2^+} f(x)=1$

Left Hand Limit = Right Hand Limit

Run your pencil along the graph (Line*Curve) as x approaches 2. *APPROACHES*!

One Sided Limits

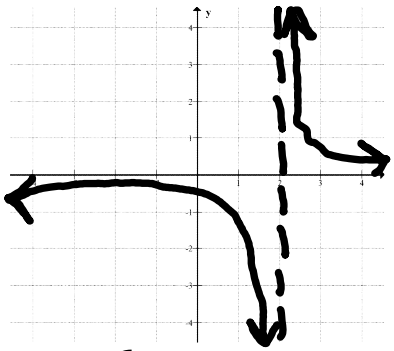
$\lim_{x\to c^+} f(x)=L$ *The Limit of $f(x)$, as x approaches c, from the positive side (right), equals L.*

$\lim_{x\to c^-} f(x)=L$ *The Limit of $f(x)$, as x approaches c, from the negative side (left), equals L*

Limit Exists if and only if: $\lim_{x\to c} f(x)=L$ *or* $\lim_{x\to c} f(x)=DNE$ *Limit Does Not Exist*

Left hand Limit = Right Hand Limit

$f(x)=\frac{1}{x-2}$ $\lim_{x\to 2} f(x)=DNE$ *The Limit of $f(x)$, as x approaches 2, Does Not Exist*



$\frac{1}{2.001-2}$

$\frac{1}{.001}$

$\frac{1}{\frac{1}{1000}}\times\frac{1000}{1}$

$\frac{1000}{1000}$

$\frac{\#^*}{0^+}\approx+\infty$ $\frac{\#^*}{0^-}\approx-\infty$

$\frac{1}{2^--2}=\frac{1}{0^-}=-\infty$

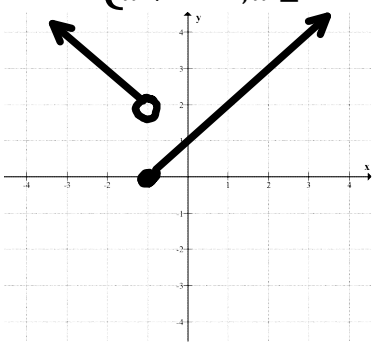
$\frac{1}{2^+-2}=\frac{1}{0^+}=\infty$

x	y
1.9	-10
1.999	-1000
2	DNE
2.001	1000
2.1	10

$\lim_{x\to 2^-} f(x)=-\infty$ *and* $\lim_{x\to 2^+} f(x)=+\infty$

Left Hand Limit ≠ Right Hand Limit

$g(x)=\begin{cases} -x+1 & ;x<1 \\ x+1 & ;x\geq -1 \end{cases}$



$\lim_{x\to -1} g(x)=DNE$

The Limit of $g(x)$, as x approaches -1, DNE.

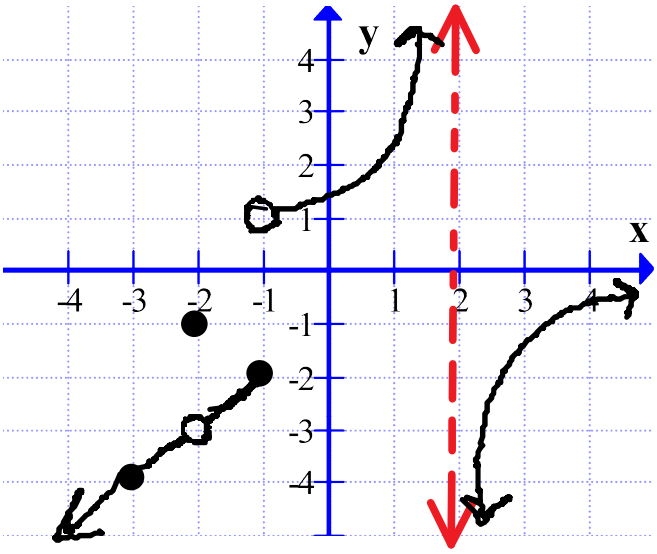
$\lim_{x\to -1^-} g(x)=2$ *and* $\lim_{x\to -1^+} g(x)=0$

Left Hand Limit ≠ Right Hand Limit

x	y
-1.1	2.1
-1.001	2.001
-1	0
-.999	.001
-.9	.1

C12 - 1.1 - Limits VA/Find k Notes

Lim x-inf/-inf



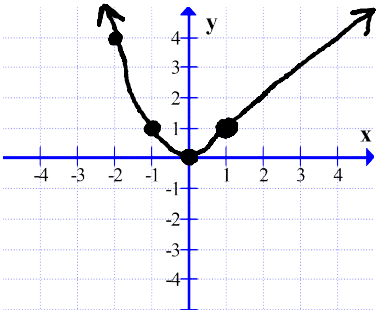
$\lim_{x\to -3^-} f(x) = -4$
$\lim_{x\to -3^+} f(x) = -4$
$\lim_{x\to -3} f(x) = -4$
$f(-3) = -4$
Continuous

$\lim_{x\to -2^-} f(x) = -3$
$\lim_{x\to -2^+} f(x) = -3$
$\lim_{x\to -2} f(x) = -3$
$f(-2) = -1$
Discontinuous (Point)

$\lim_{x\to -1^-} f(x) = -2$
$\lim_{x\to -1^+} f(x) = 1$
$\lim_{x\to -1} f(x) = DNE$
$f(-1) = -2$
Discontinuous (Jump)

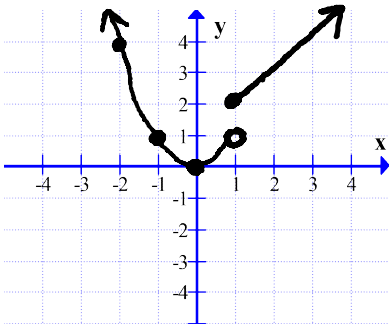
$\lim_{x\to 2^-} f(x) = \infty$
$\lim_{x\to 2^+} f(x) = -\infty$
$\lim_{x\to 2} f(x) = DNE$
$f(2) = DNE$
Discontinuous (Infinite)

$$f(x) = \begin{cases} x^2 ; & x < 1 \\ x ; & x \geq 1 \end{cases}$$



$\lim_{x\to 1} f(x) = 1$

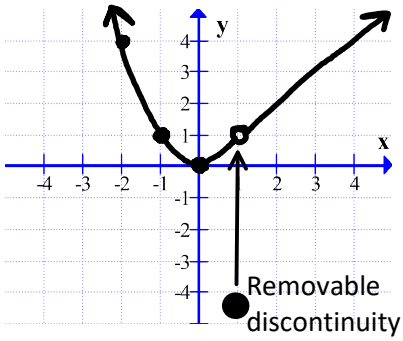
$$f(x) = \begin{cases} x^2 ; & x < 1 \\ x+1 ; & x \geq 1 \end{cases}$$



$\lim_{x\to 1} f(x) = DNE$

$$f(x) = \begin{cases} x^2 ; & x < 1 \\ x ; & x > 1 \\ f(x) = 1 ; & x = 1 \end{cases}$$

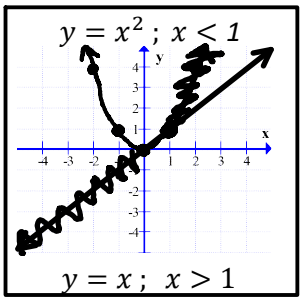
Discontinuous ↓ Continuous



Removable discontinuity
$f(x) = 1, x = 1$

$\lim_{x\to 1^-} x^2 = 1$ $\lim_{x\to 1^+} x = 1$

$\lim_{x\to 1^-} x^2 = 1$ $\lim_{x\to 1^+} x+1 = 2$



Find K so continuous.

$$f(x) = \begin{cases} x + k ; x \geq 1 \\ x^2 ; x < 1 \end{cases}$$

$x + k = x^2$
$(1) + k = (1)^2$
$1 + k = 1$
$k = 0$

Set Equations Equal to Each other & substitute boundary.

$$f(x) = \begin{cases} 2x + k ; x \leq 2 \\ kx^2 - x ; x > 2 \end{cases}$$

$2x + k = kx^2 - x$
$2(2) + k = k(2)^2 - (2)$
$4 + k = 4k - 2$
$k = 2$

C12 - 1.1 - Limits Graph Info Notes

Multiple Answers

$f(-1) = 2$
$f(1) = -1$
$f(2) = DNE$

$\lim_{x \to 2} f(x) = 1$

$\lim_{x \to -\infty} f(x) = 0$

$\lim_{x \to \infty} f(x) = \infty$

$\lim_{x \to 1^-} f(x) = 3$

$\lim_{x \to -2} f(x) = \pm\infty$

Chapter 2

$f'(3) = 1$

$; f''(-3) = -ve$
$; f''(-1) = +ve$